Traceability of Voltage Measurements for Non-Sinusoidal Waveforms

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This paper describes the result of work performed at the Laboratoire National de Métrologie et d'Essais (LNE) aiming at developing a standard system to measure RMS value and harmonic contents of distorted voltage waveforms by means of a sampling voltmeter. Thermal converters are used to trace the RMS value to the SI units. The error of the DVM has been generally found less than 10 µV/V up to 2 kHz but can reach about 50 µV/V at 2.5 kHz for RMS voltage measurements for sine waves. For distorted waveforms, deviations within 15 µV/V have been obtained whatever the total harmonic distortion of the waveforms.

Keywords: distorted voltage waveforms, thermal converters, sampling techniques, analog to digital converters.

1. INTRODUCTION

THE MEASUREMENTS of root mean square (RMS) of sinusoidal signals with uncertainties of a few µV/V are carried out in most National Metrology Institutes (NMIs) by using thermal converters. Indeed, these thermal converters are widely used to trace AC quantities (AC currents and voltages) to the corresponding DC quantities with the best level of uncertainty [1]. Agreement of some parts in 10^7 to 15 part in 10^6, in the frequency range from 1 kHz to 1 MHz, has been found between various NMIs at voltages ranging from 1 V to 3 V [2]. Below 1 kHz frequency, uncertainties of 1 or 2 parts in 10^6 are currently obtained and are higher down to 10 Hz. Although they are the most accurate true RMS measuring devices available, they suffer from many drawbacks: their low impedance, their low dynamic range and their long measuring times. Moreover, they do not give information about the harmonic contents of the signals such as amplitude and phase of each harmonic, total harmonic distortion…

All these quantities are really very important since over the past few years new requirements for electricity supply quality including low harmonic distortion have appeared. As a consequence power analyzers are increasingly being used to insure the quality of the grids. This has created a need for national laboratories to provide calibration facilities and traceability for the harmonic related quantities measured by these new instruments.

For all these reasons, one alternative to thermal converters is the use of sampling methods and analog-to-digital converters (ADCs) of high accuracy with small integration times, especially those from the HP 3458 A* sampling digital voltmeter (DVM), which is commonly used in most NMIs. The objective of the study is to measure the total RMS value U_{eff} of distorted waveforms by means of a sampling voltmeter as well as the RMS value of each harmonic and the total harmonic distortion. Thermal converters are only used to trace the quantity U_{eff} to the SI units.

2. DEFINITIONS

The distorted voltage waveforms have a dominant fundamental component at 50 Hz and the harmonic content is limited to the first 50 stationary harmonics (periodic signals). These waveforms can be decomposed into Fourier series as:

$$u(t) = \sum_{k=1}^{\infty} U_k \sin(2\pi f_k t + \varphi_k) \tag{1}$$

U_1, U_k are the amplitudes of the fundamental and the kth harmonic component which frequencies are respectively f_1 and $f_k = k f_1$.

The two quantities of interest for our work are :

- the RMS value U_{eff} of the voltage signal which is defined as:

$$U_{eff} = \left(U_1^2 + U_2^2 + ... + U_k^2 + ...\right)^{\frac{1}{2}} \tag{2}$$

- the total harmonic distortion of the signal, often abbreviated as *THD* which can be calculated in one of two different ways:

$$THD = \left(\frac{\sum_{k>1}^{\infty} U_k^2}{U_{eff}^2}\right)^{\frac{1}{2}} \text{ or } THD = \left(\frac{\sum_{k>1}^{\infty} U_k^2}{U_1^2}\right)^{\frac{1}{2}} \tag{3}$$

The first definition is based on comparing the rms value of the harmonics to the total rms value while the second definition compares the rms value of the harmonics to the rms value of the fundamental component. In most situations, the difference will be negligible, but for highly distorted signals, it may be of importance.

In this paper, we have adopted the second definition which is the most commonly used.

3. MEASUREMENT METHOD

A. Basic measuring principle of the AC-DC thermal transfer

The principle of AC-DC thermal transfer measurements is based on the comparison of the heating of a resistor produced by the successive application of AC and DC signals and measured by the means of a thermocouple fixed on the resistor. The procedure is the following : the AC signal to be measured is first applied to the heater resistor and the output voltage of the thermocouple is measured. Then the AC signal is replaced by a DC signal which is adjusted to produce the same output voltage of the thermocouple as previously. When this condition is fulfilled the DC signal is measured and it can be concluded that the RMS value of the AC signal is equal to the measured value of the DC signal.

