Sensitivity of the RF magnetic field receivers with superconducting quantum amplifiers for NMR spectroscopy and tomography

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Abstract. *The contribution gives results of the theoretical analysis of the noise properties of the RF magnetic field receivers using the Superconducting Quantum Magnetometer (SQM) for NMR systems with the low magnetic field (Low-Field-NMR). The relation for the equivalent spectral sensitivity of the magnetic induction of the receiver, which covers the influence of the intrinsic noise of SQM, antenna system resistance loss components, the influence of measured sample and their noise temperatures, is given.*

Keywords: Sensitivity of NMR SQUID systems, NMR spectroscopy, signal-to-noise ratio

1. Introduction

Nuclear Magnetic Resonance (NMR) systems became the base for one of most important diagnostic method in medicine. The same importance they have in solid state physics in NMR spectroscopy. In medical conditions the cost of the equipment and the operating expenses significantly depend on the value of DC magnetic field used in tomograph. The smaller is the field, the smaller are the frequency and the intensity of the RF magnetic field excited by high-frequency impulse. This contribution deals with theoretical sensitivity limits of the receivers with SQMs [1].

2. Analysis

The simplified diagram of the RF magnetic field receiver is in Fig. 1. The receiver consist of

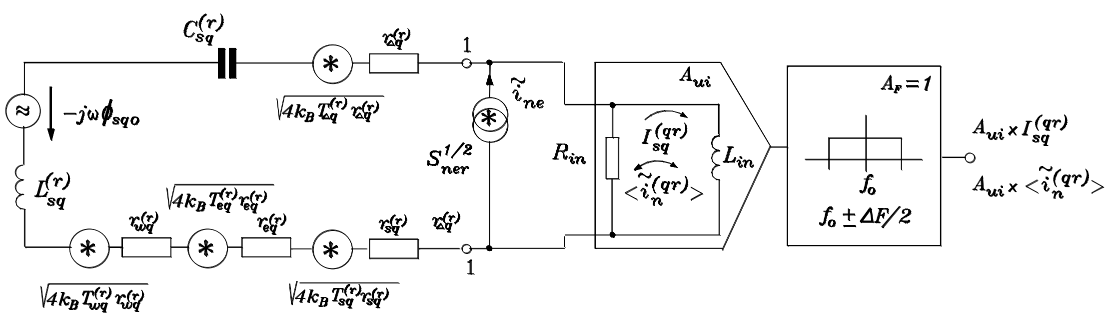

Fig. 1. Equivalent diagram of the receiver with superconducting quantum magnetometer.

receiving antenna in resonance mode, active amplifying element – electronic unit of the superconducting quantum magnetometer (EU SQM) and of frequency filter conditioning the transmission bandwidth. A_{ui} (V/A) and A_F (V/V) are the transfer constants of the EU SQM and the frequency filter respectively. Detailed theoretical analysis of the transmission properties of such circuit and comparison of its parameters with the receiver using the classic semiconductor amplifier was given in [1]. The signal-to-noise ratio on the receiver with the SQM is given by relation $p_{s/n}^{(qr)} = \left|I_{sq}^{(r)}\right| / \langle \tilde{i}_n^{(qr)} \rangle_{\Delta F}$, where $I_{sq}^{(r)}$ is the amplitude of current in the EU SQM input coil, which is induced by the magnetic flux with amplitude $\Phi_{sqo}^{(r)}$ and $\langle \tilde{i}_n^{(qr)} \rangle_{\Delta F}$ is RMS value of equivalent noise in the input coil with the inductance L_{in} . The next relation follows from the analysis [1]:

$$p_{s/n}^{(qr)} = \frac{\omega \Phi_{sqo}^{(r)} \left| j\omega L_{in} + Z_{1q}\left(j\omega L_{in}/R_{in}+1\right)\right|^{-1}}{\left[\int\limits_{f_o-\Delta F/2}^{f_o+\Delta F/2} S_{nrq}^{(r)} \left| j\omega L_{in} + Z_{1q}\left(j\omega L_{in}/R_{in}+1\right)\right|^{-2} df + S_i \Delta F\right]^{1/2}}. \quad (1)$$

