An Optical Method for the Measurement of Shape Deviations of Elliptical Mirrors

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Abstract. *The contribution deals with the fast optical method of shape deviations measurement of elliptical mirrors. The method for the shape deviations measurement utilises a scanned laser beam reflected from the elliptical mirror surface and a digital CCD camera for the beam position measurement in focal plane of the mirror under test. A mathematical evaluation including Fast Fourier Transformation (FFT) of the measured data was used to evaluate the shape deviations and waviness of the mirror's surface. Results of the optical measurement were compared with the data obtained from the CMM (Coordinate Measuring Machine) measurement.*

Keywords: *contactless measurement, elliptical mirror, shape deviations, waviness*

1. Introduction

Elliptical optical mirrors are high performance optical components playing important role in many optical experimental setups and measuring systems. A limiting factor to expand more their use is high cost of the manufacturing and also difficulties of the form measurement of aspherical surfaces. Preferred method of shape measurement of flat or spherical optical surfaces is full-aperture interferometry, which offers fractional nanometer resolution. This interferometric method utilises a null optics – flat or spherical surface as the reference wavefront and usually it is difficult to apply this method in the case of aspherical – elliptical surfaces. Common method for the measurement of the aspherical surfaces is the use of coordinate measuring machine CMM. Use of CMM brings some difficulties, e.g. mechanical probing is time consuming, can harm the surface under test, and comparing to the optical interferometric measurement CMM is not so sensitive.

2. Elliptical mirror and optical method of its shape deviations measurement

From mathematical point of view ellipse is one type of conic section and is defined as set of points in a plane satisfying the condition that the sum of distances from two fixed points – foci is given positive constant 2a. If the major axis lie along the x-axis and coordinates of foci are $F_1[-f,0]$, $F_2[f,0]$ the equation of ellipse in Cartesian coordinates following from definition is

$\sqrt{(x+f)^2+y^2}+\sqrt{(x-f)^2+y^2}=2a$ and after simplifying we obtain $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is major semiaxis, $b=\sqrt{a^2-f^2}$ is minor semiaxis, $2f$ is distance between two foci.

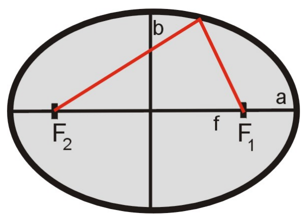


Fig. 1 Concave elliptical mirror

Important characteristic of the ellipse is also eccentricity defined as $e=\frac{f}{a}=\sqrt{1-\frac{b^2}{a^2}}$

If the ellipse is rotated about the major axis we obtain a prolate ellipsoid. It can be mathematically proved that light ray that passes through one focus point of an elliptical mirror is reflected in such direction that it passes through the second focus point. This property will be exploited for the elliptical mirror shape deviations measurement.

Intensity profile of a Gaussian laser beam

It is obvious that if we want to evaluate the position of the laser beam spot it is essential to know the laser beam transverse intensity profile and laser beam divergence. In the ideal case the Gaussian laser beam has in the transverse direction (to the propagation) intensity profile described by the equation [2]

$$I(r) = I_0 . e^{-\frac{2r^2}{w^2}} = \frac{2P_0}{\pi w^2} \cdot e^{-\frac{2r^2}{w^2}} \tag{1}$$

where w is radius of the laser beam defined as the radial distance r from axis where the intensity I has

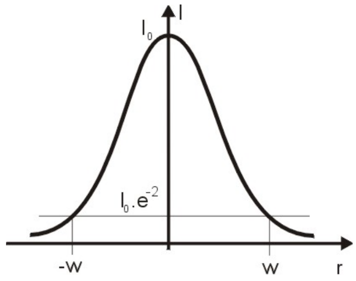


Fig.2 Intensity profile of a Gaussian laser beam

fallen to the value e^{-2} =0.135 of the axial (maximal) value I_0 (see Fig. 2).

The center of the laser beam spot can be evaluated by several mathematical methods [4]. We have used method of "center of gravity", which evaluates the center of the beam by the equation

$$r = \frac{\sum_k I_k r_k}{\sum_k I_k} \tag{2}$$

where I_k is intensity measured by *k*-th CCD pixel with coordinate r_k and $|r_{k+1} - r_k|$ is equal to a dimension of a single CCD pixel. Using this evaluation method, the final uncertainty of the laser beam spot center evaluation can be less than the fractional part of a single pixel size [4].