Nevertheless since the response of a thermal converter is different for AC and DC signals, a correction called the AC-DC transfer difference has to be applied. The AC-DC transfer difference δ of a thermal converter is defined by :

$$\delta = \frac{E_{ac} - E_{dc}}{E_{dc}} \qquad (4)$$

where E_{ac} is the RMS value of the AC signal applied to the thermal converter and E_{dc} is the value of the DC signal which produces the same output voltage of the thermocouple. In practice, DC signal is applied successively in both polarities to eliminate the reversal error arising from asymmetry in the construction of the thermal converter. E_c then represents the mean value of E_c+ and E_c-.

The AC-DC transfer difference δ of the thermal converter used for the experiment is plotted in Fig.1 as a function of the frequency f of the signal measured. The uncertainties attainable by this technique are of the order of 1 µV/V for measurements at voltages of about 1 V at low frequency.

B. Experimental setup

The block diagram of the set-up is given in Fig.2. The sampling voltmeter (DVM) used for the measurement of the RMS value U_{eff} and the reference thermal converter are connected in parallel and can be feeded by an AC or a DC source by means of an AC-DC switch. The output voltage of the thermal converter is measured by a precision nano-voltmeter. The sampling voltmeter is periodically calibrated against a Zener source as it also serves to measure the voltage delivered by the DC source (reference voltage).
The sequence AC, DC+, DC- and AC is then applied. Voltages measured by both the DVM and the precision nano-voltmeter (output voltage of the thermal converter) are each time recorded. The error of the DVM is computed from this set of data.

The AC source used to generate the distorted waveforms is a FLUKE 6100A* Electrical Power Standard Master unit which also provides a "sample reference" (TTL) at its rear panel to trigger the sampling DVM (Fig.3). The TTL reference signal is a harmonic of the phase reference signal and is phase locked to it. Thus the frequency of the power source and the sampling rate are synchronized, in order to take an integer number of periods.
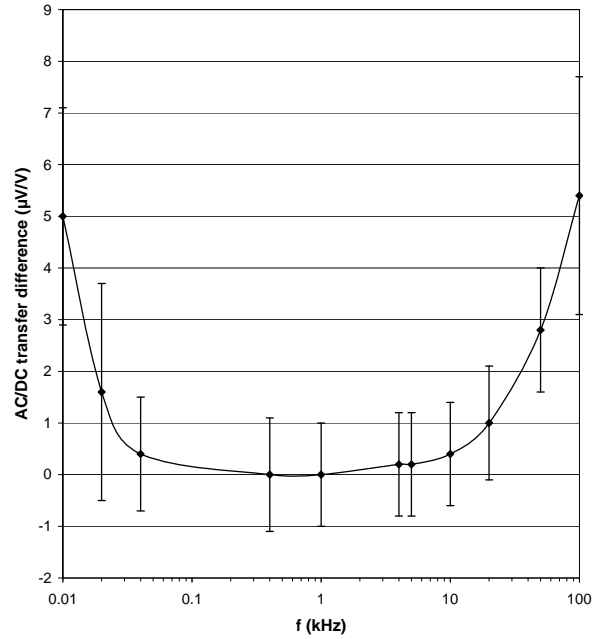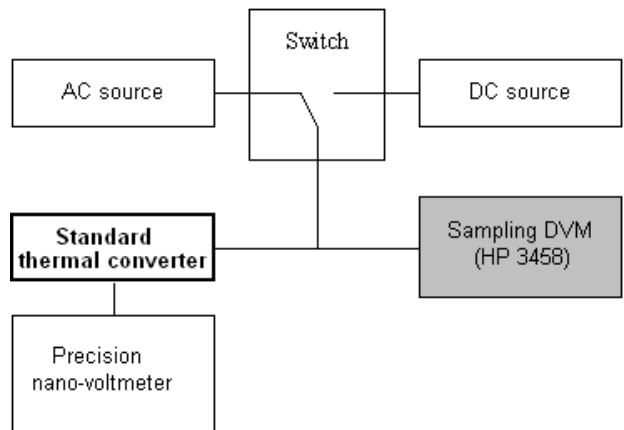

Fig.1. AC-DC transfer difference δ of the thermal converter
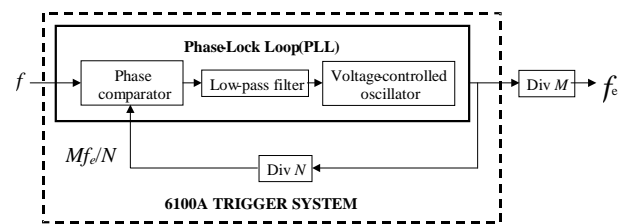

Fig.2. Experimental set-up.

