

## Uncertainty Measurement of Weighing Results from an Electronic Analytical Balance

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**Abstract:** *Uncertainty of a measurement result is a major issue in all fields of measurement. Weighing is a critical step in any chemical laboratory and analytical methods. It's an important uncertainty sources in uncertainties measurement in an analysis.*

*In this paper we tried to present the influence factors that affect in uncertainty measurement of a mass determination. Technical specifications of a balance such as: Readability, Repeatability, Non-linearity, Sensitivity Tolerance, Temperature Coefficient of Sensitivity and effects of environmental factors such as: air humidity, air pressure and air buoyancy were estimated. In the absence of some disturbance factor like water adsorption phenomena, evaporation, magnetic and electrostatic effects, the measurement uncertainty for an electronic balance in our laboratory is in the  $10^{-4}$  to  $10^{-3}$  range (100 to 1000 ppm).*

**Keyword:** *Weighing, Uncertainty Measurement, Non-linearity, Electronic Balance*

### 1. Introduction

Weighing is a common task in any chemical laboratory and its results may determine the acceptability of a products or the outcome of a test. Weighing data are associated with some uncertainty [1], as this is common with all other working procedures and their data. Also all manufacturer specifications are based on “idealized“ weighing conditions. Otherwise, comparisons could not be made between different instruments because the methods actually used in the field often differ from those used by the manufacturer. Hence it is important to have procedures for assuring the quality of these weighing devices in our laboratory conditions.

Estimation of measurement uncertainty [2] has become a key requirement when aiming to obtain a laboratory accreditation is desired. Uncertainty measurements are very important for quality control and quality assurance programs (QA/AC) and quantifying uncertainty in analytical measurement in a laboratory or acceptability of a products or the outcome of a test in industry.

Weighing is an important step in an experiment and its results, from electronic balances or other weighing devices are often of critical importance for next steps of an analysis.

Many influences affect accuracy and precision of weighing results. These influences provide a bias, an average value different from zero and contribution to uncertainty.

### 2. Experimental equipment

Electronic Balance: B154-S Mettler Toldo with a maximum load of 151g

Calibration weight: Scale range from 200 mg to 150 g

Humidity measuring instrument: Testo 635, for measurement of temperature (°C) and relative humidity (%)

Barometer: Mercury barometer, Lufft

We checked the temperature, air humidity and pressure of room balance in morning and evening for six months. We also calibrate the electronic balance every week. Calculations and evaluations that may be used to calculate uncertainty are done by calculations from a series of repeated observations, using statistical methods or by evaluations from calibration data and balance specification technical sheet.

### 3. Results and Discussion

The influences that affect accuracy and precision of weighing results are divided to three types. Influences originate from balance, such as readability, repeatability, non-linearity, sensitivity accuracy and temperature coefficient, influences that caused by environment such as: air humidity, air temperature, air pressure, heat radiation and direct sunlight and influences that affect the weighing object such as: air buoyancy, way of loading the sample, operator and so on.

The uncertainty in the result of a measurement, or the uncertainty of the measured value of the specific quantity subject to measurement, expresses doubt about how well the result represents the value of the measured value. The uncertainty reflects the lack of exact knowledge of the value of the measured values owing to random and systematic effects. Whereas uncertainty designates a general concept, its quantitative measure is called standard uncertainty. The standard uncertainty is an estimate of the standard deviation, the positive square root of the variance, of the probability distribution of the possible values of the measured values.

There are two types of evaluations that may be used to calculate uncertainty: Type A evaluations of uncertainty are done by calculations from a series of repeated observations, using statistical methods. Type B evaluations of uncertainty are derived from other sources e.g from calibration data.

We described and calculated these influence parameters and their interplay to the combined standard uncertainty for an electronic balance in our laboratory.

#### 3-1. Readability

Readability is a measure of an instrument's ability to display incremental changes in its output value. Rounding of the weighing value to the last digit inherently introduces quantization noise. This uncertainty is plus-minus half of the last digit (d).

