Fractal complexity of EEG signal

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Abstract. *The paper deals with the presence of exponential or power-law decay in the power spectra of electroencephalogram (EEG). About 2300 EEG time series recorded during relaxed wakefulness were analysed. The whole spectrum of EEG was studied and power-law decay of about 2.28 prevailing over the exponential falling off was established. Correspondence between spectrum power-law decay and correlation dimension estimated for EEG was also validated.*

Keywords: electroencephalography, EEG, relaxation, spectral power, spectral decay; correlation dimension

1. Introduction

Recent investigations indicate that although, EEG does lose information rapidly from second to second, there is a significant level of long-range correlations that is evident after at least five seconds [1]. This correlation, indicated by the presence of very low frequencies, is characteristic of the so called fractal or scale-invariant processes. They are particularly relevant in the context of phase transition in physics, as scaling phenomena are known to occur in many physical systems near a critical state. Even though in this paper we do not ask whether the brain works in critical state, let us mention that some findings suggest that power law scaling is characteristic of healthy cerebral activity and the breakdown of this scaling may lead to incapability of quick reorganization during processing demands [2, 3].

Regarding spectral properties of different types of data the next statements are generally accepted:

Stochastic behaviour - The power spectrum decays via a power law $P(f)\sim f^{-\gamma}$ (γ can be obtained as the slope of linear part when plotted on a log-log scale) [4]. As an example, white noise with $\gamma=0$ or random walk time series with $\gamma=2$ (power spectrum decreases as $1/f^2$ with increasing frequency) can be mentioned. γ came to be called spectral decay, fractal exponent, or power-law exponent.

Periodic or quasi-periodic behaviour - The power spectrum consists of discrete spikes corresponding to distinct frequencies. In the case of noisy experimental data the power spectrum falls polynomially in the region of higher frequencies.

Chaotic behaviour - The power spectrum falls exponentially at high frequencies [5]. Exponential decay of power spectrum is a decay of the form of $P(f)\sim a.e^{-bf}$, where a and the b are positive constants. (The decay manifests itself as linear when plotted on a log-linear scale.) In the case of real chaotic systems, one can again observe only a finite region of exponential decay. Then the spectrum settles into the power law decay characteristic of noise.

The above summary shows that the falloff of the power spectrum helps us to answer the question, whether the observed erratic behaviour is essentially deterministic or stochastic.

In the case of EEG, Pereda et al. [6] investigated the spectral exponent. They selected the frequency range of *3–30 Hz* and found, that EEG exhibits random fractal structure with $1/f^{\gamma}$ spectrum. The authors estimated the decay rate (computed for frequencies less then alpha activity) to be in a range *0,98-2.18*.

In [2] the power spectrum of EEG of an epileptic patient recorded far away from any seizures was calculated. For a range between *20–100 Hz*, a spectral slope $\gamma=2$ can be observed, suggesting the existence of Brownian noise in this domain. Over a range from *0.1–10 Hz*, the power spectrum shows a scaling exponent approximately $\gamma=1.2$ interpreted by the authors as a "compromise" between the complete unpredictability of white noise and the very smooth landscape of Brownian noise.

The above cited papers suggest that the power of EEG spectrum may be expected to decrease polynomially.

The main goal of this work is to check the presence of exponential or power-law decay in the power spectra of EEG. As there seems to be no consensus regarding the choice of the regions of power-law decay in the literature, we studied the whole spectrum of EEG to find the sections of clear power-law (or exponential) decay.

In the case of polynomial decrease of the spectrum we are going to verify declarations about the relation between D_2 and spectral decay exponent.

2. Methods and Results

Eight healthy volunteers (3 females and 5 males) took part in EEG recording. Participants ranged in age from 24 to 39 years, with a mean of 25.5 years, s.d. 5.1 yrs.. They attended 2 measurements per each of 25 days. During recording subjects were lying in a darkened, electrically shielded room. They were instructed to keep their eyes closed and relax both physically and mentally. Data of 3-minute length were recorded, digitized at *500 Hz*. 8-channel EEG system with scalp-electrode impedances kept below *5 kΩ* was used for data recording. From the 8 signals (6 active electrodes and 2 reference electrodes) six difference signals were derived by. A digital high pass FIR filter with cut-off at *0.75 Hz*, with the width of 3000 data points, and with a Blackman window was utilized. For the purpose of this study, about 2300 electroencephalograms were analyzed. Following digital filtering the first and the last 1500 points were omitted and 87000 data point EEGs remained.

These data sets were used to look for the presence of exponential or power-law decay in the power spectra of EEG.