Fig.3. Trigger control with phase-locking.

Using this "sample reference", it is not possible to select any number N of samples and any number M of periods. For example, at 53 Hz, we get by default exactly 1024 samples over 1 period or 2048 samples over 2 periods and so on. The sampling frequency is 54.3 kHz and can not be modified. Therefore, the analog-to-digital converter's integration time (or aperture time T_a) which corresponds to the integration of the signal over a finite time is limited. So, some frequency

dividers have been inserted in the experimental set-up in order to divide the sampling frequency for getting exactly 1024 samples over 3, 5 or 7 periods (Fig.2) which gives sampling rates respectively at about 18 kHz, 10.8 kHz and 7.7 kHz. This allows to increase the aperture time and therefore amplitude resolution.

For AC voltage measurements, the DVM is used in the DCV sampling mode. N samples are taken at equally spaced discrete instants, covering an integer odd number M of periods of the input waveform. The sampling frequency F_e is 10.8 kHz. It is assumed that, at half the sampling frequency and above, the amplitudes of all spectral components are smaller than the resolution of the sampler. This is the mandatory condition for aliasing to be avoided. Then, the DVM records all the samples in reading memory using DINT format (32 bit integer format) and transfers the data to the controller using the DINT output format. The maximum number of samples is limited by the 20kBytes storage capacity of the multimeters. During the sampling, the autozero function and the display of the multimeters are disabled.
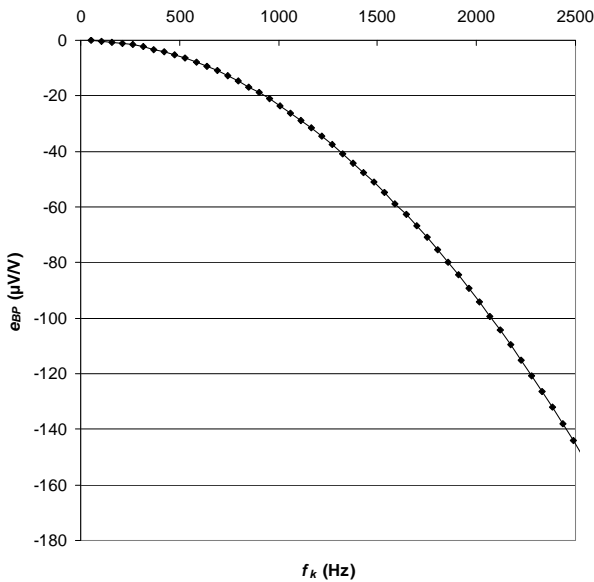

Fig.4. Error e_{BP} due to limited bandwidth according to the frequency f_k of the harmonics.

The samples are then analyzed by applying discrete Fourier transform (DFT) and all the components of the signal are deduced from the amplitude spectrum. Using the DVM in the DCV sampling mode, the main amplitude errors are due to the limited bandwidth and the non zero aperture time T_a:
- because of the limited bandwidth of the DVM, the behavior of its input channel is similar to a second order low pass filter composed of two 5 kΩ resistances and two 82 pF capacitances [3]. This filter changes the magnitude of each sample and the resulting error has been corrected by applying a factor e_{BP} given by:

$$e_{BP} = \frac{1}{\sqrt{1 + 7(2\pi f_k RC)^2 + (2\pi f_k RC)^4}} \tag{5}$$

Fig.4 shows this error according to the frequency f_k of the harmonics. The harmonic content being limited to 50 stationary harmonics, the higher frequency considered is 2500 Hz and the resulting error is 145 µV/V.
- the other error source is due to the non-zero aperture time T_a. The mean value of the signal integrated over this finite time does not correspond exactly to its value at the centre of the integration interval.

