Uncertainty Assessment of the Geometric Accuracy of the 2D Machined Profile

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Abstract. *Statistical properties of the assessments of the geometric accuracy of the machined profile are investigated in this work. The cases when a distribution of the measured deviations differs from the normal distribution are especially considered. Among such cases, of fundamental importance is the case when the deviations of the profile described in the form of the Fourier series expansion represented by a sum of few harmonics. A set of the point positions within the minimum zone is considered as a random sampling, and the width of the minimum zone presents the sampling range. Additionally, information about the power spectrum of the measured profile is applied for numerical estimations of the uncertainty of the measured profile. As an application example, statistical properties of the deviations of the 2D machined (milled or ground) profile are considered. This case is characterized by an interesting interrelation of the uncertainty components caused by cumulative error and cycle error deviations.*

Keywords: Geometric Accuracy, Uncertainty, Profile Machining

1. Introduction

Modern international and national standards use the tolerance-zone-based models for assessments of geometric accuracy. The assessments are based on a set of real-life measurements on coordinate measuring machines and calculated as a range of the measured sample. Since a calculated value of the estimated range depends on the number and location of points on the measured part, the standard assessment represents a sample-based random variable and must be accompanied by an estimation of the uncertainty associated with the sample [1].

It is a common knowledge that the normal distribution model is usually used for description of the *dimensional* accuracy. This model is also often applied for assessments of the *form* accuracy. In a number of cases, however, the normal distribution model corresponds poorly to actual measurements of geometric accuracy [2]. These data testify that the distribution model calls for further investigation. Obviously, alternative models have to be considered for these cases. Since the distribution is completely specified by a physical model of the estimated random variable, the investigation is closely related to a specific type of geometric accuracy.

In this paper, the geometric accuracy assessment of circular profile machined by 2D tracing on milling or grinding machine tool is considered. The profile deviations are described in the form of the Fourier series expansion. The resulting distributions are studied using the theory of random functions. Important practical cases, in which a spectrum of roundness deviation includes few harmonics, need special consideration since their distributions differ essentially from the normal distribution. An uncertainty-related analysis is based on interpretation of the minimum zone as a sample range, which, in turn, depends on the distribution function and the number of measurements.

1. Presentation of circular profile deviations

Since locating a set of points of the coordinate measuring machine measurements is fixed independently of the phase of profile deviations, the current deviation in the ith point of measurements ($i = 1,...,N$) presents a function of the random phase angle φ uniformly distributed between 0 and 2π. Therefore, the set may be interpreted as a statistical sample of size N consisting of elements δ_i. If an eccentricity component is eliminated, the roundness deviation may be represented as a centered polyharmonic function composed of n random-phase sine waves over the profile angle φ:

$$\delta(\varphi) = \sum_{k=1}^{n} a_k \sin(p_k\varphi + \zeta_k) \qquad (1)$$

where n is the number of harmonics describing the profile deviations; a_k is the magnitude of the kth sine wave, $p_k > 1$ is its frequency, and ζ_k is the non-random phase angle. The deviation δ_i presents the current value of function (1), i.e., $\delta_i = \delta(\varphi_i)$.

In the case of equal magnitudes ($a_1 = \ldots = a_n = a$), an uncertainty estimation for random variable (1) may be calculated using a confidence coefficient $P\{|\delta| \le \lambda_n a\}$ corresponding to a condition that the random variable δ lies within the interval $\pm\lambda_n a$ with the probability:

$$P\{|\delta| \le \lambda_n a\} = \frac{\lambda_n}{n} + \frac{2}{\pi}\sum_{j=1}^{\infty}\left[\frac{1}{j}J_0\left(\frac{j\pi}{n}\right)\right]^n \sin\frac{j\pi\lambda_n}{n} \qquad (2)$$

where $J_0(z)$ is the Bessel function of order zero and λ_n is the magnitude factor.

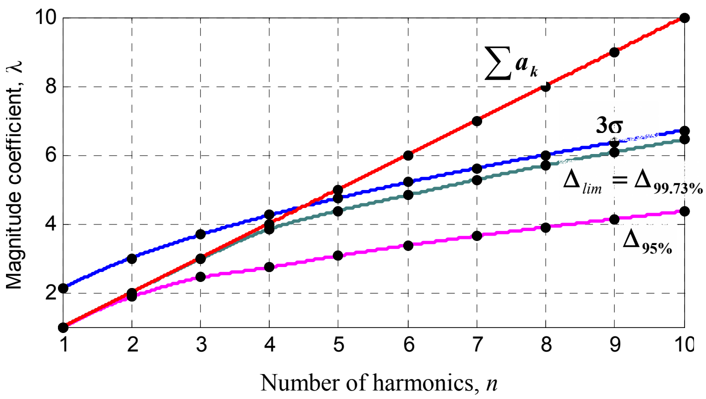


Fig. 1 Magnitude factor vs. the number of harmonics n .