Divergence of the laser beam

Due to a diffraction of light waves the laser beam spreads transversally and as a result we cannot have ideally collimated laser beam. Transverse spreading of the Gaussian laser beam can be described by the equation [1], [2]

$$w(z) = w_0 \cdot \sqrt{1 + \left(\frac{\lambda}{\pi w_0^2} \cdot z \right)^2} = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2} \tag{3}$$

where $z_R = \frac{\pi w_0^2}{\lambda}$ is Rayleigh range (it is distance over which the beam radius spreads by a factor $\sqrt{2}$), λ is wavelength of the laser beam and w_0 is radius at so called waist which is the most narrow part of the beam, z is coordinate along the beam propagation. Equation (3) is valid for an ideal Gaussian laser beam, for real beam the propagation equation is given by

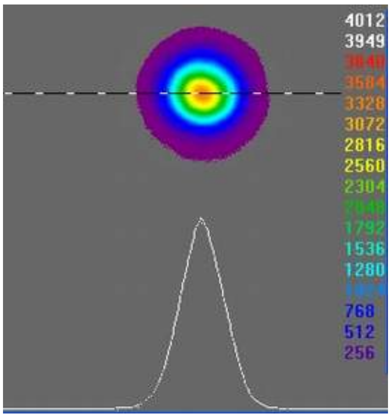
$$w_R(z) = w_{0R} \cdot \sqrt{1 + \left(\frac{\lambda M^2}{\pi w_{0R}^2} \cdot z \right)^2} \tag{4}$$

where propagation factor M^2 was introduced

$$M^2 = \frac{w_{0R} \cdot \theta_R}{w_0 \cdot \theta} \quad (5)$$

R denotes real beam parameters, θ is far-field divergence angle.

To ensure that the laser beam in the optical setup is fairly collimated, the working distance in the optical setup should not exceed the Rayleigh range. If the HeNe laser path is in the range of 1 m, than the diameter of the laser beam waist should be around $2w_0 = 2\sqrt{\lambda z / \pi}$ =0.9mm



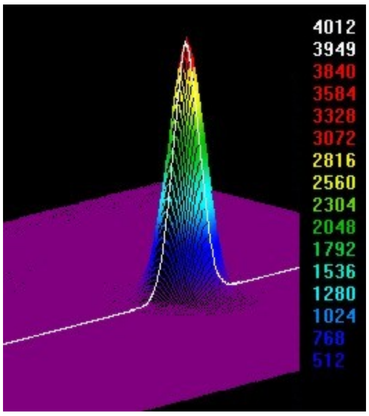


Fig. 3a,b HeNe TEM_{00} Gaussian laser beam profile measured by the CCD camera. 2D (left) and 3D (right) representation of the laser beam profile.

Optical setup

Basic optical scheme is shown in the Fig. 4. Measuring optical beam is generated by a HeNe laser (λ=633 nm, P=5 mW). Laser beam is incident on a small plane mirror located in the first focal point