Readability is a bias thus with triangular distribution and two reading, its variances can be approximated:

$$S^2_R = \left(\frac{1}{\sqrt{3}} \cdot \frac{1}{2} \cdot d\right)^2 = \frac{1}{12} \cdot d^2 \quad (1a)$$

And with considering two rounding and differences between two values, the uncertainty variance for readability is:

$$U^2_R = 2S^2_R = \frac{1}{6} \cdot d^2 \quad (1b)$$

### 3-2. Repeatability

Repeatability is the variation in measurements taken by a single person or instrument on the same item and under the same conditions. Deviating results when the same object is weighed under repeatability conditions is power dissipated by electronic circuits, wind draft at the site of the balance (especially with resolutions of 1mg and below), vibrations of the support on which the balance stands, temperature differences, pressure fluctuations of the ambient air are produced this parameter.

The standard uncertainty  $S_{Rp}$  due to repeatability is normally evaluated as the simple standard deviation of the balance readings for  $n$  successive loadings of the same weight. Repeatability is a random contribution therefore the variance of repeatability is:

$$U^2_{Rp} = S^2_{Rp} = \frac{1}{(n-1)} \cdot \sum_{i=1}^n (R_{pi} - \overline{Rp})^2 \quad (2)$$

Where  $R_{pi}$  is the  $i$  th balance reading, and  $R_p$  is the average of the  $n$  balance readings.

### 3-3. Non-linearity

Non-linearity is a change that is not based on a simple proportional relationship between cause and effect. This parameter is caused by load dependent deformations of the weighing cell, the electro-dynamics or transducer's inherent non-linearity between current and force [2]. Linearity indicates how much a balance or a scale deviates from the theoretically linear slope of the characteristic calibration curve (The relationship between the input (the load) and the output (the reading) of a balance is called its characteristic curve). An ideal balance exhibits a perfectly linear relationship ( $y=x$ ) between displayed value and the load on the platform and the equation between displayed and load values. Non-linearity is the deviation of the characteristic curve from this straight line. Thus it concerns the term:  $y_{NL2}-y_{NL1}$

With assumption of rectangular distribution and in order them to standard uncertainties the non-linearity contribution of single reading is:

$$S^2_{NL} = \left(\frac{1}{\sqrt{3}} \cdot NL_{max}\right)^2 \quad (3a)$$

The non-linearity contribution of a complete weighing operation, obtained by taking the differences of two readings, adds up to thus the non-linearity variances is:

$$U^2_{NL} = 2S^2_{NL} = \frac{2}{3} \cdot NL^2_{max} \quad (3b)$$

### 3-4. Eccentric Load

This parameter, also called “off-center load error” or “pan position error”, means the change in readout when the same load is placed in various positions on the weighing pan or load plate. It caused by finite tolerances in manufacturing, assembly and adjustment, deformations of the weighing cell due to the position of the weighing

object relative to the weighing pan and load dependent deformations of the weighing cell.

This effect can be easily avoided by the careful placing of the object centrally onto the pan. Therefore eccentric load is not considered in estimation of total uncertainty.

### 3-5. Sensitivity Accuracy

This parameter caused by adjustment tolerance, or determination accuracy of the calibration weights, buoyancy of the calibration weight, spontaneous and temperature induced drift, mainly of the lever's mechanical advantage, mechanical shocks.

The sensitivity is the change in a displayed value divided by the change in the load signal generated by the mass on the pan. If a balance or scale with a digital display has been correctly adjusted, the sensitivity must always be exactly 1.

Long-term drift, that means the sensitivity over the time may change spontaneously, and zero drift, that take place within a short interval of time without re-zeroing the balance, are two other aspects of sensitivity accuracy. When a balance is adjusted with calibration standards the long-term drift is eliminated. Zero point drift is only important for long-term measurements involving a constant load, as in sorption and thermo gravimetric measurements. Thus fore a simple weighing we can eliminate these two terms for sensitivity and consider the three most important influences such as:

#### Sensitivity Tolerance

The sensitivity of the balance has some tolerance or uncertainty. The sensitivity tolerance is a permissible deviation of the actual sensitivity from the nominal sensitivity. It is given as a percentage with respect to the nominal sensitivity.