Four curves for estimation of limit deviations corresponding to four different distribution models vs. the number n of harmonics constituting the deviation, Eq. (1), are shown in Fig. 1. The first curve presents the sum of the amplitudes $R_{ex}=2\Sigma a_k$, i.e., the physically possible boundaries. The second curve describes the six-sigma deviation when the normal distribution hypothesis is applied, $R_{lim} = 6[(\sum_{k=1}^{n} a_k^2)/2]^{1/2}$. This case may be practically used when the number of harmonics in the Eq. (1) is at least more than 8 – 10. The third and fourth curves show the exact limit deviations of the sum of n harmonics calculated by Eq. (2) for two confidence levels P = 0.95 and 0.9973, respectively. The 99.73% confidence level is assumed to be the exact tolerance value. A comparison shows that the limit magnitude factor $\lambda_{n,0.9973}$ converges to the normal-based limit estimation 3σ when the number n increases. For example, if profile is formed by single-harmonic disturbance and the normal distribution model is applied, the limit estimation will be overestimated by 110%, furthermore, when n = 2 overestimation is about 50%, and for n = 4 it is about 10%.

2. 2D Contour machining: the profile error model

Let us consider the profile deviations for the practically important case of the 2D profile machining. The machined profile is assumed to be formed by two coordinate motions. The issues of the motion errors affected on the machined profile deviations are the lead screw pitch errors δ_s. It may be represented as a sum of cumulative error δ_{cum} and periodical error δ_{per} (Fig. 2):

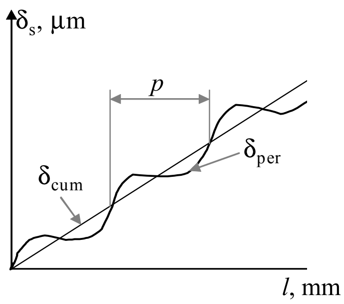


Fig. 2 Lead screw error model

$$\delta_s = \delta_{cum} + \delta_{per}, \text{ with } \delta_{cum}(x) = a_0 + a_1 x + a_2{}^2 x^2, \text{ and } \delta_{per}(x) = a_p\left(\sin\frac{2\pi x}{p} + \xi_x\right), \tag{3}$$

where x is the displacement along the X-axis, $0 \le x \le l$; l is the interval of machining. The lead screw errors are actual along the interval l, i.e. sum $D_w + D_g$ of diameters of wheel and workpiece for external machining, and difference $D_w - D_g$ of these diameters for internal machining.

Affect of the errors of the lead screw on the machining profile deviations is described by following formula:

$$\delta_c = \delta_{sx}\cos\varphi + \delta_{sy}\sin\varphi = c_0 + c_1\cos(\varphi - \zeta_1) + c_2\cos 2\varphi + c_3\cos(3\varphi - \zeta_3) + \\ + a_p\cos\varphi\sin(k\pi\cos\varphi + \lambda_1) + b_p\sin\varphi\sin(k\pi\sin\varphi + \lambda_2) \tag{4}$$

The cumulative error of the screw results in regular low-frequency harmonics, while the periodical error may be presented as double-sine function $f(\varphi) = \sin(k\pi\sin\varphi + \lambda)$.

A comparison of the three density distribution functions is given in Fig.3. The cases when deviation function (1) has $n = 1$ and $n = 2$ are given in Figs. 3a and 3b, respectively. The density of double-sine function $f(\varphi) = \sin(\pi\sin\varphi + \lambda)$(Fig. 3c) is distinguished from the arcsine distribution (Fig. 3a) by the peak in the center area from the arcsine distribution.

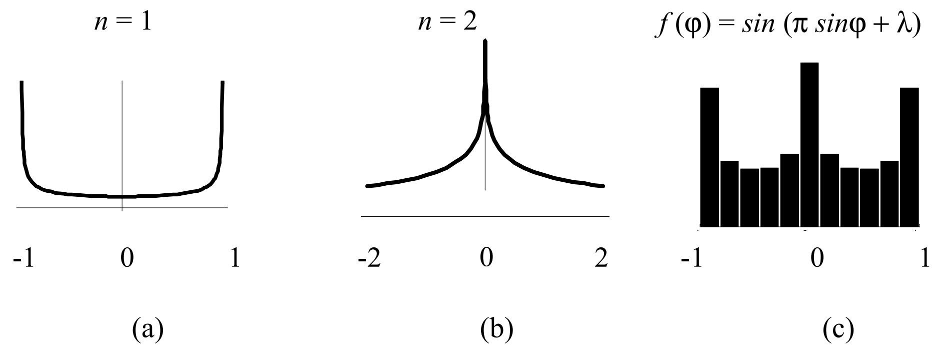


Fig. 3 Distribution density functions: (a) arcsine distributions, (b) distribution of sum of two harmonics with equal amplitudes, (c) double-sine function $f = \sin(\pi \sin \varphi + \lambda)$

