Evaluation of Uncertainties by Matrix Form for Standard Platinum Resistance Thermometer

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Abstract. *Calibration of the standard platinum resistance thermometer (SPRT) in accordance with the ITS-90 (International Temperature Scale 1990) should be provided with the corresponding uncertainties. The SPRTs are calibrated in defining fixed points (DFPs) from the triple point of water (TPW) to freezing point of aluminium.. Evaluation of the uncertainties takes into account corresponding covariances. Matrix form for the propagation of the uncertainties is presented.*

Keywords: Temperature scale, ITS-90, SPRT calibration, Uncertainty, Covariance

1. Introduction

Results of the SPRTs calibration at the DFPs should be provided with the corresponding uncertainties. In many cases calibration certificates do not include the necessary information for precise evaluation of temperature measurements with corresponding uncertainties taking into consideration covariances. (see [3], [4], [5], [6]). For that reason there is necessary to evaluate the SPRT calibration taking into account uncertainties and covariances as well.

In calibration proces covariances between the resistnaces measured at DFPs influence on the resulting calibration uncertainty. Some possible solutions were presented in [7], [8], [9], [10].

This contribution presents evaluation of the uncertainties of the SPRT calibration at the DFPs in accordance with [2]. Procedure for the evaluation of the coefficients of the deviation function and corresponding uncertainties and covariances, evaluation of the resistance ratio and its uncertainty are in [10].

2. Calibration of the SPRT in the range from the triple point of water to the freezing point of Aluminium

Calibration of the SPRT leads to the value of the deviation function ΔW of the SPRT. The inverse function is valid for the evaluation of the measured temperatures T by the SPRT in accordance with the ITS-90 in the corresponding temperature range [1]

$$T - 273{,}15 = D_0 + \sum_{i=1}^{9} D_i \left[\frac{W_r - 2{,}64}{1{,}64} \right]^i \tag{1}$$

where the values of the constants D_i are given in [1] and the values W_r are determined from the deviation function for the corresponding temperature range

$$W - W_r = \Delta W = a[W-1] + b[W-1]^2 + c[W-1]^3 \tag{2}$$

it means from equation

$$W_r = W - \Delta W \tag{3}$$

where

ΔW is the deviation function from the calibration of the SPRT,

W is ratio of the resistances defined as follows

$$W = \frac{R}{R_{\mathrm{TPW}}} \tag{4}$$

where R - the resistance of the SPRT corresponding to the temperature T,
R_{TPW} - the resistance of the SPRT corresponding to the temperature of TPW T_{TPW} from the calibration (arithmetic mean of the SPRT resistances measured in the TPW after the measurements in the DFPs (Al, Zn, Sn) which should be taken from calibration).

In order to determine the measured temperature and its uncertainty by the SPRT measurement it should be determined by calibration of the SPRT the values R_{TPW} and ΔW and their uncertainties and covariances between them. We consider that the SPRT was calibrated in the temperature range from 0 °C to 660 °C in accordance with the ITS-90 at the temperatures of TPW (T_{TPW}), freezing points of tin (T_{Sn}), zinc (T_{Zn}) and aluminium (T_{Al}). We suppose that we know the measured resistances of the SPRT in the corresponding defining fixed points

$$\boldsymbol{R}_{\mathrm{DFP}} = \left(R_{\mathrm{TPWSn}}, R_{\mathrm{TPWZn}}, R_{\mathrm{TPWAl}}, R_{\mathrm{Sn}}, R_{\mathrm{Zn}}, R_{\mathrm{Al}}\right)^{\mathrm{T}} \tag{5}$$

We can see the example of calibration data in the Table 1.

Table 1: Measured values

DFP	$\mathrm{TPW_{Sn}}$	$\mathrm{TPW_{Zn}}$	$\mathrm{TPW_{Al}}$	Sn	Zn	Al
Resistance (R/Ω)	24,8002001	24,8001927	24,8001872	46,9397533	63,7056752	83,7191875

We suppose the knowledge of variance-covariance matrix associated with the resistance measured at the DFP. Review of the correlations between the measured SPRT resistances are in the Table 3.

