Influence of noncoherent sampling upon an error of power quality parameters measurement

I. Szolik, K. Kováč, V. Smieško

Department of Measurement, Faculty of Electrical Engineering and Inform. Technology, Slovak University of Technology, Bratislava, Slovakia. E-mail: I.Szolik@kmer.elf.stuba.sk

Abstract: The paper deals with errors, which rise up during discrete realisation of harmonic analysis for power quality measurements. By this the coherence of sampling interval has a significant influence upon the error of measurement. Presented paper analyses the influence of window function type upon the resulting error of analysis as well as the influence of length of analysing interval. The analysis is performed for frequency fluctuations allowed according to conditions given in technical standards.

Keywords: power quality measurement, harmonic analysis, noncoherent sampling, window functions.

1. Introduction

By discrete realisation of harmonic analysis, which is described in e.g. [1], there may rise up the whole series of errors. These may be caused by different reasons: sampling coherency error, analogue/digital conversion errors and truncation or rounding errors during calculation of harmonic analysis [2]. The overall error is the summation of these error components. In this paper we concentrate on error caused by noncoherent sampling of periodic signals and metrology aspects of techniques used for suppression of this error, especially some window functions as Hanning, Hamming and Blackmann windows are. Our main task is to show the influence of the ratio between window length and signal period. We analyse the described topic from the point of view of power quality measurement, so we omit specialities used in other areas of digital signal processing.

Measuring process within power quality measurement is strictly determined by power frequency, which is obviously 50 or 60 Hz. According to standards which concern power frequency within power grid, the power frequency may change within interval <-6 %; +4 %> in open networks and even <-15 %; +15 %> in closed networks. These tolerance intervals create problems with keeping coherency of harmonic analysis sampling process.

2. Noncoherency of sampling

Under the term noncoherency of sampling process ones usually mean a situation, when sampled signal sequence entering the process of harmonic analysis does not represent exactly the integer multiple of measured signal time period [3]. Noncoherency may be represented by parameter $\alpha$ called noncoherency factor according following formula:

$$ T_p = (P + \alpha)T_s $$

(1)

where $T_p$ is processing time interval i.e. length of sampled sequence, $P$ is an integer number of signal periods, $T_s$ is a time period of measured signal. According to facts already mentioned noncoherency factor may obtain values from interval <-0.06; 0.04> or maximally <-0.15; 0.15>. For coherent sampling the value of the factor is zero.
3. Window functions used for power quality harmonic analysis

For suppressing of noncoherency there exist different techniques [4]. There are many types of window functions, but only some of them are recommended as suitable for power quality measurements. In our paper we will concentrate on window functions, which are recommended for noncoherency suppression also by some standards. Those are preferably cosine window functions, which may be defined by cosine functions. This group of functions may be generally expressed by formula:

\[ w(t) = \sum_{i=0}^{m} (-1)^i a_i \cos \left( \frac{i \cdot 2 \pi t}{T_p} \right) \]  
(2)

where by help of weighting parameters \( a_i \) we may change the properties of window function.

In our paper we will consider Hanning, Hamming and Blackman windows. Their weighting coefficients are shown in Table 1 [4]. Typical shape of window function looks as is shown in Fig. 1.

<table>
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<tr>
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<th>( a_0 )</th>
<th>( a_1 )</th>
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<tr>
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<td>0.50</td>
<td>0.50</td>
<td>---</td>
</tr>
<tr>
<td>Hamming</td>
<td>0.52</td>
<td>0.48</td>
<td>---</td>
</tr>
<tr>
<td>Blackman</td>
<td>0.42</td>
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Table 1.: Weighing coefficients of typical window functions.

During the process of harmonic analysis the resulting frequency spectrum is mostly computed numerically. Then it means that resulting spectrum is discrete and frequency bin is indirectly proportional to the length of processing interval and then to the number of analysed signal periods. So the larger is the number of signal periods the more precise the result should be.

4. Errors of harmonic analysis by windowing functions noncoherency suppression

During the error analysis of harmonic analysis process we will use single frequency signal harmonic signal (only the first harmonic) as input. Then the resulting error of harmonic analysis may be expressed by the relation:

\[ \varepsilon(\alpha, \phi, P) = \sqrt{\varepsilon_0^2 + \varepsilon_1^2 + \sum_k \varepsilon_k^2} = \sqrt{A_0^2 + (A_I - A_M)^2 + \sum_{k=2} A_k^2} \]  
(3)

where \( \varepsilon_0 \) is an error of dc component, \( \varepsilon_1 \) is an error of the first harmonic, \( \varepsilon_k \) is an error of \( h \)-th harmonic, \( A_0, A_k \) is the resulting value of dc resp. \( k \)-th component, \( A_I \) is measured value of the first harmonic and \( A_M \) is its theoretical value. As is indicated the resulting error is dependent on sampling process parameters: \( \alpha \) - noncoherency factor, \( \phi \) - beginning of sampling process within period of input signal and \( P \) - the number of input signal periods covered by sampling interval.
The values of resulting error according to (3) were numerically calculated for values of $\alpha$ from full interval $<-0.15;0.15>$ and values of $\phi$ from the interval $<0;2\pi>$. The results are shown in Fig. 2.

The graphs displayed in Fig. 2 show overall error consisting from errors all 50 harmonic components. If we need to know which component errors are dominant we need to compare the values of error of separate components. The dependence of error value upon the order of harmonics for 10 periods sampling interval are shown in Fig. 3.

![Graph showing the dependence of overall error upon noncoherency factor $\alpha$ and $\phi$ - phase angle of beginning of sampling interval for number of signal periods $P=10$ for Hanning window.]

5. Numerical calculations

Obtained results of numerical calculations of resulting error we presented in Fig. 2 as 3D graph for Hanning window. If we want to compare influence of different window functions and different numbers of periods we need to use some evaluation criterion. We decided to use for this purpose integral evaluation function:

$$\varepsilon(P) = \int_{0}^{\pi/4} \int_{-0.15}^{0.15} \varepsilon(\alpha, \phi, P) d\alpha d\phi$$

Resulting criterion values for the most typical window functions and for several values of processing interval length are shown in Table 2.
6. Discussion

From Table 2 it is evident that for one period analysis interval the Hamming window is the best, but the resulting error is significant. Even the error value makes the measurement unusable. For higher number of analysed periods the most complicated window function (Blackman window) is the most precise. The second important fact is that it has no sense to enlarge the analysed time interval because increasing the length of interval over 10 periods does not lower down the resulting error. Only for Hamming window the optimum interval length is about 15 signal periods.

<table>
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</table>

Tab 2.: Resulting overall errors for different window functions and several durations of processing interval of harmonic analysis.

7. Conclusions

In the paper the analysis of influence of several window functions on an error of the harmonic analyses was presented. The analysis was realised for single frequency harmonic signal and it was oriented to power quality measurements so the analysed conditions were taken according to power standards. The results show that Blackman window is the most advantageous for discrete realisation of harmonic analysis. In addition to this it was shown that 10 periods of input signal are the optimum length of analysing interval. Increasing the length of analysed interval above this optimum does not bring any increase if measurement precision.

Acknowledgements

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References