This parameter also treated as a rectangular distribution and is proportional to the net weighing value [3]. From balance specification the maximum deviation is  $3 \cdot 10^{-4}$ .

$$U^2_{(ST)} = \left( \frac{1}{\sqrt{3}} \cdot W_{\text{net}} \cdot ST_{\text{max}} \right)^2 \quad (4a)$$

$$U^2_{(ST)} = 3 \cdot 10^{-8} \cdot W_{\text{net}}^2 \quad (4b)$$

#### Temperature Coefficient

The temperature coefficient is the relative change of a physical property when the temperature is changed by 1 K. The weighing transducer of the balance may exhibit a temperature dependent characteristic. If a weight is divided by the change in temperature, this will yield the value of the temperature coefficient [3]. This influence is expressed by the (sensitivity) temperature coefficient (TC) of the balance and is defined as the ratio of relative change of sensitivity  $\Delta S/S_0$  and the temperature difference  $\Delta T$ :

$$TC = \left( \frac{\Delta S}{S_0} \right) / \Delta T \quad (5a)$$

The TC of sensitivity only states how sensitive the slope of the balance's characteristic curve reacts to temperature.

As earlier mentioned  $S_0$  for a correctly adjusted balance is 1. Therefore the variance of TC with assumption of rectangular distribution can be obtained as follows:

$$U^2_{TC} = \left( \frac{1}{\sqrt{3}} \cdot TC \cdot \Delta T \cdot W_{net} \right)^2 \quad (5b)$$

TC is described by temperature drift of sensitivity. From balance specification TC is  $1.5 \cdot 10^{-4}/^\circ\text{C}$  and there are  $3^\circ\text{C}$  temperature changes in our laboratory. Therefore the equation 5b can be rewritten as:

$$U^2_{TC} = 6.75 \cdot 10^{-8} \cdot W_{net}^2 \quad (5c)$$

### Air Buoyancy

If a weighing takes place in a vacuum on the surface of earth, a body with mass  $m$  produces a weight force of  $G = mg$ . If the same body is weighed in the atmosphere, it will be subject to an air buoyancy force, which is opposed to the weight force and diminishes the latter.

The buoyancy correction factor "Bu" is defined by the following equation using the densities of sample  $\rho_s$  reference weight  $\rho_r$  and air  $\rho_a$  [4, 5]:

$$\text{Bu} = \frac{1 - (\rho_a / \rho_r)}{1 - (\rho_a / \rho_s)} = \frac{\rho_a (\rho_r - \rho_a)}{\rho_r (\rho_s - \rho_a)} \quad (6)$$

Mass of reference weights (usually a steel alloy with a density of  $8000 \text{ kgm}^{-3}$ ) is known with low uncertainty. Both the reference weights, as well as the sample are affected by buoyancy. The mass of a sample that has the same density can be determined without the necessity of a correction because the influence of buoyancy onto the sample and the calibration weight cancel each other. The weighing value of objects with a density other than  $8000 \text{ kgm}^{-3}$  deviate from their mass. Therefore if the true, physical mass of the sample is to be determined, buoyancy must be corrected.

The variance resulting from buoyancy at the time of weighing the reference standard can be derived by taking the partial derivatives. Its three partial derivatives are:

$$\frac{\partial \text{Bu}}{\partial \rho_s} = \frac{\rho_a (\rho_r - \rho_a)}{\rho_r (\rho_s - \rho_a)^2} \quad (7a)$$

$$\frac{\partial \text{Bu}}{\partial \rho_r} = \frac{\rho_s \cdot \rho_a}{\rho_r^2 (\rho_s - \rho_a)} \quad (7b)$$

$$\frac{\partial \text{Bu}}{\partial \rho_a} = \frac{\rho_s (\rho_r - \rho_s)}{\rho_r (\rho_s - \rho_a)^2} \quad (7c)$$

Then the variance resulting from buoyancy can be calculated as follows:

$$U_{Bu}^2 = \left(\frac{\partial Bu}{\partial \rho_s}\right)^2 U^2(\rho_s) + \left(\frac{\partial Bu}{\partial \rho_r}\right)^2 U^2(\rho_r) + \left(\frac{\partial Bu}{\partial \rho_a}\right)^2 U^2(\rho_a) \quad (8)$$

As shown in equation 8, for obtaining variance of buoyancy it is necessary to calculate the uncertainty of density of reference weights, density of the sample to be weighed and air density. Most reference weights are made today from a steel alloy with density  $8000 \text{ kgm}^{-3}$  with a reasonable standard uncertainty  $10 \text{ kgm}^{-3}$ .