Table 2: Review of the uncertainties in the DFPs (Ω)

Source	1	2	3	4	5	6	7	8	9	10	11	B*	A*	C*
$u(R_{\mathrm{TPWSn}})$	-	4,00 .10^{-7}	2,00 .10^{-6}	2,00 .10^{-6}	-	-	4,90 .10^{-6}	1,50 .10^{-9}	2,0 .10^{-7}	2,00 .10^{-6}	9,89 .10^{-6}	1,258 .10^{-5}	1,98 .10^{-6}	1,273 .10^{-5}
$u(R_{\mathrm{TPWZn}})$	-	4,00 .10^{-7}	2,00 .10^{-6}	2,00 .10^{-6}	-	-	4,90 .10^{-6}	1,50 .10^{-9}	2,0 .10^{-7}	2,00 .10^{-6}	9,89 .10^{-6}	1,258 .10^{-5}	1,98 .10^{-6}	1,273 .10^{-5}
$u(R_{\mathrm{TPWAl}})$	-	4,00 .10^{-7}	2,00 .10^{-6}	2,00 .10^{-6}	-	-	4,90 .10^{-6}	1,50 .10^{-9}	2,0 .10^{-7}	2,00 .10^{-6}	9,89 .10^{-6}	1,258 .10^{-5}	1,98 .10^{-6}	1,273 .10^{-5}
$u(R_{\mathrm{Sn}})$	1,934 .10^{-5}	1,66 .10^{-6}	1,84 .10^{-6}	4,60 .10^{-6}	3,13 .10^{-6}	7,37 .10^{-6}	-	-	2,1 .10^{-7}	1,01 .10^{-5}	2,39 .10^{-5}	3,378 .10^{-5}	1,842 .10^{-5}	3,848 .10^{-5}
$u(R_{\mathrm{Zn}})$	3,12 .10^{-5}	1,91 .10^{-6}	1,73 .10^{-6}	4,33 .10^{-6}	3,81 .10^{-6}	6,93 .10^{-6}	-	-	2,7 .10^{-7}	9,53 .10^{-6}	3,21 .10^{-5}	4,67 .10^{-5}	1,734 .10^{-5}	4,981 .10^{-5}
$u(R_{\mathrm{Al}})$	3,974 .10^{-5}	1,03 .10^{-6}	1,59 .10^{-6}	7,95 .10^{-6}	5,64 .10^{-6}	7,95 .10^{-6}	-	-	3,6 .10^{-7}	1,03 .10^{-5}	3,97 .10^{-5}	5,854 .10^{-5}	2,385 .10^{-5}	6,321 .10^{-5}

*A- type A evaluation of standard uncertainty; *B-Total type B evaluation of standard uncertainty from the contributions 1-11; *C-combined standard uncertainty

Contributions of the uncertainty budged for the type B evaluation of uncertainty of the SPRT calibration are in the Tables 2, and 3. In thee case of the measurements of the output signal by AC resistance bridge the components of the uncertainty budget are caused by following effects:
– chemical impurities of the substance in the DFP /column 1 in the Table 2 and 3/
– hydrostatic-head effect (corresponding to location of SPRT sensing element in DFP) /column 2/
– self heating effect (error) of the SPRT /column 3/
– immersion effect (error) of the SPRT /column 4/
– effect of gas pressure in the (metal) DFPs /column 5/
– choice of fixed point value from plateau /column 6/
– isotopic variations (for TPW only) /column 7/
– residual gas pressure in the TPW cell /column 8/
– changes of resistances of standard resistor caused by its temperature changes /column 9/
– non linearity of the resistance bridge /column 10/
– uncertainty of calibration of resistance standard /column 11/

Table 3: Correlation coefficients

Source	1	2	3	4	5	6	7	8	9	10	11
$r(R_{TPWSn},R_{TPWZn})$	-	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWSn},R_{TPWAl})$	-	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWZn},R_{TPWAl})$	-	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWSn}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWSn}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWSn}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{Sn},R_{Zn})$	0	0	1	1	0	0	-	-	1	1	1
$r(R_{Sn}, R_{Al})$	0	0	1	1	0	0	-	-	1	1	1
$r(R_{Zn}, R_{Al})$	0	0	1	1	0	0	-	-	1	1	1