where $\Phi_{sqo}^{(r)} = N_q^{(r)} A_{qN}^{(r)} B_{so}$ is the amplitude of the magnetic flux (acting on the antenna coil) induced by harmonically changing RF magnetic field with the axial component $B_s = B_{so}\sin(\omega t)$, where B_{so} is mean value of its amplitude in the planes $N_q^{(r)}$ of the turns, $A_{qN}^{(r)}$ is the area of one turn, t is the time and $\omega = 2\pi f$ is the angular frequency. R_{in} is the resistance loss component of the SQM input impedance and for the impedance Z_{1q} it holds that

$$Z_{1q} = r_{saq}^{(r)} + r_{\Delta q}^{(r)} + j\omega L_{sq}^{(r)} + \frac{1}{j\omega C_{sq}^{(r)}}, \quad (2)$$

where $L_{sq}^{(r)}$ is the inductance of the antenna coil and $C_{sq}^{(r)}$ is the capacitance of the tuning capacitor. The resistance components $r_{saq}^{(r)} = r_{wq}^{(r)} + r_{eq}^{(r)} + r_{sq}^{(r)}$ and $r_{\Delta q}^{(r)}$ are explained below. The spectral density $S_{nrq}^{(r)}$ of total Johnson noise of these resistance loss components of the resonance circuit is given by the relation

$$S_{nrq}^{(r)} = 4k_B\left(T_{wq}^{(r)} r_{wq}^{(r)} + T_{eq}^{(r)} r_{eq}^{(r)} + T_{sq}^{(r)} r_{sq}^{(r)} + T_{\Delta q}^{(r)} r_{\Delta q}^{(r)}\right). \quad (3)$$

$T_{wq}^{(r)}$, $T_{eq}^{(r)}$, $T_{sq}^{(r)}$, $T_{\Delta q}^{(r)}$ are the noise temperatures of the resistance components respectively. The resistance $r_{wq}^{(r)}$ characterizes the antenna coil intrinsic losses, $r_{eq}^{(r)}$ covers the summary losses due to further elements of the resonance circuit and its coupling on environment (losses caused by cryostat, constructional, etc.). The resistance $r_{sq}^{(r)}$ is equivalent loss resistance caused by measured sample. Equivalent quality factors and loss bandwidths referred to antenna coil with inductance $L_{sq}^{(r)}$, corresponding to pertinent resistances are as follows:

$$Q_{wq}^{(r)} = \omega_o L_{sq}^{(r)} / r_{wq}^{(r)}, \quad Q_{eq}^{(r)} = \omega_o L_{sq}^{(r)} / r_{eq}^{(r)}, \quad Q_{sq}^{(r)} = \omega_o L_{sq}^{(r)} / r_{sq}^{(r)} \quad Q_{saq}^{(r)} = \omega_o L_{sq}^{(r)} / r_{saq}^{(r)} \quad (4)$$

$$\Delta f_{wq}^{(r)} = f_o / Q_{wq}^{(r)}, \quad \Delta f_{eq}^{(r)} = f_o / Q_{eq}^{(r)} \quad \Delta f_{sq}^{(r)} = f_o / Q_{sq}^{(r)} \quad \Delta f_{saq}^{(r)} = f_o / Q_{saq}^{(r)}. \quad (5)$$

The resistance $r_{\Delta q}^{(r)}$ is an additional damping resistance which can be used for adjusting the frequency bandwidth of the resonance circuit on desired value $\Delta F_a = f_o / Q_a$, where

$$Q_a = \frac{1}{\omega_o C_{sq}^{(r)}} \left\{ r_{saq}^{(r)} + r_{\Delta q}^{(r)} + \frac{R_{in}}{1 + \left[R_{in}/\omega_o L_{in}\right]^2} \right\}^{-1}. \quad (6)$$

In the equation (1) the S_i (A^2/Hz) is spectral density of the input equivalent noise current of the SQM (practically independent of input load) and ΔF is frequency bandwidth of the transmission band given by parameters of the output frequency filter. In Fig. 1, $S_{ner}^{1/2}$ and $\tilde{\imath}_{ner}$ are spectral density and instantaneous value of the noise current of equivalent source of the SQM intrinsic noise. Denoting the sources by the asterisk means that they are characterized by spectral densities of the noise voltage or current.