Because even for many pure chemicals the density is unknown or the reliability of the published data is difficult to estimate and nobody has the time and interest to determine it, we assume that the object density is only rarely known to an uncertainty of  $\pm 1\%$  and that  $\pm 10\%$  or even worse is what can be expected in reality.

### Air Density

The density of air increases with increasing pressure and decreasing temperature and decreasing relative humidity. The following empirical equation can be used [5, 6]:

$$\rho_a (\text{kg.m}^{-3}) = 10^{-3} \frac{A.P - B.h_r \cdot \exp(C.T)}{273.15 + T} \quad (9)$$

P: Air pressure in Pa, hr: Relative air humidity in %, T: Air temperature in °C  
A, B and C are constant (A: 3.4848, B: 9.024, C: 0.0612)

This equation is valid for the conditions  $90 \text{ KPa} \leq P \leq 110 \text{ KPa}$ ,  $10^\circ\text{C} \leq T \leq 30^\circ\text{C}$ , and  $h_r \leq 80\%$ .

Air density depends largely on air pressure and temperature rather than air humidity. Because the second term in equation 9 ( $B.h_r \cdot \exp(C.T)$ ) compare to first term ( $A.P$ ) is very small. Thus the variance of air density can be determined:

$$U^2(\rho_a) = (\rho_a)^2 \left[ \left(\frac{\partial P}{P}\right)^2 + \left(\frac{\partial T}{T}\right)^2 \right] \quad (10)$$

The mean air density in our laboratory is  $1.027 \text{ kgm}^{-3}$  with extremes of  $1.033$  and  $1.021 \text{ kgm}^{-3}$  ( $86936 \text{ Pa}$ ,  $19^\circ\text{C}$ ,  $35\%$  air humidity and  $87950 \text{ Pa}$ ,  $25^\circ\text{C}$ ,  $45\%$  air humidity, respectively). The standard uncertainty of the air density according to equation 10 is  $0.019 \text{ kgm}^{-3}$ .

### 3.6. Combined weighing standard uncertainty

To obtain the combined variance of a sample weighing, all individual variances earlier discussed are added. This provides a reasonable estimate of the combined variance if the individual contributions are mutually independent.

Some of the influences, such as readability, repeatability, non-linearity, zero drift, are *additive* influences and their contribution simply adds to the reading, because they are independent of sample mass and some of influences are *multiplicative* in their effect as they are proportional to the sample mass and therefore can be considered to influence the sensitivity of the transfer characteristic such as air buoyancy, temperature coefficient and sensitivity tolerance [7].

We assume that all individual variances that discussed earlier for both type of additive and multiplicative are independent and statistically uncorrelated. Therefore to

obtain combined variance of a sample weighing, all individual variances are added. Thus fore combined variance of additive influences we have:

$$U^2_{(Tadit)} = U^2_R + U^2_{RP} + U^2_{NL} \quad (11)$$

Temperature:  $22 \pm 3$  °C

Air humidity:  $40 \pm 5$  %

Air pressure:  $863 \pm 5$  mbar

Calibration Weight Tolerance: 1.5 ppm

Calibration Weight Density:  $8006 \text{ kgm}^{-3}$

Std. Dev. of Cal. Weight Density:  $10 \text{ kgm}^{-3}$

**Table1.** Data sheet specifications for B154-S balance

Property	Specification
Readability	0.1 mg
Repeatability	
	Up to 10 g
	Up to 150 g
	0.065 mg
	0.102 mg
Non-linearity	
	Within 10 g
	Within 150 g
	0.12 mg
	0.3 mg
Temperature coefficient	$1.5 \cdot 10^{-4} / \text{°C}$
Sensitivity Tolerance	$3 \cdot 10^{-4}$

