Iterative Method and Dithering with Averaging used for Correction of ADC Error

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Additive iterative method in combination with averaging of dithered samples is designed for self-correction of ADC linearity error in the paper. Iterative method is one of the automated error correction techniques. Dithering is a special tool for quantizer performance enhancement. Dither theory for Gaussian noise and averaging has been used for exhibition of method abilities in ADC characteristic improvement.

Keywords: ADC Error Correction, Iterative Correction Method, Dithering

1. INTRODUCTION

F OR MODERN devices self-correction functions have become an essential feature. Usually analog-to-digital converter (ADC) is used for signal level measurements. So obviously ADC is the basic part of a general digital measurement channel, which determines the overall precision of the channel. It is often not difficult to make correction of offset and gain error of measurement transducer (MT) such as ADC. But correction of nonlinearities of the static transfer characteristic is problematic especially if they vary in time.

Methods for automatic correction of ADC have been employed as discussed below. Proposed correction is focused on ADC nonlinearities. Additive iterative method (AIM) is suitable for nonlinear error correction in the case of analog MT. But the ideal characteristic of ADC is fundamentally nonlinear reflecting the quantization error. Quantization error limits efficiency of AIM, therefore in the designed measurement system AIM is combined with nonsubtractive dithering (ND).

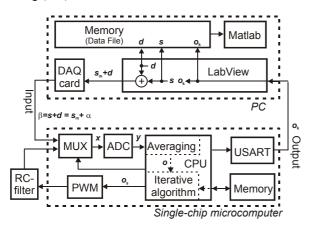


Fig.1 Block diagram of the workplace.

Block diagram in Fig.1 shows the experimental system. Diagram of the tested measurement unit (MU) is located in the lower part of the figure. Designed measurement system consists only of a single-chip microcomputer (with some basic peripherals like power supply etc.) and low-pass RC-filter. The correction method is implemented there. The upper side is devoted to the main PC components used for testing and experiments.

2. CORRECTION METHOD PRINCIPLE

This method utilizes four main blocks of the system: MT - in Fig.1 it is represented by ADC; block of processing (BP) – CPU (processor) with memory; inverse element (IE) – Pulse Width Modulation (PWM) with RC-filter; switch (SW) – multiplexer (MUX). The correction is performed iteratively in several steps. The BP controls the whole process. It receives input value from the MT and according to the implemented algorithm and data in the memory it calculates next input o_s to the IE. Evaluation of o_s is given by correction formula [1]

$$o_{s,i} = o_{s,i-1} + \left(o_{s,0} - h [h_{IE}(o_{s,i-1})] \right)$$
(1)

Function h(x) is transfer characteristic of MT and it determines initial value $o_{s,0} = h(s_m)$. Argument s_m represents measured value, which is in these measurements the mean value of input β (Fig.1). After the initial step, SW controlled by BP switches the input of ADC from measured signal $x = \beta$ to signal from IE $x = h_{\text{IE}}(o_{s,i})$. With each following step of iterations $o_{s,i}$ should become a more accurate representation of the measured value s_m . After appropriate number of steps actual $o_{s,i}$ could be sent to the output of the MU as a result o_k of correction.

The iterative process is convergent if the condition of convergence is satisfied [2]. Then it tends to value given by characteristics of IE $h_{\rm IE}^{-1}(s_{\rm m})$. Therefore the aim is to have an ideal IE with inverse characteristics equal to ideal characteristics of MT $h_{\rm I}(x) = h_{\rm IE}^{-1}(x)$.

Inverse element operation

Transfer characteristic of IE determines the best or theoretically reachable accuracy of measurement output corrected with the iterative method. Therefore this element must be designed thoroughly. Inverse element for ADC is digital-to-analog converter (DAC) and it has been built by means of pulse width modulation output of microprocessor. PWM circuits are naturally precise but to get the mean $a_{\text{PWM},0}$ ($a_{\text{PWM},k}$ denotes *k*-th frequency component) of PWM output corresponding to precise DAC result, low-pass filter should be added. Simple RC-filter has been used with frequency characteristics $A_{\rm RC}(\omega)$ [2], which is influenced by the time constant $\tau_{\rm RC}$. The filter slows down the correction process because after each step the process has to wait until the filter output settles.

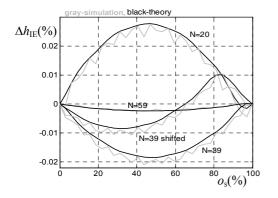


Fig.2 Theoretical error of IE mean evaluated through averaging of samples.

To speed up the process we proposed to use a combination of analog and digital filter. Output of the analog RC-filter oscillates in the range of several LSB. Sampling and averaging of *N* samples in each step of the iterative correction is used as digital filtering. The best way is to use synchronous sampling here but there was no possibility to synchronize ADC and PWM circuits. Generally, sampling gets samples from rectangular window $T_{RW}=N.T_s$ wide (T_s is sampling period) with frequency characteristic $A_{RW}(\omega)$. Denoting $\omega_{RW} = 2\pi/T_{RW}$ and $\omega_{PWM} = 2\pi/T_{PWM}$ (T_{PWM} is period of PWM output), mathematical model of IE error caused by non-synchronous sampling is [2]

$$\Delta h_{\rm IE}(o_{\rm s}) = \left[\frac{1}{T_{\rm RW}}\sum_{k=-\infty}^{\infty}a_{\rm PWM,k}(\{o_{\rm s}\})A_{\rm RC}(k\omega_{\rm PWM},\{\tau_{\rm RC}\})\right]$$
$$A_{\rm RW}(-k\omega_{\rm PWM},\{t_{\rm s1},T_{\rm RW}\}) - a_{\rm PWM,0}$$
(2)