The spectrum was computed using standard FFT with frequency step of *0.029 Hz* and variance reduction factor of 10. Exponential decay is expected to manifest as a straight line when spectra are plotted in a log-linear coordinate system, and the corresponding exponent would be given by the slope of this line. Power-law decay can be obtained accordingly as a slope of linear regression applied to the power spectrum in log-log coordinate system. Both spectrum presented in log-linear graph and spectrum presented in log-log graph show apparent peak of about *10 Hz* corresponding to brain alpha frequency.

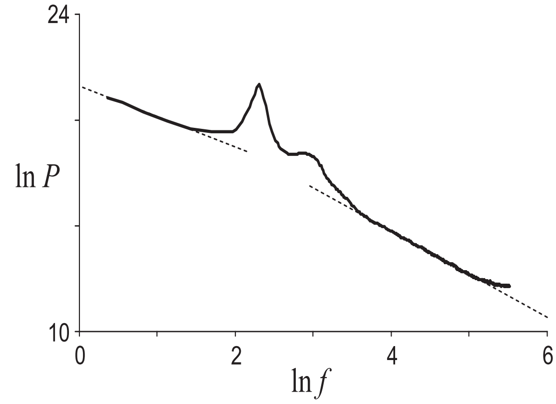

Fig. 1. Typical graph of spectral exponent estimation for EEG frequency range before alpha activity and following alpha activity.

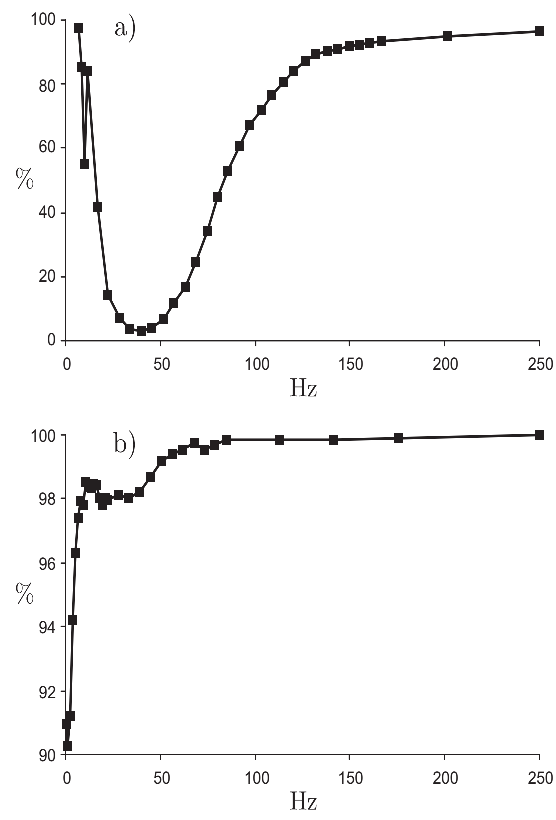

Fig. 2. Efficiency in % of power-law model in contrast to exponential model
a) computed for 2300 power spectra from fixed 5 Hz to ascending right limit (values of the horizontal axis) of the spectral range
b) computed for 2300 power spectra from fixed 250 Hz to descending left limit of the spectral range

However, as we are going to show, in the case of log-log graph frequencies before and after alpha peak fall linearly in contrast to log-linear graph indicating a presence of power-law behaviour. Fig. 1 shows a typical spectrum displayed in log-log graph. Consequently different regions of the whole power spectra (including alpha range) were taken to assess the efficiency of power-law model in comparison to exponential model. Intervals of fixed left frequency limit to ascending right limits resp. of descending left limits to fixed right bound were investigated. Fig. 2 illustrates the efficiency (with significance $P<0.05$) of power-law model in contrast to exponential model for particular parts of spectra.

As a result, power-law decay prevailing over the exponential falling off was established. The mean value of the power-law decay rate for relaxed EEG was estimated to be about 2.28.

As we confirmed the power of EEG spectrum to be decreasing polynomially, let us mention a result that advocate the use of the order of the polynomial falloff as a tool for correlation dimension [7] estimate. EEG has been a matter of interest for the dimension computation since the first correlation dimension (D_2) estimates for sleep cycles were made by Babloyantz et al. [8]. This attempt was inspired by the chaos hypothesis, i.e. it was assumed that the EEG could be described by a deterministic chaotic system and therefore the corresponding attractor could be characterized by the fractal dimension of relatively low value.

After all, the understanding of low values of the correlation dimension as a sign of deterministic chaos has been challenged. There are two remarkable paper questioning the view that stochastic time series lead to a non-convergence of the correlation dimension [9, 10]. The authors showed that some stochastic systems may result in spuriously low estimates of correlation dimension. The method of the dimension estimate cannot distinguish between fractal attractors of deterministic systems and self-affine fractal random curves if their dimensions equal.