Variance-covariance matrix associated to the resistances measured at the fixed points (in Ω^2)

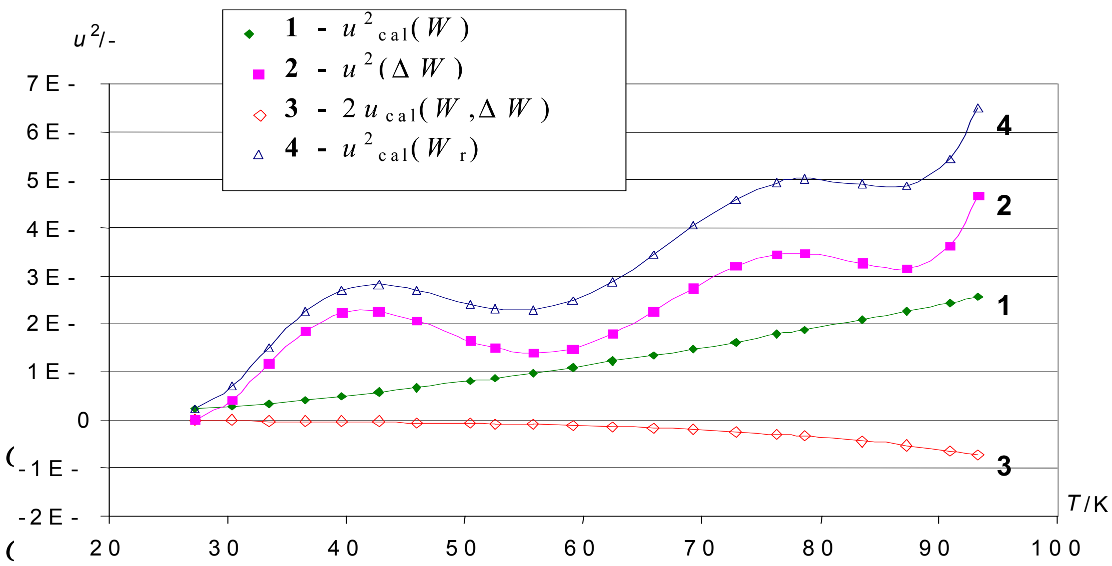
$$\boldsymbol{U}_{\boldsymbol{R}_{\rm DFP}} = \begin{pmatrix} 1{,}38.10^{-10} & 1{,}34.10^{-10} & 1{,}34.10^{-10} & 2{,}52.10^{-10} & 3{,}31.10^{-10} & 4{,}01.10^{-10} \\ 1{,}34.10^{-10} & 1{,}38.10^{-10} & 1{,}34.10^{-10} & 2{,}52.10^{-10} & 3{,}31.10^{-10} & 4{,}01.10^{-10} \\ 1{,}34.10^{-10} & 1{,}34.10^{-10} & 1{,}38.10^{-10} & 2{,}52.10^{-10} & 3{,}31.10^{-10} & 4{,}01.10^{-10} \\ 2{,}52.10^{-10} & 2{,}52.10^{-10} & 2{,}52.10^{-10} & 1{,}48.10^{-9} & 8{,}87.10^{-10} & 1{,}10.10^{-9} \\ 3{,}31.10^{-10} & 3{,}31.10^{-10} & 3{,}31.10^{-10} & 8{,}87.10^{-10} & 2{,}48.10^{-9} & 1{,}41.10^{-9} \\ 4{,}01.10^{-10} & 4{,}01.10^{-10} & 4{,}01.10^{-10} & 1{,}10.10^{-9} & 1{,}41.10^{-9} & 4{,}00.10^{-9} \end{pmatrix}$$

Uncertainty of calibration $u_{\rm cal}(W_{\rm r})$(for the non-random R) is (see. [10])

