# ADC Testing by Decomposition of the Error Model

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***Abstract.*** *Improvements in technology, increasing in resolution and various correction techniques of analogue-to-digital converters (ADC) makes linearity testing of ADC difficult and time-consuming task. New methods and approaches are examined. Statical and histogram tests require long time measurement and huge amount of data to be analysed. This paper deals with original approach for ADC modelling and testing that reduces time of measurement, providing information about linearity errors character with a good precision.*

*Keywords: ADC testing and modelling,*

## 1. Introduction

Testing of transfer function errors as differential nonlinearity (*DNL*) and integral nonlinearity (*INL*) characteristics becomes with increasing resolution of ADC more time consuming. Higher accuracy measurement requires more samples for data processing and precise input signals. Generation of high purity harmonic signal is easier than generation of full scale triangular signal. Triangular signal from the standard generator with reduced amplitude satisfies requirements for low distortion. The proposed method utilises advantages of these two testing signals for estimation of parameters in unified ADC error model for modelling of ADC integral nonlinearity.

## 2. Design of unified error model

The unified error model is described by the INL function splitted in two components [1]:

a) Low code frequency (LCF) component represented by modelled polynomial approximation of ${}^{LCF}INL_m(k)$ covers errors caused by analog pre-processing circuit. Function ${}^{LCF}INL_m(k)$ can be expressed by polynomial.

$$ {}^{LCF}INL(k) = A_0^{'} + A_1^{'}k + A_2k^2 + \cdots + A_Lk^L \tag{1} $$

b) High code frequency (HCF) component ${}^{HCF}INL_m(k)$ which describes the discontinuities of *INL* and is calculated from *DNL*($k$). It is possible because of relation between significant values of *DNL* and code $k$. This component is architecture dependent and use of this component is convenient for bit oriented architectures as for example successive-approximation-register (SAR) ADC is.

The modelled shape of the integral nonlinearity using both components is expressed as follows

$$ INL_m(k) = {}^{LCF}INL_m(k) + {}^{HCF}INL_m(k) = {}^{LCF}INL_m(k) + \sum_{l=0}^{k} DNL_m(l) \tag{2} $$

Main advantage of unified error model is its possibility to concentrate typical *DNL* manifestation of various ADC architectures in a relatively small number of parameters. Measurement of $INL(k)$ or $DNL(k)$ over full scale is a difficult task giving negligible contribution to the ADC description comparing to the information provided by unified error model. Using error model with low number of error parameters represented by coefficients in formulae (2) is a balance compromise between accuracy and testing load.

## 3. Identification of LCF component

A weakness in ADC testing by end user is availability of quite accurate signal generator for dynamic testing of LCF component. Harmonic signal generator meets best requirement for low distortion. The proposed method for identification of LCF component is based on the spectrum analysis for the harmonic stimulus signal covering ADC full scale.

### *Identification of LCF component by harmonic analysis*

This method is based on spectral analysis of ADC output signal for calculation of ${}^{LCF}INL_m(k)$. The harmonic input signal containing one harmonic component (low distortion) $x(t) = X_0 \cos(\Omega t + \varphi_1) + X_{offset}$ is implemented at the ADC input. Distortion of the real transfer function from its ideal linear function has impact at output signal spectrum. Output signal can be expressed by linear combination of several harmonic components, and these components contain information about ADC transfer function nonlinearities.

Let consider the low code frequency component ${}^{LCF}INL_m(k)$, equation (1) [2], and output signal as Fourier series expansion

$$y(iT_S) = H_0 + \sum_{m=1}^{L} H_m \cos(m\Omega i.T_S + \varphi_m) \tag{3}$$

Harmonics coefficients $H_0 \dots H_L$ of ADC output signal (3) are related to ${}^{LCF}INL_m(k)$ (1) coefficients $A_0 \dots A_L$ by the **P** matrix (4).

$$\mathbf{H = P.A} \tag{4}$$

where

$$\mathbf{H} = \begin{bmatrix} H_0 \\ H_1 \\ H_2 \\ \vdots \\ H_L \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} \begin{matrix} C(1,1) & 0 & \dfrac{C(3,2)}{2} & \cdots & \dfrac{1+(-1)^{j+i}}{2} \dfrac{C(j,\frac{i+j}{2})}{2} \\ 0 & C(2,2) & 0 & \cdots & \dfrac{1+(-1)^{j+i}}{2} C(j,\dfrac{j+i}{2}) \\ 0 & 0 & C(3,3) & \cdots & \dfrac{1+(-1)^{j+i}}{2} C(j,\dfrac{j+i}{2}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \dfrac{1+(-1)^{j+i}}{2} C(L,L) \end{matrix} \end{bmatrix} . \begin{bmatrix} 1 \\ \dfrac{X_0}{2} \\ \dfrac{X_0^2}{2^2} \\ \vdots \\ \dfrac{X_0^L}{2^L} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \vdots \\ A_L \end{bmatrix}$$

where i - row, j – column and $C(j,i) = \dfrac{j!}{i!(j-i)!}$.

The unknown coefficients $A_0 \dots A_L$ are determined by inversion of matrix relation (4).

The spectral components $H_m$ achieve positive or negative value. The sign depends on relation between $\varphi_m$ and $m\varphi_1$. Because information of amplitude and phase of every output spectral component is required a convenient method of spectrum calculation need to be chosen. After $A_i$ coefficient calculation (4) the equation (1) can be expressed as

$${}^{LCF}INL(k) = (A_0 - X_{offset}) + (A_1 - X_0)k + A_2 k^2 + \dots + A_L k^L \tag{5}$$

### *Spectrum components calculation*

Because of typically low values of integral nonlinearities for ADCs with mean resolution (typically less than 1LSB for 10-12 bit ADC), the harmonic components in the output spectrum are hidden in the quantisation noise background. Similarly to other dynamic ADC testing methods, the distortion of testing generator should be below level of expected ${}^{LCF}INL_m(k)$.

Harmonic coefficients $H_i$ calculated by Fast Fourier Transformation are valid only with coherently sampled signals. In case of uncoherent sampling, which represents the most probable event in real measurement, the harmonics coefficients are corrupted by leakage effect. Application of window functions causes better amplitude estimation of harmonic components, but phase calculation is less precise. Moreover the ratio between the fundamental harmonic and higher harmonics caused by ADC nonlinearity is extremly high (70-110dB). The three suitable methods of harmonic coefficients calculation for noncoherently sampled record have been tested out.

The first method for spectra estimation is based on the estimation of the first harmonic by four parametric best fitting method. Besides known amplitude of the $H_1$ and $H_0$ the best fitted cosinusoid at the ADC input $h_1(\mathrm{i})$ serves for the recovering of residual function $res_1(\mathrm{i})=k_{\mathrm{real}}(\mathrm{i})-h_1(\mathrm{i})$ from the digital output record $k_{\mathrm{real}}(\mathrm{i})$. The second harmonic component $h_2(\mathrm{i})$ is obtained in the next phase from the residuals $res_1(\mathrm{i}) = k_{\mathrm{real}}(\mathrm{i})-h_1(\mathrm{i})$ by the three parametric fitting method. Estimation of other higher harmonic $h_j(\mathrm{i})$, can be done by the recurrence of estimation process for the previous harmonic using new partial residuals.

The second method makes using of FFT possible by the time-domain signal interpolation [3] and its coherent resampling. For coherent resampling the frequency of fundamental harmonic is needed. First estimation of frequency is precised by four-parameter method [4]. This method gives reliable results when a record with a sufficient number of signal periods is analysed. The results of the first and second method application are showed in fig.1.