3. Experimental investigation: accuracy of the 2D machined profile

The results of the standard control tests of contour grinding machine (Fig. 4) were used for investigation of real deviations distributions. This machine tool grinds two-dimensional curves by cylindrical grinding wheel of diameter D_g = 100 mm. Diameter of the machined cylinders are $D_{w,e}$ = 165 mm and $D_{w,i}$ = 125 mm, for the external and internal machining, correspondingly. The horizontal motions are carried out by lead screws with pitch p = 12 mm. The schemes of the external and internal profile machining, profiles deviations and probability density function of these deviations are shown in Fig. 5a and 5b, correspondingly.

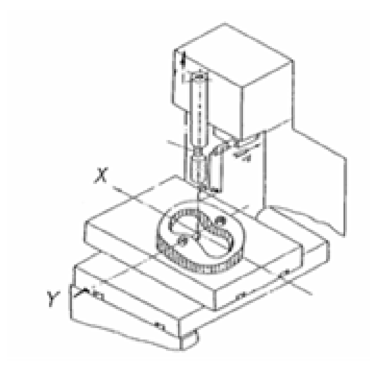
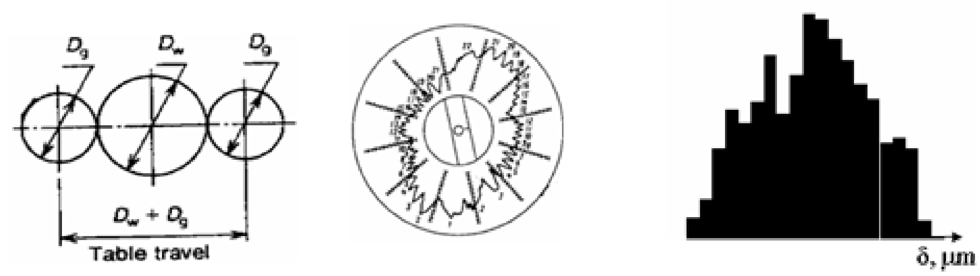
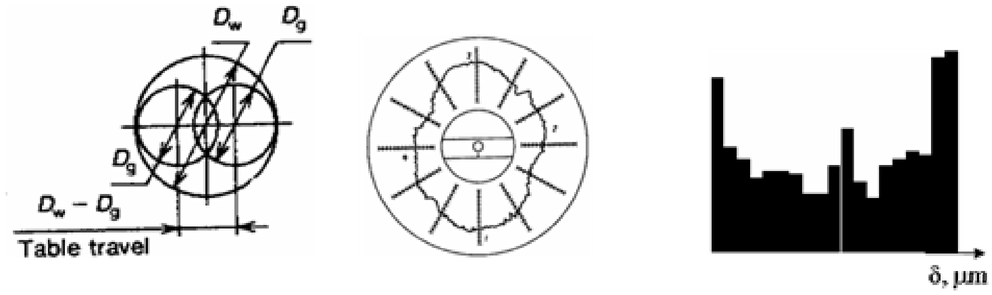


Fig. 4 Contour-Grinding Machine [3]



(a)



(b)

Fig. 5 Schemes of the machining, profile deviations and probability density function for (a) external , and (b) internal profile machining

Referring to Fig. 5a, external machining of the circle profile gives the elliptical basic deviation caused by cumulative error with periodical error caused waviness. The corresponding density curve is typical for profile with about two dominant harmonics (Fig. 5a). In the case of internal grinding (Fig. 5b), there is only one dominant harmonic in the profile deviations. The reason is that the active interval of table motions is relatively small. For comparison, the basic frequency of periodical error for the external machining is $2k = 2(D_{w,e}+D_g)/p = 2(165+100)/12 = 44.2$, while the corresponding frequency for the case of internal machining is $2k = 2(D_{w,i}-D_g)/\mathrm{p} = 2(125-100)/12 = 4.2$. Note that the simulation results (Fig. 3c) are close to the obtained distribution.

4. Conclusions

The normal distribution model is valid only when the measured profile consists of wide range of harmonics with magnitudes of the same order. However, in many important practical cases, a spectrum of roundness deviation includes few harmonics and, therefore, the resulting distribution differs essentially from the normal one. Distribution histograms of the deviations of the measurements results offer important information about the number of the dominant harmonics, and, therefore, about uncertainty of the geometric accuracy assessments. However, a model of assessment uncertainty may be formed if the physical model of error issues is known. As an example, the uncertainty model for the 2D machining of the circular profile is investigated. In this case, the total error includes both regular harmonics and double-sine harmonics.

4. References

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