EXAMINATION OF METROLOGICAL PROPERTIES OF THE WAVE THERMOANEMOMETER SYSTEM

Zygmunt Biernacki¹⁾, Tadeusz Zloto²⁾

 ¹⁾ Institute of Electronics and Control Systems, Czestochowa Technical University, e-mail: biernac@el.pcz.czest.pl
 ²⁾ Institute of Machine Technology and Production Automation, Czestochowa Technical University, Poland, e-mail: zloto@itm.pcz.czest.pl

Abstract: The paper deals with the Wave Thermoanemometer System developed and constructed on the basis of the author's own design. The Wave Thermoanemometer (WTS) is based on Kovasznay's idea of measuring the transit time of thermal markers drifted by the flowing medium (e.g. gas) on the a priori known interval Δl , which is a construction constant of the measuring converter. The scope of the investigation includes model studies with the use of computer simulation techniques as well as experiments on a laboratory stand. The paper concentrates mainly on the model studies with the use of adequate mathematical methods.

1. Introduction

The Wave Thermoanemometer System (WTS) is a modification of Kovasznay's idea [1], which provides the basis for various versions and configurations of sensors and thermoanemometric systems often referred to as wave thermoanemometers [2,3]. The thermoanemometer system discussed here employs one of the three constructional variants of the thermoresistive sensor [2]. The system has been developed to minimize all the drawbacks characteristic of traditional convective thermoanemometers [4,5]. The functioning of the WTS is based on measuring the transit time Δt of thermal markers drifting on a flowing medium along the known section Δl , accordingly with the formula:

$$W_{G} = \frac{\Delta l}{\Delta t} \left[\frac{m}{s} \right]$$
(1)

The value of Δl assumed is about several millimeters, and the frequency of generating thermal markers is about several tens of Hertz. In Fig. 1 the two application versions of the three-wire WTS measuring sensor are presented.



Fig. 1. Three-wire sensor, (a) segmented version, (b) integrated version

In the three-element version applied in the WTS (Fig. 2), the sensor consists of a transmitter N and two detectors C_1 and C_2 . The configuration of the system is presented in Fig.2. To evaluate the metrological properties of the WTS it was thoroughly investigated both analytically and empirically.



Fig. 2. Diagram of the Wave Thermoanemometer System.

2. Theoretical considerations

As described in [4], the formula were found corresponding to the thermal waveforms occurring in the measuring sensor of the WTS. They provide an adequate mathematical model which can be used for the modelling and simulations of the following elements:

- the transmitting element N:

$$\Theta_{NI}(t) = \Theta_{I0} + \frac{b}{a} \left(I - e^{-at} \right) \text{ where } t \in \langle 0, t_i \rangle$$
(2)

$$\Theta_{N2}(t) = \Theta_{10} + \frac{b}{a} \left(1 - e^{-\frac{aT}{2}} \right) \cdot e^{\frac{a(T-t)}{2}} \text{ where } t \in \langle t_i, T \rangle$$
(3)

- the detector C1:

$$\Theta_{CII}(t) = \Theta_{I0} + k_{CI} \left[\frac{b}{a} + \left(-\frac{b}{a} - b(t - t_0) \right) e^{-a(t - t_0)} \right] \text{ where } t \in \langle t_0, 2t_i \rangle$$
(4)

$$\Theta_{C12}(t) = \Theta_{10} + k_{C1} \left\{ e^{-a(t-t_0)} \left[\frac{b}{a} e^{\frac{aT}{2}} - b^{\frac{T}{2}} - \frac{b}{a} + b \left(1 - e^{-\frac{aT}{2}} \right) e^{\frac{aT}{2}} \left(t - t_0 - \frac{T}{2} \right) \right] \right\} \text{ where } t \in \langle 2t_0, 2t_i + T \rangle$$
(5)

- the detector C_2 :

$$\Theta_{C21}(t) = \Theta_{20} + k_{C2} \left\{ \frac{b}{a} + \left[-\frac{b}{a} - b(t - 2t_0) \right] e^{-a(t - 2t_0)} \right\} \text{ where } t \in \langle 2t_0, 2t_0 + T \rangle$$
(6)

$$\Theta_{C22}(t) = \Theta_{20} + k_{C2} \left\{ e^{-a(t-2t_0)} \left[\frac{b}{a} e^{\frac{aT}{2}} - b\frac{T}{2} - \frac{b}{a} + b \left(1 - e^{-\frac{aT}{2}} \right) e^{\frac{aT}{2}} \left(t - 2t_0 - \frac{T}{2} \right) \right] \right\}$$
(7)

where $t \in \langle 2t_0 + \Delta t, 2t_i + \Delta t + T \rangle$

The mathematical model presented above provided the basis for developing a simulation program. The program, called "docent" was created in Turbo Pascal with the use of the Turbo Pascal compiler package (Borland). As can be seen in Fig.3, even though the amplitudes of the impulses are decreasing, the impulses are successfully detected. The wires in N,C₁ and C₂ [4,7] are treated as homogeneous cylinders. Under the real conditions, however, after a longer period of the operation of the transmitter in a real gas, various kinds of particles are deposited on the surface of the transmitter N and the detectors C₁ and C₂. Thus, the wires should be treated as non-homogeneous (two-layer) cylinders, appropriate formula should be adopted accordingly, and the RC model should follow (Fig.4).



