Evaluation of the positional deviation of numerically controlled axes

[1]R. Palenčár, [1]M. Halaj, [1]E. Kureková

[1]Department of Automation, Informatics and Instrumentation, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Nám. Slobody 17, 812 31 Bratislava, Slovak Republic

Email: rudolf.palencar@stuba.sk, martin.halaj@stuba.sk, eva.kurekova@stuba.sk

Abstract. *Nowadays many modern production machines require positioning in thousandths of millimeter. Therefore a big attention must be paid to prepare such a controlling software that drives individual axes of the production machine which minimises deviations between the desired position and actually reached position. The so called positional deviation (difference between the actual and target position) belongs to the important criteria that describe the performance of numerically controlled axes. The procedure for determination of such deviation is described in the international standard ISO 230-2:1997. This standard provides calculation of the positional deviation only in several discrete (measuring) points. Moreover it does not consider effects of the measuring instrument on the obtained results. Thus the new methodology must be adopted that enables estimation of the positional deviation in any point of the axis travel, together with the uncertainty of such estimate. Obtained results can be incorporated into a control system in the form of corrections enhancing positioning possibilities of individual axes.*

Keywords: numerically controlled axis, positional deviation, measurement uncertainty, estimation theory

1. Introduction

Testing of the positional deviation of the numerically controlled axis (either rotary or longitudinal) is ruled by the international standard ISO 230-2:1997 [1]. This standard provides guide for design of the test, testing conditions and also evaluation procedure for processing the measured data. In general, the testing procedure is based on repeated measurements of the actual position of the tested axis in several discrete points (target positions), located equally along the axis travel.

2. Evaluation of measured data according to the standard

The above mentioned standard introduces also procedures for evaluation of the measured data. The focus of presented evaluation is aimed namely at determining the maximum positional deviation over the whole axis (measurement) travel [3]. The evaluation of results covers calculation of the parameters related to the positional deviation in each of the measurement points P_i, covered also by the deviation boundaries $\bar{x}_i \uparrow +2 s_i \uparrow$; $\bar{x}_i \uparrow -2 s_i \uparrow$ (respectively $\bar{x}_i \downarrow +2 s_i \downarrow$; $\bar{x}_i \downarrow -2 s_i \downarrow$ in reversal direction).

Note that the deviation boundaries are calculated only from measured data, considering their variation as caused only by a tested machine and not influenced by the measurement instrument itself. Moreover, the standard assumed linear course of the positional deviation among the measurement (testing point), as well as linear course of the deviation boundaries.

Taking these limitations into account, the estimation theory [2] enables to determine the estimate of the positional deviation in any point along the axis travel, provided with the measurement uncertainty of such estimate (see Fig. 1). This enables to incorporate estimate of the positional deviation into a control program of the machine, thus enhancing its positioning accuracy.

3. Evaluation of the positional deviation in any point of the axis travel

Following positional deviations are measured in individual measurement points:

$$\begin{array}{l} P_1: \ \Delta_{11}, \Delta_{12}, ..., \Delta_{1j}, ..., \Delta_{1n} \\ P_2: \ \Delta_{21}, \Delta_{22}, ..., \Delta_{2j}, ..., \Delta_{2n} \\ \quad \vdots \\ P_m: \ \Delta_{m1}, \Delta_{m2}, ..., \Delta_{mj}, ..., \Delta_{mn} \end{array} \tag{1}$$

where

i 1 to m is the running number of the measurement point,

Δ_{ij} are positional deviations*,

j 1 to n is the running number of the measurement of positional deviation in a given measurement point. It is assumed that the same number n of measurements of the positional deviation is performed in each measurement point.

(*Remark: the positional deviation that is designated in the standard [1] as x_{ij}, is for sake of better clarity and understandability designated by different symbol Δ_{ij})

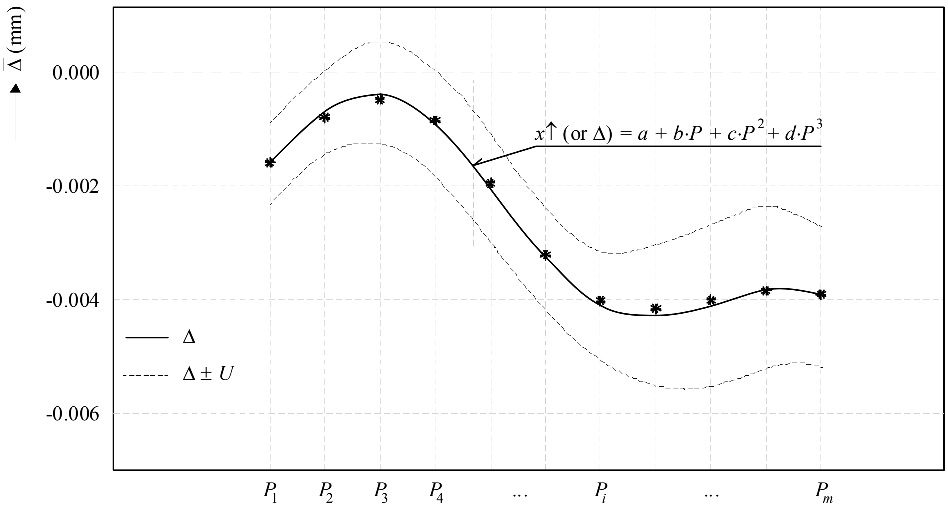

Fig. 1. Estimation of the positional deviation in any point of the axis travel

If we want to obtain the estimates of the positional deviations also in other points than the measurement ones, we must approximate course of estimates. The least squares method is suitable for such approximation. The curve in a form of polynomial of the third order will be placed over the points (P_1, $\overline{\Delta_1}$), (P_2, $\overline{\Delta_2}$), ..., (P_i, $\overline{\Delta_i}$), ..., (P_m, $\overline{\Delta_m}$):

$$\Delta = a + b \cdot P + c \cdot P^2 + d \cdot P^3 \tag{2}$$

where

Δ is the positional deviation of the target position and actual position in any point P and $P \in \langle P_1; P_m \rangle$,

a, b, c, d are unknown parameters of the polynomial.