By {} brackets dependency on parameters of system is expressed. According to simulation the RC-filter time constant was set to $\tau_{\rm RC}$ =0,1 s. By choosing appropriate number of sample *N* or value $T_{\rm RW}$ =*N*. $T_{\rm s}$, a special case of quasisynchronous sampling could be achieved. Suitable values are *N*=20, 39, 59 (etc.) for which IE error dependencies are shown in Fig.2. As could be seen, also time shift $t_{\rm s1}$ – shift from the start of PWM counter to the first sample – influences the accuracy. Expecting error of measurement after correction near to 0,01% of scale, the value *N*=59 has been chosen.

3. NOISE AVERAGING

ADC represents real quantizer and its quantization error limits measurement accuracy of an ideal converter or of a real ADC corrected with AIM. Averaging of samples is the way to overcome this limitation employing natural noise present in the measured signal. In many cases intentionally added noise (dither) d could help. A process of resolution improvement is called nonsubtractive dithering (ND), if the noise is not subtracted from the signal after quantization.

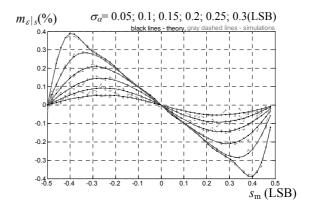


Fig.3 Mean error curves - dependency from standard deviation of Gaussian noise.

Noise is present in real applications and usually it is of Gaussian nature, but its dispersion may be too small to get significant resolution improvement. For evaluation of noise influence on accuracy of our system, where dithering with averaging is implemented, the appropriate error parameter must be chosen. Theory [3][4] yields for mean error ε of noisy samples m_{els}

$$m_{\varepsilon|s} = q \sum_{k=1}^{\infty} \frac{(-1)^k}{\pi k} \exp\left[-2\pi^2 k^2 \left(\frac{\sigma_a}{q}\right)^2\right] \sin\left(\frac{2\pi k s_m}{q}\right)$$
(3)

Fig.3 exposes dependency of the mean error from measured value $s_{\rm m}$ depicted in the range of one quantization step q. Theoretical curves show that with increasing standard deviation σ_{α} of total input noise α – composed from natural noise and dither together – the mean error decreases. Gray lines are obtained from simulation as a mean from P=20 results of averaging of N=59 samples. The mean is estimated through averaging. The bigger the input noise dispersion is the noisier the curves are. It suggests that an optimal noise variance exists in the system with ND and averaging. The formula of theoretical mean error (3) does not include this fact.

The mean-square error (MSE) was chosen as a suitable parameter of dithering and averaging performance rating for finding optimal noise dispersion. It is theoretically evaluated as mean for one whole quantization step [3]. Using (3) and theory from [3] for Gaussian noise it holds

$$\mu_{a}^{2}(\sigma_{\alpha}, N) \cong \frac{\frac{q^{2}}{12} + \sigma_{\alpha}^{2}}{N} + \left(1 - \frac{1}{N}\right) \frac{q^{2}}{2\pi^{2}} e^{-4\pi^{2} \left(\frac{\sigma_{\alpha}}{q}\right)^{2}}$$
(4)

This formula embodies influence of both the mean error (3) and the occurrence of noise in measurement results. For Gaussian dither and N=59 according to theoretical relation from [3] the optimal value of dither disperse σ_d is 0,347*q*, if natural noise is not present in the input signal.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Experiments were performed with the designed measurement system, where AIM and ND with averaging of N=59 samples was implemented. In 51 levels of input voltage P=20 correction processes were accomplished.

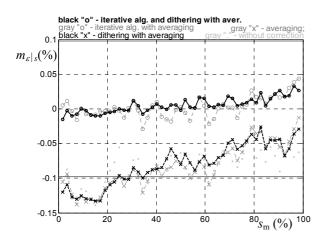


Fig.4 Mean error before and after correction.

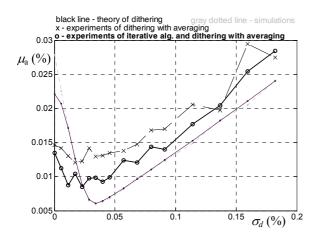


Fig.5 RMSE before and after correction.

5. CONCLUSIONS

A combination of two methods for measurement error correction has been implemented in the experimental measuring device. Additive iterative method based on precise inverse element has automatically corrected integral nonlinearity in every process of measurement result evaluation. Nonsubtractive dithering with averaging has enabled correction under level of 1 LSB of used 10-bit ADC. The mean of periodic IE output has been evaluated through averaging in our system. Analysis of quasi-synchronous sampling has been performed here, leading to negligible IE error. Dispersion of natural noise present in real signal is usually smaller than optimal for dithering. Theoretical dependence of root mean square error (RMSE) upon standard deviation of added noise has been proved through measurements in the whole range. The best experimental dispersion of dither has been found, it was less than theoretical. The RMSE has been reduced from 0.036 % to 0.0085 % and adequately ENOB has been improved from 9.64 bit to 11.73 bit using proposed correction methods.

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