Sensitivity of the measuring system

The sensitivity is defined by a minimum measurable magnetic induction $B_{s\,min}^{(qr)}(\Delta F)$ determined by the condition $p_{s/n}^{(qr)max} = 1$ (signal-to-noise ratio at frequency f_o). It holds

$$B_{s\,min}^{(qr)}(\Delta F) = \frac{\langle \tilde{\imath}_n^{(qr)} \rangle_{\Delta F}}{\omega_o N_q^{(r)} A_{qN}^{(r)} \left| F_{qr}(f_o) \right|}, \text{ where} \quad (7)$$

$$\langle \tilde{\imath}_n^{(qr)} \rangle_{\Delta F} = \sqrt{\int_{f_o-\Delta F/2}^{f_o+\Delta F/2} S_{nrq}^{(r)} \left(\left| F_{qr}(f) \right| \right)^2 df + S_i \Delta F} \quad \text{a} \quad F_{qr}(f) = \left[j\omega L_{in} + Z_{1q} \left(j\omega L_{in} / R_{in} + 1 \right) \right]^{-1} .$$

Assuming $\Delta F < \Delta F_a << f_o$ the noise spectrum in the band ΔF is approximately uniform and has a character of the white noise. For the equivalent spectral sensitivity $S_{Be}^{(qr)}(\Delta F)^{1/2}$ it holds:

$$S_{Be}^{(qr)}(\Delta F)^{1/2} = B_{s\,min}^{(qr)}(\Delta F) / \Delta F^{1/2} . \tag{8}$$

The influence of above considered noise sources on its value can be expressed by components of the spectral density $S_{Beq}^{(r)}(f)^{1/2}$ of the input equivalent noise magnetic induction acting on the antenna coil. It holds:

$$S_{Beq}^{(r)}(f)^{1/2} = \left[S_{Bwe}^{(qr)}(f) + S_{Bs}^{(qr)}(f) + S_{B\Delta}^{(qr)}(f) + S_{Bq}^{(r)}(f) \right]^{1/2} , \tag{9}$$

where

$S_{Bwe}^{(qr)}(f)^{1/2}$ – component corresponding to thermal noise of the loss resistances $r_{wq}^{(r)}$ and $r_{eq}^{(r)}$ of the resonance circuit;

$S_{Bs}^{(qr)}(f)^{1/2}$ – component corresponding to resistance $r_{sq}^{(r)}$ (losses caused by measured sample);

$S_{B\Delta}^{(qr)}(f)^{1/2}$ – component corresponding to additional damping resistance $r_{\Delta q}^{(r)}$;

$S_{Bq}^{(r)}(f)^{1/2}$ – component corresponding to SQM noise contribution.

For these components it holds:

$$S_{Bwe}^{(qr)}(f)^{1/2} = \left[4k_B \left(r_{wq}^{(r)} T_{wq}^{(r)} + r_{eq}^{(r)} T_{eq}^{(r)} \right) \right]^{1/2} / \omega N_q^{(r)} A_{qN}^{(r)} \qquad S_{Bq}^{(r)}(f)^{1/2} = S_i^{1/2} \left\{ \omega N_q^{(r)} A_{qN}^{(r)} \left| F_{qr}(f) \right| \right\}^{-1}$$

$$S_{Bs}^{(qr)}(f)^{1/2} = \left(4k_B r_{sq}^{(r)} T_{sq}^{(r)} \right)^{1/2} / \omega N_q^{(r)} A_{qN}^{(r)} \qquad S_{B\Delta}^{(qr)}(f)^{1/2} = \left(4k_B r_{\Delta q}^{(r)} T_{\Delta q}^{(r)} \right)^{1/2} / \omega N_q^{(r)} A_{qN}^{(r)} \tag{10}$$

3. Results of the analysis

Fig. 2 shows an example of the frequency dependence of spectral density of the input equivalent noise and the frequency dependence of its components. The parameters of considered measuring systems are:

