Matching of RF Coils for NMR Measurements Using Inductors

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Abstract: For Nuclear Magnetic Resonance (NMR) experiments radiofrequency tuned coils are used for both the RF excitation and NMR signal detection. Matching RF coils with circuits using capacitors is lossless but some conditions must be achieved. If it is not possible a modification of the circuit with an inductor instead of one capacitor can be used sometimes. Elements of the circuit are calculated and some parameters of the circuit are graphically depicted. The resulting signal-to-noise ratio is calculated and compared with the circuit consisting of capacitors.

1. Introduction

Matching circuit is still important part of an RF coil. It can be designed and realised in more ways. Circuits consisting of capacitors are often preferred for they provide with lossless matching. The paper [1] showed two types of capacitive matching circuits used in NMR scanners. The second type of the both is more frequently used for it provides more easily realisable values of capacitors. The paper [2] analysed some qualities of the circuit from the point of view of tuning. Nevertheless the calculated values of the matching capacitors are not always realisable. Therefore some elements can be replaced with inductors or more complex circuits. The purpose of this paper is to analyse the matching circuit with matching inductor and compare the resulting quality of matching with that achieved by the capacitive matching. The results of the study can be used in NMR experiments and practice.

2. Results

The matching circuit with RF coil is depicted in generic form in the Fig. 1. The RF coil is represented by its real \( R_c(\omega) \) and reactive \( X_c(\omega) \) parts of the impedance. The coil is matched to the receiver input impedance \( Z_{in}(\omega) \) and tuned to resonance together with matching elements \( C_1 \) and a reactance \( X_m \). Consider \( Z_{in}(\omega) = R_{in} \) for all calculations. At the resonance frequency \( \omega = \omega_o \) the output impedance of the matching circuit with coil has its desired value so the system of two equations can be determined and solved. There can be two sets of the matching elements calculated. The first solution is given by

\[
C_1 = \frac{R_{in}X_c(\omega_o) - \sqrt{R_c(\omega_o)R_{in}(R_c(\omega_o)^2 - R_c(\omega_o)R_{in} + X_c(\omega_o)^2)}}{R_{in}(R_c(\omega_o)^2 + X_c(\omega_o)^2)\omega_o} \quad \text{and}
\]

\[
X_m(\omega_o) = -\frac{\sqrt{R_c(\omega_o)R_{in}(R_c(\omega_o)^2 - R_c(\omega_o)R_{in} + X_c(\omega_o)^2)}}{R_c(\omega_o)}.
\]

The solution must fulfil the following condition...
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\[
\frac{X_c(\omega_c)^2}{R_c(\omega_c)^2} \geq \frac{R_{in}}{R_c(\omega_c)} - 1 \geq 0.
\]

The value of \( R_{in} \) should be higher than that of \( R_c(\omega_c) \). The reactance \( X_m \) corresponds to a capacitance. The solution has already been described in [2, 3].

The second solution is given by

\[
C_1 = \frac{R_{in} X_c(\omega_c) + \sqrt{R_c(\omega_c) R_{in} (R_c(\omega_c)^2 - R_c(\omega_c) R_{in} + X_c(\omega_c)^2)}}{R_{in} (R_c(\omega_c)^2 + X_c(\omega_c)^2) \omega_c} \quad \text{and} \quad \frac{X_m(\omega_c)}{R_c(\omega_c)} = \frac{\sqrt{R_c(\omega_c) R_{in} (R_c(\omega_c)^2 - R_c(\omega_c) R_{in} + X_c(\omega_c)^2)}}{R_c(\omega_c)}.
\]

The solution must fulfill the following condition

\[
\frac{X_c(\omega_c)^2}{R_c(\omega_c)^2} \geq \frac{R_{in}}{R_c(\omega_c)} - 1.
\]

The value of \( R_{in} \) can not only be higher but even smaller than that of \( R_c(\omega_c) \). The reactance \( X_m \) corresponds to an inductance \( X_m(\omega_c) = \omega_c L_m \).