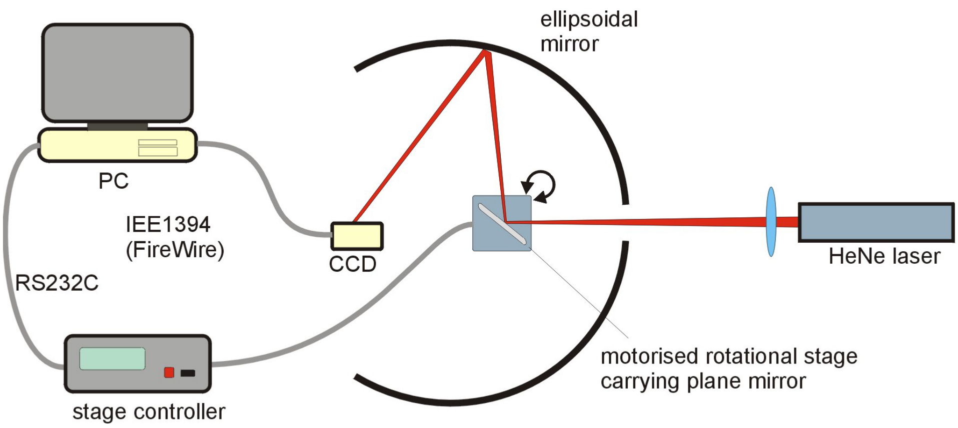


Fig. 4 Basic setup of an elliptical mirror measurement

of the elliptical mirror. Plane mirror is gripped to the motorized rotational stage, controlled by a stage controller. The beam is deflected in to desired direction and after reflection from the elliptical mirror is incident on a matrix CCD camera located in the second focal point. The deviation of the laser beam from the focus point position is measured by the CCD and than evaluated in the PC.

3. Experimental results and comparison with the CMM measurements

The basic measurement geometry is shown in the Fig. 5. Laser beam passing through one focus and reflected from the ideal elliptical surface should pass through the second focus point. If the laser beam is reflected from a deformed surface which derivative y'(x) differs from the ideal surface derivative y_i'(x) at Δ_α (≈ tan Δ_α), than the reflected beam is deflected at $2.\Delta_\alpha$ and the beam incident on CCD is shifted at Δ_x (see Fig.5)

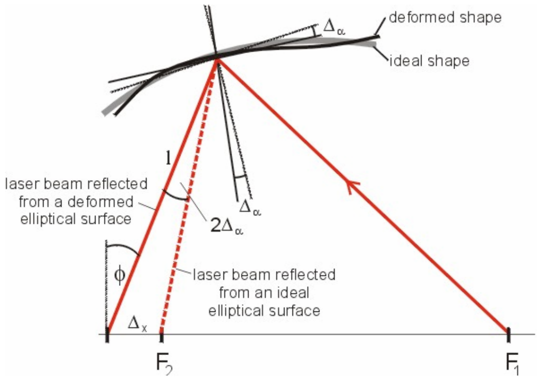


Fig. 5 Measurement geometry

$$\Delta_x = \frac{2\Delta_\alpha l}{\cos\phi} \tag{6}$$

and because $\Delta_\alpha \approx y'(x) - y_i'(x)$, the searched shape deviation can be obtained by the integration

$$y(x) - y_i(x) = \int [y'(x) - y_i'(x)]dx + c = \int \frac{\Delta_x \cos\phi}{2l} dx + c \tag{7}$$

The measurement of Δ_x is done in a discrete way (using finite step $\Delta\phi$) and the integral in the equation (7) has to be calculated numerically.

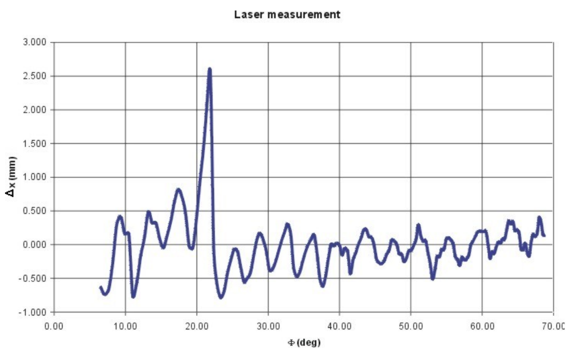


Fig.6 Graphic representation of the measured data

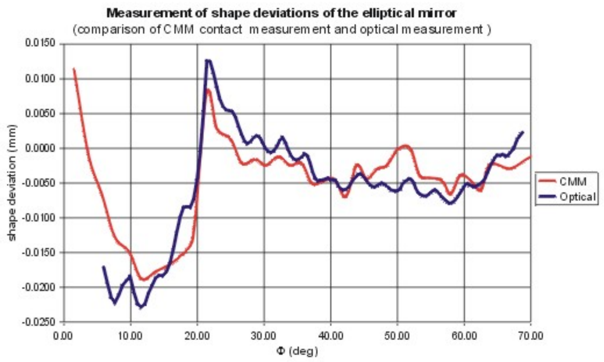


Fig.7 Shape deviations of GEM mirror – evaluated optical measurement and CMM measurement