Antenna coil: diameter 23.7 (cm); length 15 (cm); diameter of Cu wire 3 (mm); number of turns $N_q^{(r)} = 10$; $L_{sq}^{(r)}$ =22.6 (μH); SQM: $S_i^{1/2}$ =1×10^{-11} (A Hz$^{-1/2}$), $L_{in} = 2\times10^{-6}$ (H), $R_{in} = 10^4$ (Ω)

Further parameters of the system: antenna coil $Q_{wq}^{(r)} = 50000$, $T_{wq}^{(r)} = 4.2$ (K); remaining damping effects $Q_{eq}^{(r)} = 10000$, $T_{eq}^{(r)} = 4.2$ (K); measured object $Q_{sq}^{(r)} = 10000$, $T_{sq}^{(r)} = 4.2$ (K); additional damping resistance $r_{\Delta q}^{(r)} = 0$; tuning capacitance for $f_o = 2.2$ MHz (base magnetic field $B_0 \approx 0.05$ T) $C_{sq}^{(r)} = 213.1$ (pF); bandwidth $\Delta F_a = 0.94$ (kHz). These parameters illustrate the case close to optimum conditions for application of SQM and cryogenic cooling. Equivalent input spectral sensitivity of the magnetic induction is $S_{Beq}^{(r)}(f_o)^{1/2} = 7\times10^{-19}$ (T Hz$^{-1/2}$) and achieved s/n ratio at the amplitude $B_{so} = 2\times10^{-15}$ (T) of acting RF magnetic field and frequency bandwidth $\Delta F = 0.1$ (kHz) is $p_{s/n}^{(qr)} = 285.7$.

From Fig. 2 it is seen that total spectral density $S_{Beq}^{(r)}(f)^{1/2}$ (curve 1) is in the band ΔF practically constant and it holds:

$$S_{Be}^{(qr)}(\Delta F) \approx S_{Beq}^{(r)}(f_o) . \tag{11}$$

From equation (8) for the sensitivity it follows:

$$B_{s\,min}^{(qr)}(\Delta F) = S_{Beq}^{(r)}(f_o)^{1/2} \Delta F^{1/2} . \tag{12}$$

At above mentioned bandwidth $\Delta F = 100$ Hz in given example $B_{s\,min}^{(qr)}(\Delta F)$ is equal to 7×10^{-18} T. These parameters correspond to conditions for the samples of non-biological origin. In MRI of living objects usually it is necessary to display relatively large volume. This is connected with large dimensions of the magnet and greater inhomogeneity of the magnetic field in measuring space. It

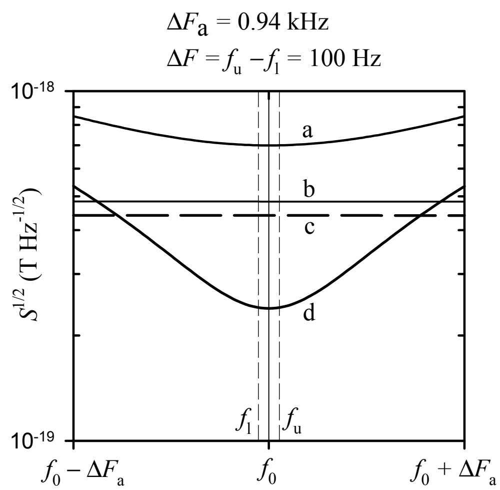

Fig. 2. Frequency dependence of the spectral density of the equivalent input noise magnetic

requires to increase the frequency bandwidth and damping of the resonance circuit. This is usually accompanied with greater RF losses of the biological samples. It results in fact that the total noise is significantly greater than SQM noise. Generally, using the SQM is reasonable primarily in NMR spectroscopy of solid state.

4. Conclusions

The contribution gives the basic information on results of the theoretical analysis enabling an exact comparison of the noise parameters of RF receivers in MRI systems with superconducting quantum magnetometers with the noise parameters of the receivers using conventional semiconductor active elements. The example in Fig. 2 is for orientation, what are the limits of achievable sensitivity in the systems with SQM. An important result is the possibility to determine the separate influence of single noise sources and during optimization to concentrate on dominant components.

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