Fig. 3. Thermal characteristics of the elements of the thermoanemometric sensor for $I_N = 0.06$ [A]; $t_0 = 5$ [ms], $T_{max} = T = 400$ [ms], $\Theta_N(t) \Rightarrow m_x = 30$ [deg/cm], $\Theta_1(t) \Rightarrow m_0 = 0.5$ [deg/cm], $\Theta_2(t) \Rightarrow m = 0.3$ [deg/cm], $\vartheta_0 = 293$ K, $\vartheta(t) = \vartheta_0 + l_\vartheta \cdot m$ [K]



Fig. 4. Discrete RC model of the two-layer cylinder

The components of this model are determined on the basis of the following expressions:

$$R_{\alpha} = \frac{l}{2\Pi \cdot R_{2} \cdot \alpha \cdot l} \left[\frac{W}{K} \right] \qquad ; R_{z1} = \frac{0.5l}{\Pi \cdot \lambda_{1} R_{1}^{2}}$$

$$R_{r2} = \frac{\ln R_{2} / R_{1}}{2\Pi \cdot \lambda_{2} \cdot l} \left[\frac{W}{K} \right]; R_{z2} = \frac{0.5l}{\Pi \cdot \lambda_{2} (R_{2}^{2} - R_{1}^{2})}$$

$$C_{1} = \frac{\Pi \cdot \lambda_{1} \cdot R_{1}^{2} \cdot l}{a_{1}} \left[\frac{Ws}{K} \right] \qquad ; C_{2} = \frac{\Pi \cdot \lambda_{2} (R_{2}^{2} - R_{1}^{2}) \cdot l}{a_{2}} \left[\frac{Ws}{K} \right] \qquad (8)$$

In the model assumed (Fig. 4) the thermal system under consideration can be approximated by means of a second order inertial element with the operator transmittance

$$K(s) = \frac{1}{(T_1 s + 1) \cdot (T_2 s + 1)}$$
(9)

The response of the detector to the jump forcing is the following:

$$\vartheta(t) = \vartheta_M \left[I - \frac{T_1}{T_1 - T_2} e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} e^{-\frac{t}{T_2}} \right]$$
(10)

In practical applications [4,5,6] (Fig. 4), the following transmittances are used for description

$$K(s) = \frac{K}{ch\sqrt{sT}} \tag{11}$$

as well as the response to a unitary temperature jump

$$\vartheta(t) = K \left[1 - \frac{4}{\Pi} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cdot \exp\left(-\frac{\Pi^2}{4T} (2n+1)^2 \cdot t\right) \right]$$
(12)

Occasionally, a RC model of the continuous type may be useful for the analysis of transient temperature changes in the two-layer cylinder. The transmittance of the model is

$$K(s) = K \exp\left(-\sqrt{sT}\right) \tag{13}$$

The response of the system with the transmittance (13) to the unitary jump is the following:

$$\vartheta(t) = \frac{2K}{\sqrt{\Pi}} \cdot \int_{\frac{1}{2}\sqrt{T}}^{\infty} \exp(-\gamma^2) d\gamma$$
(14)

A graphic representation of the models described by the formulas (12) and (14) is presented in Fig. 5.



Fig. 5. Waveforms of the jump responses in Continuum-type models (formulas 12 and 14)

The function (14) is associated with the error function $erf(\gamma)$, defined as

$$erf(\gamma)^{df} = \frac{2K}{\sqrt{\Pi}} \cdot \int_{0}^{x} \exp\left(-\gamma^{2}\right) d\gamma = \frac{2K}{\sqrt{\Pi}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^{n} \gamma^{2n+1}}{(2n+1)n!} \bigg|_{0}^{x} d\alpha$$
(15)

Taking into consideration e.g. four terms of the series (formula 15) the following expression is obtained

$$erf(\gamma) \approx \frac{2K}{\Pi} \cdot \left[\gamma - \frac{\gamma^3}{3} + \frac{\gamma^5}{10} - \frac{\gamma^7}{42}\right]$$
 (16)

The actual construction of a laboratory prototype of the WTS followed its computer modelling, in which the specialized software called MIKRO - CAP II was used. The software is intended for the analysis of electronic systems.

3. Conclusions

- Defining a mathematical model for the measuring sensor in a WTS has enabled the simulation experiments of the sensor as well as of the whole WTS.
- The results of simulations on models, performed with the use of specialized software, confirmed the adequacy of metrological characteristics of the WTS.
- The idea of basing the functioning of the WTS on the transit time of thermal markers has a number of advantages over traditional methods, concerning the exploitation and metrological properties [4].

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