Fig. 2. shows the both \( C_1 \) and \( L_m \) varying as quality factor of the coil \( Q = \frac{X_c(\omega_c)}{R_c(\omega_c)} \).

![Fig. 2](image)

The output impedance of the RF coil with the matching elements \( Z_{out}(\omega) \) is expressed as

\[
Z_{out}(\omega) = R_{out}(\omega) + jX_{out}(\omega) = Z_{out}(\omega)e^{j\phi_{out}} = \frac{R_c(\omega)}{\omega^2 R_c(\omega)^2 C_1^2 + \omega^2 X_c(\omega)^2 C_1^2 - 2\omega C_1 X_c(\omega) + 1} + \frac{X_c(\omega) + X_m(\omega) - \omega C_1 R_c(\omega)^2 - \omega C_1 X_c(\omega)^2 - 2\omega C_1 X_c(\omega) X_m(\omega) + \omega^2 C_1^2 R_c(\omega)^2 X_m(\omega) + \omega^2 C_1^2 X_c(\omega)^2 X_m(\omega)}{\omega^2 R_c(\omega)^2 C_1^2 + \omega^2 X_c(\omega)^2 C_1^2 - 2\omega C_1 X_c(\omega) + 1}.
\]

Fig. 3. shows the real and the imaginary parts of the matched coil impedance varying as frequency. It is evident that the matched coil has its resonance at two frequencies. The question is how to determine the proper resonance when tuning the coil. The output impedance of the matched coil depicted in another way can be found in Fig. 4. The both matching elements \( C_1 \) and \( L_m \) are sharing on the coil tuning but the influence of the capacitor value is more significant. If varying the output impedance as \( C_1 \) two resonances appear and the proper one is that with the higher value of the capacitance. The influence of the coil

![Fig. 3](image)

![Fig. 4](image)
inductance $L_m$ is more significant for the proper impedance matching. In a real practice a coil never can be made without a resistive part of its impedance creating another source of noise. The interesting task is evaluating the influence of the noise source. The equivalent circuit of the matched coil from the point of view of the noise sources can be seen in Fig. 5.

![Fig. 3. The real and the imaginary parts of the matched coil impedance.](image1)

$f_\omega = 4.45 \text{ MHz}; \quad X_c(\omega) = 75 \Omega; \quad R_c(\omega) = 0.25 \Omega; \quad R_m = 50 \Omega;$

![Fig. 4. Output impedance of the matched coil varying as the matching elements values.](image2)

$f_\omega = 4.45 \text{ MHz}; \quad X_c(\omega) = 75 \Omega; \quad R_c(\omega) = 0.25 \Omega; \quad R_m = 50 \Omega;$

![Fig. 5. The equivalent circuit of the matched coil from the point of view of noise.](image3)
Each of the three sources of noise voltages $v_{n1}, v_{n2}, v_{n3}$ created by the three resistances $R_x, R_m, R_{in}$ has its influence on the resulting noise in the load impedance $Z_{in}(\omega) = R_{in}$. The possible way how to express the influence is the signal-to-noise ratio (SNR) calculation for the output signal $v_{out}$. Fig. 6. depicts SNR varying as frequency of the signal. Figure a) shows the SNR of the coil matched and tuned with inductors with quality factors: 50; 150 and 300, the higher Q, the higher SNR. Figure b) shows the SNR of the same coil matched and tuned with the circuit consisting of two capacitors (solution 1 described in [2, 3]). Some deterioration of sensitivity is evident if using the matching inductor depending on its quality factor. Nevertheless if the condition for successful capacitive matching can not be achieved the circuit using an inductor can be an alternative.

3. Conclusion

Purpose of the paper was a brief analysis of a matching circuit with an inductor. The analysed circuit can be used in the case it is not possible to apply the matching with capacitors and lower sensitivity is acceptable. The described theory supported by graphical characteristics can be used for the first decision.

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References: