Conditional averaging of a delayed signal module in measurements of the phase shift of sinusoidally noised voltages

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Abstract: *That study presents a new measuring method of a phase shift angle measurement with use of conditional averaging of interfered signals.*

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1. Introduction

There are various ways to measure a phase angle shift, unfortunately not all of them are useful when measuring noised signals [3].

An experiment with phase shift measuring techniques consists of: a measuring process under real conditions according to a selected measurement model, or in simulating the examination of a measurement model e.g. with use of computer modelling with signal generators of the selected parameters. The development of modern technologies enables one to perform simulations, modelling and the examination of complex phenomena which occur with measuring techniques and which in practise is not necessarily possible.

Simulating new measuring principles are an important experience stage, enabling one to obtain information about the properties of the applied measuring models quickly, economically and technically accurately.

The integrated environments for programming of measuring systems (labVIEW and DASYLab among others) are interesting tools for the examination of phase shift measurement with existing noises.

This study examines a new method of voltage shift for the noised sinusoidal signals. Traditional phase meters do not give the satisfactory results if used for noised signals. Phase meters average the time intervals while lowering the signal to noise ratio – a loss to their primary accuracy. To improve the accuracy of measurement of a noised signal phase shift, electronic correlative phase meters are used; but even these are not sufficiently accurate enough for many applications. The proposed method of conditional averaging of the amplitude values of sinusoidal signals in measurements of phase shift angles may enable improvements to the effectiveness of measuring algorithms in relation to the accuracy, speed and complexity of the measuring systems.

2. Subject and Methods

Mathematical modelling of signals suitable for computer modelling have been designed to examine measuring methods. The signals were established as follows: a reference signal $x(t) = A_x\cos(\omega_0 t+\varphi_x)$ and delay signal $y(t) = A_y\cos(\omega_0 t+\varphi_y)$. To simplify the analyses it has been assumed that the reference signal does not include any noise, while the delayed signal is noised $z(t) = y(t) + n(t)$ where:

$n(t)$ – noise $N(0, \sigma_n)$ independent of $x(t)$ and $y(t)$.

For modelling noise by conditional averaging of signal amplitude values when measuring a phase shift angle, normal noise probability density $p(x)$ has been used; $p(x)$ is expressed by the following formula:

$$p(x) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_n^2}} \tag{1}$$

where:

- $p(x)$ probability,
- σ_n noise standard deviation,
- x signal value.

The distribution of a noised signal probability is show in Fig. 1 [1], [2]. The diagram shows some results, depending on K value, which expresses the signal/noise ratio and is described with the following formula:

$$K = \frac{S}{N} = \left(\frac{U_{RMS}}{\sigma_n}\right)^2 \tag{2}$$

where:

- K signal/noise value ratio,
- U_{RMS} root-mean-square value of a sinusoidal signal,
- σ_n noise standard deviation.

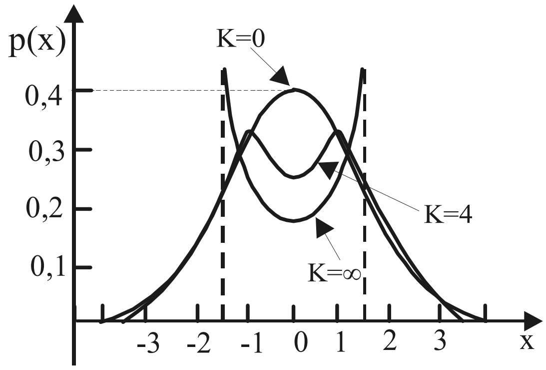

Fig. 1. Distribution of the probability density of a sinusoidal and noised signal.

The noised signal includes the additive random noises. Then the conditional expected value of a signal z(t) under condition x(t)=0 ($0 \le \varphi \le \pi$) may be expressed according to the following dependence:

$$E\left[Z\,|_{X=0}\right] = E\left[y\,|_{X=0}\right] + E(n) = E\left[y\,|_{X=0}\right] = -A\sin(\omega_0\tau + \varphi) \tag{3}$$

The dependence (3) may be the basis for evaluation of a phase shift angle φ.
An interesting solution for measurement of a phase shift is applying a straightening operation to a noised signal [4]. The conditional expected value of a straightened noised signal will be as follows:

$$E\left[|Z|\,|_{X=0}\right] \approx \left|E\left[Z\,|_{X=0}\right]\right| = \left|A\sin(\omega_0\tau + \varphi)\right| \tag{4}$$

The phase shift is proportional to the time τ_0 required to reach the minimum through the conditional expected value (4).

3. Results.

The proposed principle of a phase shift angle measurement has been modelled in the LabView environment. For input research purposes the signals of a given voltage and time parameters have also been generated there. The sinusoidal signal amplitude is 1 V and its frequency is 10 Hz, while the noising signal is in a form of noise with normal distribution of probability and standard deviation 0,1 V and zero mean value.

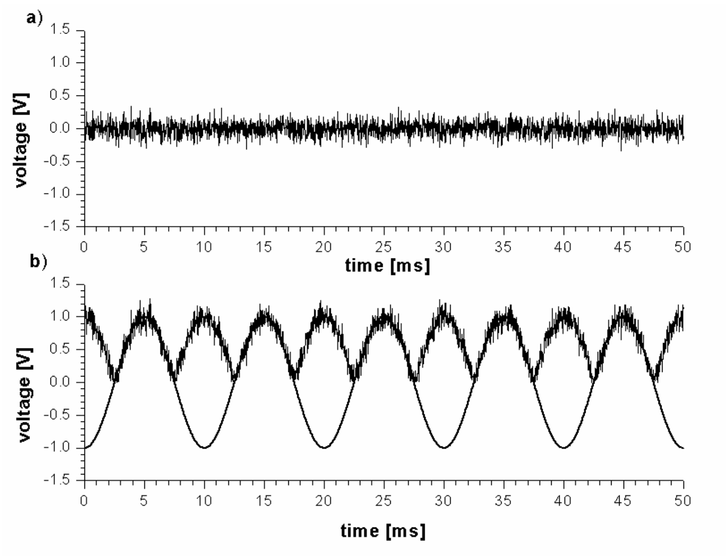

Fig. 2. Exemplary signals generated in LabVIEW program:
a) noising signal n(t),
b) sinusoidal signal y(t) and module of the signals sum |z(t)|.

The signal frequency 10 Hz sampled with frequency 5 Hz has been analyzed; the noised signal was delayed by 90°. The phase shift is proportional to time τ_0 when the function shown in the dependence (4) obtains a minimum value.

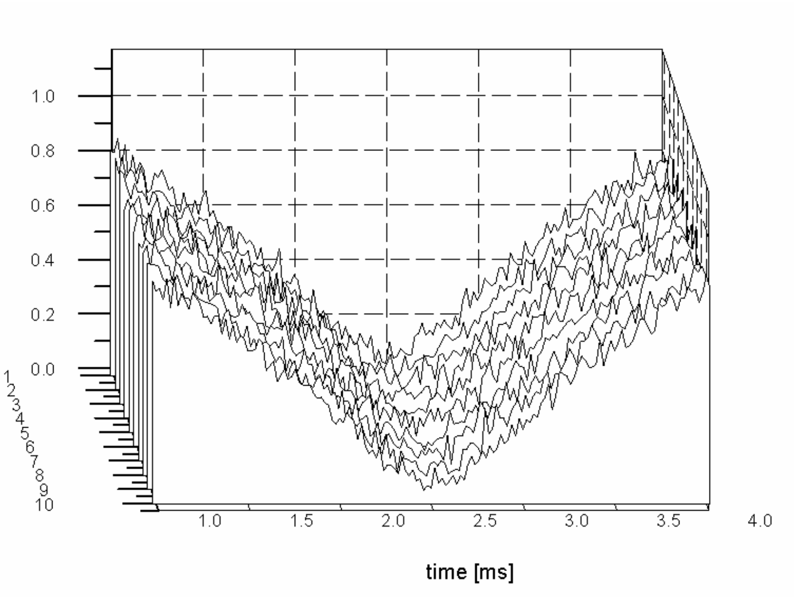

Fig. 3. The courses of 10 averaging operations of a delayed signal module.

The figure shows ten averaging operations; the results have been show in Table 1. Each averaging is a mean value of ten individual courses.

Table 1 Exemplary results of calculations of time τ_0 and a phase shift angle.

L.p.	1	2	3	4	5	6	7	8	9	10
τ_0 [ms]	2,48	2,52	2,52	2,44	2,46	2,52	2,54	2,66	2,48	2,46
$\Delta\varphi$ [°]	89,28	90,72	90,72	87,84	88,56	90,72	91,44	95,76	89,28	88,56

These values have been calculated for minimum values, which had been taken by individual averaging operations. Far more significant differences between the taken value of a phase shift angle result from the applied sampling frequency f_s=5kHz. In that case one period is related to 500 samples, so a resolution of 0.72° value may be obtained. The calculation of time τ_0 on the basis of appointment of a crossing place of the straight approximate lines, running from the selected levels – has been proposed in that study. The dependence sin(x)≈x for a given range of x variability is used there. The selection of values of those thresholds influences the accuracy of designation of a phase shift angle.

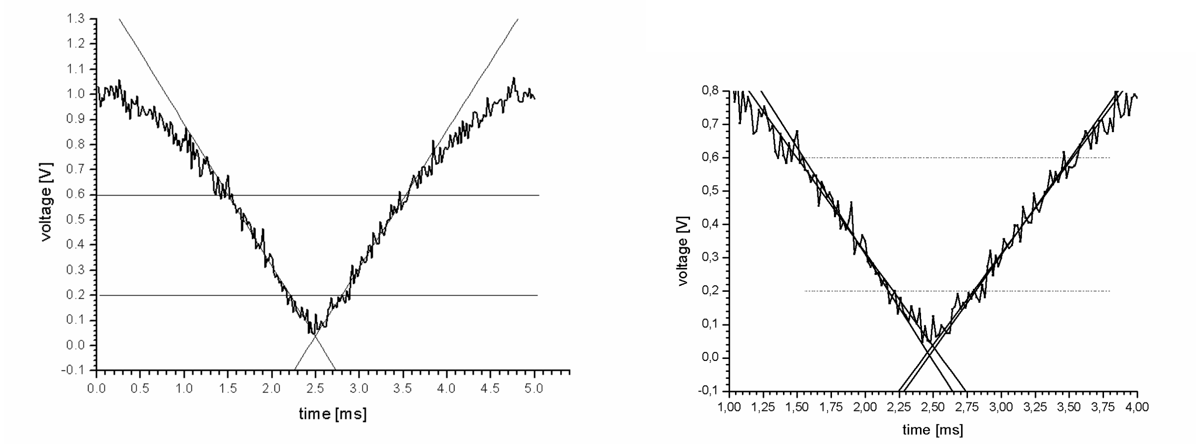

Fig. 4. The approximate straight lines designer within the limit 0,6-0,2 [V] and 0,4-0,2 [V]

Fig. 4 shows two examples of the influence of the thresholds values and various ranges selected for the purpose of calculating approximate straight lines. While analyzing the results we may confirm that the selection of the limit range applied for calculating straight lines influences the evaluation of time τ_0.

4. Conclusions

Application of the conditional averaging of signal amplitude values may enable the improvement of the accuracy of a phase shift angle measurement with a given measurement time and to shorten the measurement time with an assumed accuracy.

While designing the minimum value of the dependence (4), the width of a range designed for approximation is essential for the accuracy of time τ_0 designation. More accurate results are obtained with use of wider ranges (for a line model of the analysed module of noised signals only). The obtained results enable to search the optimal solutions in creation of a new method of a phase shift angle measurement. The essential parameter seems also to be the frequency of signals sampling, what is the subject of the study author researches.

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