The measurement was done on the double elliptical mirror GEM [3] with major semiaxis a =112.5 mm and eccentricity e=0.3333. There was used source of the measuring beam HeNe laser (Newport, λ=633 nm, P=5 mW), digital camera Point Gray Research type SCOR – 20SO (1600x1200 pixels, pixel size 4.4 x 4.4 μm), motorized stage controller type Sigma Koki SHOT-204MS connected via RS232C interface to a PC. The process of the measurement was fully automated, for this purpose we have developed our self-made program in Visual Basic, using ActiveX from Spiricon software package LBA-FW-SCOR to collect data from the CCD camera. In the Fig.6 is the graphical representation of the measured data. It shows a shift of reflected laser beam position with respect to the focal point at different angles ϕ (ϕ determines the current position on the elliptical surface under test). We can see the maximum of the deviation is reached at ϕ equal to around 22 deg. It can be explained by so-called backlash effect (it is lost of motion after reversing of direction due to play in screws) and hysteresis effect (position error when reversing direction due to relaxation of elastic forces) of the numerically controlled (NC) machine used for manufacturing of the elliptical mirror. The measured data are collected and evaluated in PC and according to the equation (7) the shape deviations are calculated. Graphical representation of elliptical mirror shape deviations measured by the optical method is shown in the Fig.7 together with results obtained by the coordinate measuring machine CMM (measuring stylus tip radius = 3 mm). Both optical and CMM measurements are well correlated regarding the "backlash" shape deviation position and periodicity of the shape deviations.

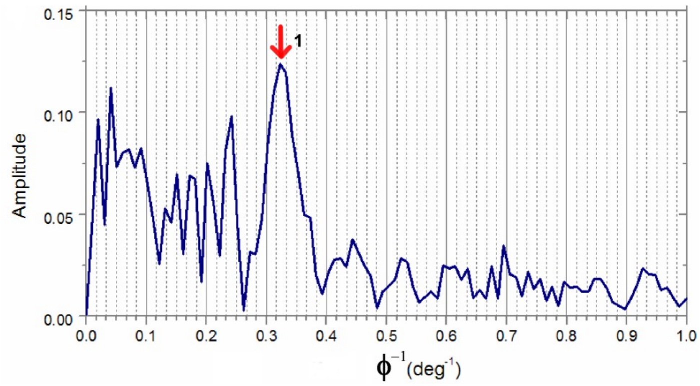


Fig. 8 Fourier analysis of the shape deviations measurement – evaluation of characteristic wavelength of the surface waviness (arrow)

To evaluate the waviness of the surface the Fourier transformation (FFT) was applied to the measured data. FFT has shown that the characteristic wavelength of the elliptical mirror waviness is:

$$L_{ch} = \frac{\pi}{180} \cdot \frac{l}{\phi_{ch}^{-1}} \cong \frac{3.14}{180} \cdot \frac{112.5}{0.33} mm \cong 6mm$$

Conclusion

Optical method for fast contactless measurement of shape deviations of elliptical mirrors was developed and successfully tested. Results of the optical measurement are well correlated with the measurements done by coordinate measuring machine CMM. Advantage of the proposed optical measuring method is very fast contactless measurement, high sensitivity and low cost of the measurement. Next improvement of the optical measuring system will add a roughness measurement of the mirror's surface based on the evaluation of the intensity distribution of the laser beam reflected from the elliptical mirror surface.

Acknowledgements

Authors are grateful to the National Institute of Advanced Industrial Science and Technology, Japan for support of this work in Japan (optical experiment) and to the Slovak Grant Agency for Science for support of the project VEGA 2/7082/27 (evaluation of the measured data).

References

[1] Donges A., Noll R.: Lasermesstechnik. Hutig, Heidelberg, 1993

[2] Saleh B. E. A., Teich, M. C.: Fundamentals of Photonics. John Wiley & Sons, New York, 1991

[3] Kawate E.: Comparison of reflection methods for determining optical constants with STAR GEM. *Measurement Science Review,* 5, 2005, 1-5.

[4] HAIN M., BARTL J., KEPPERT M.: Mathematical methods of measurement evaluation in the light-cut profile meter. In: Wave and quantum aspects of contemporary optics, 11th Slovak - Czech - Polisch Optical Conference, Stará Lesná 1998, 304-309.