To test these findings in the case of relaxed EEG, we looked for regions that lead to highest linear correlation and mutual information between correlation dimension and spectral decay.

At first, to calculate the correlation dimension, the EEG data were embedded to m-dimensional space ($m=1, 2, \ldots, 7$). Following the proposal of Takens [11] m-dimensional vectors built-up from delay coordinates were used for the reconstruction. The vectors were constructed with a time lag $\tau = 10$, which corresponds to *20 ms*. This value was chosen according to the first minimum of mutual information between the original signal and its shifted versions, meaning, that their independence is maximized [12]. Then D_2 was calculated using the GP–algorithm. The resultant estimates of D_2 "saturated", i.e. they approached a constant value as embedding dimensions increase above m=5. After this manner a significant indication of relatively low values of correlation dimension (between 3 and 6) with the mean of 4.35 was found.

To evaluate the relationship between the correlation dimension estimates and the spectral decay we computed linear correlation and mutual information of both measures. As fig. 3 shows we found the highest mutual information and also the strongest negative correlation (value of about -0.8) when spectral decay from the whole spectrum (in our case about *5 Hz* to *250 Hz*) is taken. Fig. 4 illustrates, that both D_2 and γ behave in a similar manner and they probably reflect the same features of EEG data.

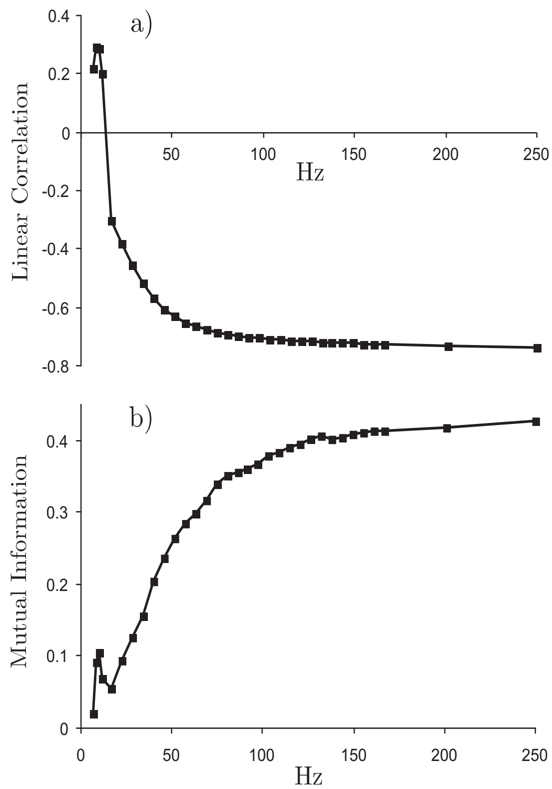

Fig. 3. a) Linear correlation between D_2 and γ
b) Mutual information between D_2 and γ
γ is computed from intervals of fixed *5 Hz* to ascending right limit (values of horizontal axes) of the spectral range.

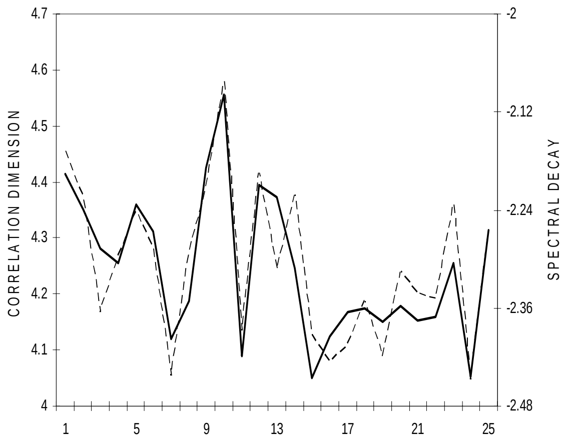

Fig. 4. Averages of spectral decay (full line) and correlation dimension (dashed line) values in the course of 25 days. Each day's value implies results of about 90 EEG time series of 87000 data.

3. Conclusions

In this study power-law decay prevailing over the exponential falling off was established from the relaxed EEG spectrum. Consequently, excellent correspondence was observed between the estimates of the correlation dimension and spectral exponent. It is obvious that, to a great extent the dimension estimate by GP-algorithm only reflects some spectral features of signal. In the case of observing low values of *D2* of EEG, the hypothesis of presence of scale-invariant fractal like structures should be preferred rather than the suggestion of low-dimensional deterministic chaos.

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