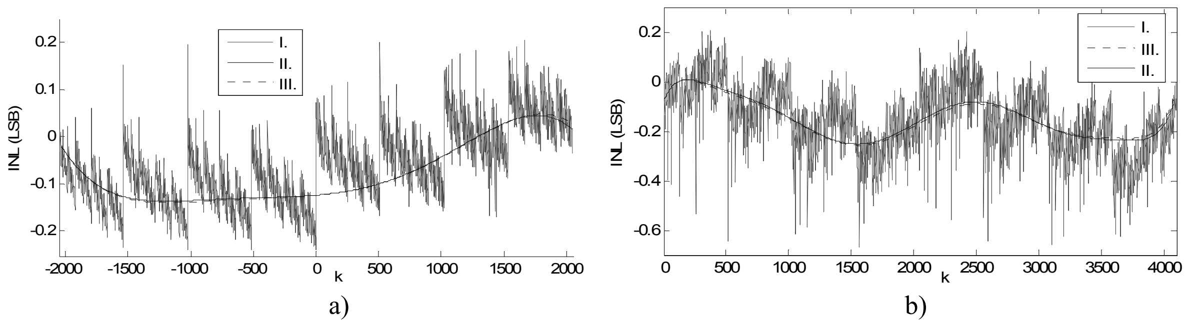


Fig. 1. Measured and modelled **INL**($k$) for 12b ADCs a) Lab-PC-1200, 5 harmonic components used b) ADuC812, 9 harmonic components used I.) **INL**($k$) measured by histogram method taking 1MS record II.) First method (recurrent best fitting) 100kS, 97.1Hz III.) Second method (spline interpolation, coherent resampling and FFT) 65.5kS, 43.2Hz

The third method is a variant of the four parameter best fitting algorithm [5]. The least mean square algorithm is implemented for simultaneous determination of all $(2L+1)$ parameters $(H_0 \cdots H_L, \varphi_1 \cdots \varphi_L, \Omega)$ in the signal (3). The optimal frequency $\Omega$ is estimated for the recursive optimalization of validity function. The ${}^{LCF}INL_m$ component in fig.2 is calculated by this method.

Advantage of the first and the third method is higher resistance to lower number of samples in the record.

## 4. Identification of HCF component

The high code frequency component is superposed on the smoothed LCF shape of the $INL(k)$ . It indicates the main discontinuities in the $INL(k)$ shape. Histogram test with reduced peak-to-peak value triangular voltage (e.g. by resistor divider) is appropriate for $DNL_m(k)$ determination. The differential nonlinearity of the code bin $k$ is calculated by

$$DNL(k) = O(k)\frac{\Delta U}{PQ'} - 1 \qquad (6)$$

where $O(k)$ is the number of samples with the digital value $k$. The input triangular voltage peak-to-peak value is $\Delta U$. The value $Q'$ is the mean code bin width value. In order to assure uniform distribution of the input signal the total number of acquired samples $P$ corresponds to an integer number of periods. Experimental results performed by this method for digital instruments with high accuracy are published in the [6]. ${}^{HCF}INL_m(k)$ component of ADC error model determined by this method is in fig.2.b.

a)
b)

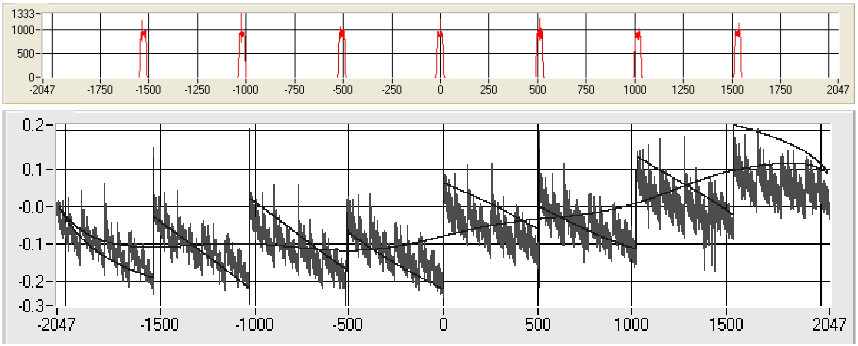


Fig. 2. *INL* of LAB-PC-1200 data acquisition card modelled by two component model. The ${}^{LCF}INL_m(k)$ obtained by spectrum analysis and ${}^{HCF}INL_m(k)$ determined by reduced amplitude triangular histogram test. a) histogram b) measured and modelled *INL*.

## 5. Conclusion

The unified error model covering large scale of ADC errors has been presented. The model composed of two components is able to describe typical manifestation of integral nonlinearity. In comparison with full scale histogram test using of unified error model reduces time of measurement approximately 10 times. It means significant testing time reduction. Methods of model parameters identification have been proposed and were tested on two 12b SAR converters. Methods of spectrum analysis based on four parameter best fitting methods are robust when shorter data records are analysed.

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