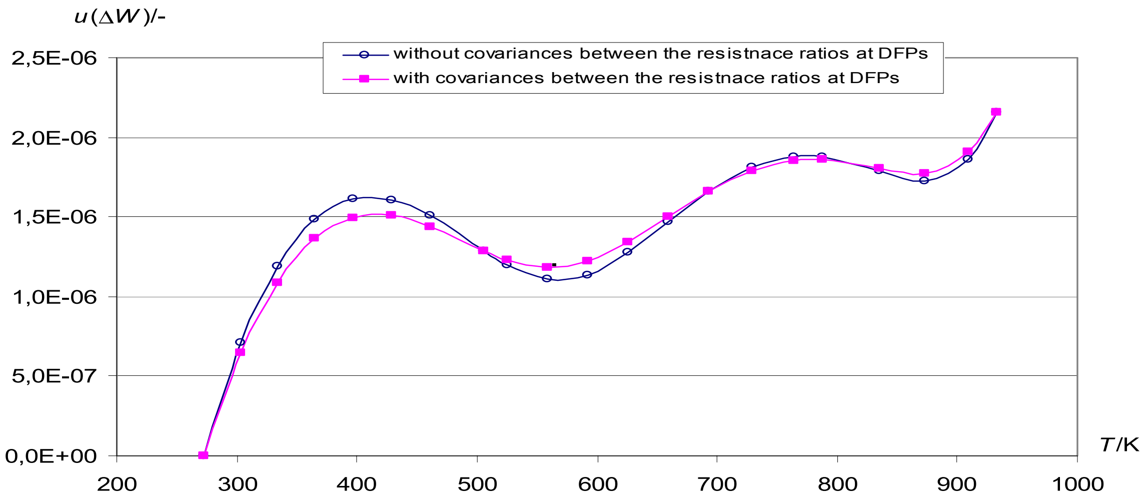
$$\begin{aligned} u^2_{\rm cal}(W_{\rm r}) = {} & u^2_{\rm cal}(W) + A^2_{W\rm Sn} u^2(W_{\rm Sn}) + A^2_{W\rm Zn} u^2(W_{\rm Zn}) + A^2_{W\rm Al} u^2(W_{\rm Al}) + \\ & 2A_{W\rm Sn} A_{W\rm Zn} u(W_{\rm Sn}, W_{\rm Zn}) + 2A_{W\rm Sn} A_{W\rm Al} u(W_{\rm Sn}, W_{\rm Al}) + 2A_{W\rm Zn} A_{W\rm Al} u(W_{\rm Zn}, W_{\rm Al}) + \\ & 2A_{W\rm Sn} u(W, W_{\rm Sn}) + 2A_{W\rm Zn} u(W, W_{\rm Zn}) + 2A_{W\rm Al} u(W, W_{\rm Al}) = \\ = {} & u^2_{\rm cal}(W) + u^2(\Delta W) + 2u_{\rm cal}(W, \Delta W) \end{aligned} \tag{6}$$

Contribution of terms of equation (6) to the calibration uncertainty of the SPRT are in graphs 1 and 2.

Graph 1:Contributions to SPRT uncertainty from terms of eq. (6)



Graph 2: Uncertainty u(ΔW) of the SPRT calibration



3. Conclusion

Procedure for evaluation of the uncertainties of SPRT calibration in temperature range from TPW to freezing point of aluminium is demonstrated in abbreviated form. The uncertainties of measurements of SPRT resistances at the DFPs and the corresponding covariances are taken into account as well. There is also considered the covariance between the deviation of resistance ratio Δ*W* and resistance ratio *W* caused by using the same value of SPRT resistance measured at the TPW in both cases. In the procedure is supposed the knowledge of the SPRT resistances corresponding to the calibration at the DFPs and the uncertainties of these resistances and covariances between them as well. Matrix form is used for simplification of the mathematical expressions.

It could be seen that the influence of covariances between resitance ratios at the DFPs and covariances between Δ*W* and *W* are not reasonable (up to 10 % of the uncertainty) under the conditions taken into account in this contribution. Uncertainties of resistance measurements are increasing from TPW to aluminium at corresponding DFPs. Between the fixed points there is an influence of mathematical definition of the temperature scale that can cause decreasing of uncertainties with local minimum.

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