According to repeatability and non-linearity in different ranges the total uncertainty of additive influences is:

$$U_{(Tadit)} = 1.244 \cdot 10^{-4} \text{ g} \quad \text{for } 0 - 10 \text{ g ranges}$$

$$U_{(Tadit)} = 2.684 \cdot 10^{-4} \text{ g} \quad \text{for } 10 - 150 \text{ g ranges}$$

As mentioned earlier multiplicative influences such as air buoyancy, temperature coefficient and sensitivity are proportional to the sample mass. Therefore for calculating these influences the sample mass, density of weighing object and the condition of balance environment such as temperature, air humidity and air pressure must be considered.

$$U^2_{(Tmulti)} = U^2_{(sensitivity-accuracy)} = U^2_{TC} + U^2_{ST} \quad (12a)$$

The total uncertainty of multiplicative influences is:

$$U^2_{(Tmulti)} = W^2_{(net)} (6.75 \cdot 10^{-8} + 3 \cdot 10^{-8}) = 9.75 \cdot 10^{-8} \cdot W^2_{(net)} \quad (12b)$$

An electronic analytical balance does not directly measure the mass of a sample; instead it measures its weight force. Therefore the display shows the so-called weighing value [9]. In most cases these data are not identical if the sample is weighed in air because the resulting air buoyancy (in the following termed buoyancy) gives rise to a deviation. Thus when the sample is weighed in air because the resulting air buoyancy its mass,  $M_s$  simply relate to its weigh,  $W_s$  as:

$$M_s = Bu \cdot W_s \quad (13)$$

$$U^2_{M_s}/M_s = \left(\frac{U_{Bu}}{Bu}\right)^2 + \left(\frac{U_{W_s}}{W_s}\right)^2 \quad (14a)$$

$$U^2_{M_s}/M_s = \left(\frac{U_{Bu}}{Bu}\right)^2 + \frac{[U^2_{(Tmulti)} + U^2_{(tadit)}]}{W^2} \quad (14b)$$

From equation 13 and considering that buoyancy is only relevant for net weight we can write:

$$U^2_{M_s} = U^2_{(Tadit)} + M_s^2 \cdot \left[\left(\frac{U_{Bu}}{Bu}\right)^2 + U^2_{(Tmulti)}\right] \quad (14c)$$

For more clear understanding, we have explained two examples:

2 g of aluminum profile with density of  $2700 \text{ kgm}^{-3}$  are weighed into a vial of 15 g (example 1) and without a vial (example 2).

Environmental conditions are: temperature:  $22 \pm 3 \text{ }^\circ\text{C}$ , air humidity:  $40 \pm 5 \%$ , atmospheric pressure:  $863 \pm 5 \text{ mbar}$ . The mean air density is  $1.027 \text{ kgm}^{-3}$  with extremes of 1.021 and  $1.033 \text{ kgm}^{-3}$ . The standard uncertainty of air density is  $0.019 \text{ kgm}^{-3}$  and we assume 1 % accuracy for density of aluminum profile, giving a standard uncertainty of  $27 \text{ kgm}^{-3}$ .

From equations 6, 7a, 7b, 7c and 8 we can calculate the buoyancy correction factor and relative uncertainty variance of buoyancy. The buoyancy correction factor and relative uncertainty variance of buoyancy are 1.00025 and  $8.33 \cdot 10^{-6}$  (8.33 ppm).

Calculation of standard uncertainty of the mass of aluminum profile in a vial or without a vial according to equation 14 in gives a combined uncertainty of 0.234 mg (relative uncertainty of  $1.17 \cdot 10^{-4}$  or 117 ppm) and 1.151 mg (relative uncertainty of  $1.0066 \cdot 10^{-3}$  or 1006 ppm) respectively.

$$U^2_{(M_s)} = (1.244 \cdot 10^{-4})^2 + 2^2 \cdot [(8.33 \cdot 10^{-6}/1.00025)^2 + 9.75 \cdot 10^{-8}] \quad \text{example 1}$$

$$U^2_{(M_s)} = (2.684 \cdot 10^{-4})^2 + 15^2 \cdot [(8.33 \cdot 10^{-6}/1.00025)^2 + 9.75 \cdot 10^{-8}] \quad \text{example 2}$$

#### 4. Conclusion

As discussed earlier weightings, the determination of the mass of a weighing object, are subject to influences from various sources. Generally, they cause a measurement bias as well as a measurement uncertainty.