Besides that we want to determine also expanded uncertainty U of estimate of the positional deviation Δ in any point P. To be able to do this, we must determine the estimates of polynomial parameters a, b, c and d, their uncertainties and covariances among them.

The positional deviation for each measurement j = 1 to n in each particular point P_i, with i = 1 to m, can be calculated as the difference between the target position and the measured actual position (see part 2 above) [1]:

$$\Delta_{ij} = P'_{ij} - P_i \tag{3}$$

where

P_i is the target (programmed) position,

P'_{ij} is the actual (measured) position.

The actual (measured) position P'_{ij} comprises two components:

$$P'_{ij} = P_{ij} + \delta_{mer.} \tag{4}$$

where

P_{ij} is the position indicated by the measuring instrument,

$\delta_{\mathrm{mer.}}$ is the measurement error in the particular point, in our case estimated as the maximum permissible error of the measuring instrument. This means that the error remains constant in any point.

According to the previous calculations of estimates of the positional deviation (as the arithmetic means), the set of equations for j = 1 to n positional deviations in m points (target positions) can be expressed as:

$$\begin{aligned} \overline{\Delta}_1 &= \overline{P}_1 - P_1 + \delta_{\mathrm{mer}} \\ \overline{\Delta}_2 &= \overline{P}_2 - P_2 + \delta_{\mathrm{mer}} \\ &\vdots \\ \overline{\Delta}_m &= \overline{P}_m - P_m + \delta_{\mathrm{mer}} \end{aligned} \tag{5}$$

where

$\overline{P}_i$ is the estimate (obtained as an arithmetic mean) of the actual (measured) positions in any given point P_i.

The set of equations describing the positional deviations (5) can be written in a matrix form:

$$\boldsymbol{x} = \overline{\boldsymbol{P}} - \boldsymbol{P} + \boldsymbol{i}\delta_{\mathrm{mer}} \tag{6}$$

where

$\boldsymbol{x}$ is the vector of the estimates of individual positional deviations (dimension m),

$\overline{\boldsymbol{P}}$ is the vector of the estimates of measured actual positions (dimension m),

$\boldsymbol{P}$ is the vector of target positions (dimension m),

$\boldsymbol{i}$ is the unit vector (dimension m) and

$$\begin{aligned} \boldsymbol{x} &= \left(\overline{\Delta}_1, \overline{\Delta}_2, \cdots, \overline{\Delta}_m\right)^{\mathrm{T}} \\ \overline{\boldsymbol{P}} &= (\overline{P}_1, \overline{P}_2, ..., \overline{P}_m)^{\mathrm{T}} \\ \boldsymbol{P} &= (P_1, P_2, ..., P_m)^{\mathrm{T}} \end{aligned}$$

When taking the matrix notation into account, the covariance matrix $\boldsymbol{U}(\boldsymbol{x})$ can be written in a form (assuming $\boldsymbol{P}$ as non-random vector, $\overline{\boldsymbol{P}}$ and δ are independent):

$$\boldsymbol{U}(\boldsymbol{x}) = \boldsymbol{U}(\overline{\boldsymbol{P}}) + u^2(\delta)\boldsymbol{i}\,\boldsymbol{i}^T \tag{7}$$

After specifying the individual terms (because P_i a P_j are independent):

$$\boldsymbol{U}(\boldsymbol{x}) = \begin{pmatrix} u^2(\overline{P}_1) & 0 & \cdots & 0 \\ 0 & u^2(\overline{P}_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & u^2(\overline{P}_m) \end{pmatrix} + u^2(\delta)\, i\, i^{\mathrm{T}} =$$

$$= \begin{pmatrix} u^2(\overline{P}_1)+u^2(\delta) & u^2(\delta) & \cdots & u^2(\delta) \\ u^2(\delta) & u^2(\overline{P}_2)+u^2(\delta) & \cdots & u^2(\delta) \\ \cdots & \cdots & \cdots & \cdots \\ u^2(\delta) & u^2(\delta) & \cdots & u^2(\overline{P}_m)+u^2(\delta) \end{pmatrix} \tag{8}$$

The uncertainties $u(\overline{P}_i)$, where i = 1, 2, ..., m, in the matrix (8) are evaluated by the type A method form measured data:

$$u(\overline{P}_i) = \sqrt{\frac{1}{n(n-1)}\sum_{j=1}^{n}(P_{i,j} - \overline{P}_i)^2} \tag{9}$$

This procedure yields to the estimates of unknown parameters a, b, c, d, uncertainties of those estimates and covariances among them.

4. Conclusion

The international standard [1] ISO 230-2:1997 provides scheme for calculation of positional deviation of the numerically controlled axis only in several discrete points. The procedure presented in this paper enables to estimate the positional deviation in any point of the axis travel, no matter whether rotational or longitudinal. Moreover it provides the estimate of the positional deviation with the respective uncertainty of such estimate. This gives the designer or programmer of the machine control system the information about behavior of the machine in any point of the axis travel. Thus appropriate corrections can be included into the control program or the adequate design corrections can be performed in the design of the machine.

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