These influences are of relevance for the weighing result depends on several conditions, which should be checked for every application. The higher the required measurement resolution of the weighing is, the more likely the influences will manifest themselves.

The measurement uncertainty is determined by the performance of the balance, properties of the weighing object and the state of the environment. The properties of the balance are partly of random nature (readability, repeatability) and partly of systematic nature (non linearity, sensitivity tolerance, temperature coefficient). However, within setting a balance properly, their expectation is zero, as these deviations get adjusted for a zero target value. Therefore the systematic deviations can be treated like random deviations and may be included into the uncertainty determination.



**Table 2.** The Balance Uncertainties

Quality	Standard Uncertainty	Aproximate Distribution	Range of Uncertainty Variance	Sensitivity Coefficient
Readability	$S_R$	Rectangular	$1.666 \cdot 10^{-3}$	-
Repeatability	$S_{RD}$	Normal	$4.22 \cdot 10^{-3}$ - 0.0104	-
Non-linearity	$S_{NL}$	Rectangular	$9.6 \cdot 10^{-3}$ -0.06	-
Sensitivity Tolerance	$S_{ST}$	Rectangular	-	$3 \cdot 10^{-8}$
Temperature coefficient	$S_{TC}$	Rectangular	-	$6.75 \cdot 10^{-8}$

The relative uncertainty, the uncertainty normalized to sample mass, remains constant for large sample masses, while it increases for decreasing sample masses. As shown in two above example and Table 2, with small sample masses, uncertainty is dominated mainly by repeatability, non-linearity and to some extent by readability (type of the balance), while with large sample masses the influences of sensitivity accuracy and air buoyancy prevail. Especially with small sample densities, variations of the ambient air density, caused by atmospheric conditions, can influence buoyancy heavily, such that this influence dominates uncertainty with large sample masses.

Relative uncertainty increases with decreasing sample mass. With large sample masses, uncertainty is dominated by sensitivity accuracy and, depending on the sample density and the conditions of the ambient air, air buoyancy. Relative uncertainty rolls off and stays essentially constant with increasing sample mass.

For more accurate and precise weighing measurement it should be used a more sensitive balance which has less value of readability, reparability, non linearity and sensitivity accuracy and for any application all of these influences should be checked.

## References

- [1] A. Kallner (2006). Uncertainty in Measurement: Introduction and Examples. Journal of the International Federation of Clinical Chemistry and Laboratory Medicine, 13, 103-109.
- [2] S.L. Ellison and M. Rossieni. (2000). Quantifying Uncertainty in Analytical Measurement. Eurachem/CITAC Guide, Second Edition, 4-5.
- [3] R. Schwartz. (2000). Determination of mass in practice: Comprehensive mass metrology. Wiley-VCH, Weinheim, 400.
- [4] A. Reichmuth1, S. Wunderli, M. Weber and V. Meyer. (2004). The Uncertainty of Weighing Data Obtained with Electronic Analytical Balances. Microchim. Acta, 148,133–141.
- [5] A. Reichmuth. (2000). Non-Linearity of Laboratory Balances And Its Impact on Uncertainty. NCSL, Workshop & Symposium, Toronto.
- [6] Conventional value of the result of weighing in air. (1979). International Organization of Legal Metrology (OIML). International Recommendations, No 33.
- [7] R.S. Davis. (1992). Equation for the density of moist air. Metrologia, 29, 67-70.
- [8] A. Picard and H. Fang. (2003). Methods to determine the density of moist air. Instrumentation and Measurement, 52 (2), 504 – 507.
- [9] F. Jones and E. Schoonover. (2002). Mass measurement. CRC Press, Boca Raton, 55.
- [10] M. Perkin. (2004). The Measurement of Mass and Weight. Stuart Davidson, National Physical Laboratory, 13-15.