The DFT of a sampled signal may be defined by:

$$X(kf) = \sum_{n=0}^{N-1} x(nT_e)\exp\left(-2\pi j\frac{kn}{N}\right) \tag{6}$$

$$k = 0, 1, 2, ..., N-1.$$

where N is the number of samples, T_e is the sampling period, F_e is the sampling rate, $x(nT_e)$ is the input signal amplitude at time nT_e and $X(kf)$ is the spectrum of x at frequency kf.
According to Poisson's formula, equation (6) can also be written as :

$$X(kf) = X(f) * F_e . \sum_{k=0}^{N-1} \delta(f - kF_e) \tag{7}$$

where δ is the Dirac function.
Taking into account the aperture time T_a, the input signal amplitude $x(nT_e)$ can be expressed as :

$$x(nT_e) = \frac{1}{T_a}\int_{nT_e-\frac{T_a}{2}}^{nT_e+\frac{T_a}{2}} x(t)dt \tag{8}$$

and equation (7) becomes :

$$X(kf) = \left[X(f)\frac{\sin(\pi f T_a)}{\pi f T_a}\exp\left(-2\pi j v\frac{T_a}{2}\right)\right] * F_e . \sum_{n=-\infty}^{+\infty} \delta(f - nF_e) \tag{9}$$

Then, the resulting amplitude error is corrected by applying the factor e_{Ta} defined as:

$$e_{T_a} = \frac{\sin(\pi f_k T_a)}{\pi f_k T_a} \tag{10}$$

As shown in Fig.5, the error due to non-zero aperture time can be very large. For example, considering a signal frequency of 2500 Hz and an aperture time of 60 µs, this error is 36602 µV/V.

These two systematic errors have been taken into account in the determination of the rms value of all harmonic components. The rms value of the distorted signal has been calculated as the quadratic sum of all harmonics.
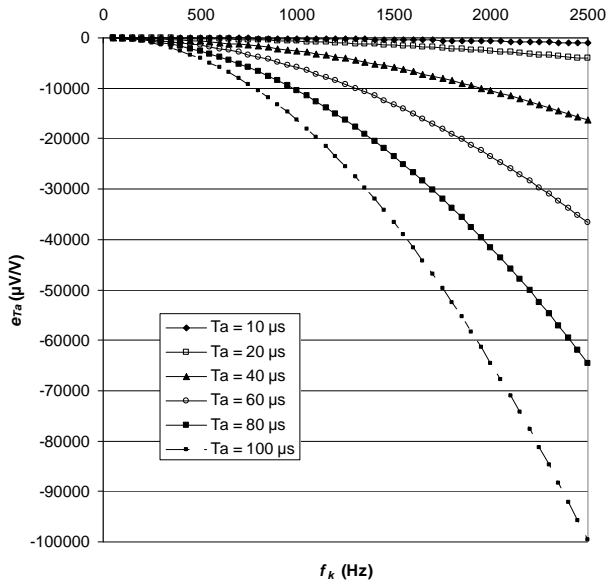
Fig.5. Error e_{Ta} due to aperture time according to the frequency f_k of the harmonics.

4. EXPERIMENTAL RESULTS

A. Characterization of the analog-to-digital converters (ADC) of the DVM

The ADCs of the DVM have already been characterized for static [4] and dynamic [5] conditions, for sine waves at frequencies up to 400 Hz. In this frequency range, selecting optimized sampling parameters, the agreement between the sampling technique and AC-DC transfer using thermal converters has been found better than 3 µV/V.
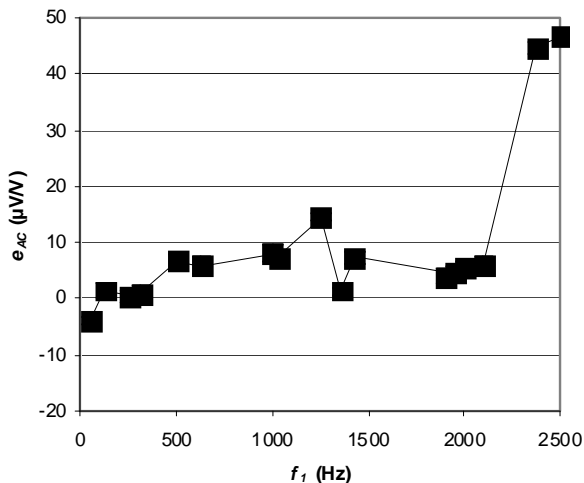

Fig.6. Relative error e_{AC} of the DVM according to the frequency f_1 of the sine wave. The sampling parameters are $f_e = 10.24$ kHz and $T_a = 60$ µs.

The highest harmonic frequency of the distorted waveform studied is 2500 Hz. Then, first experiments have been performed to characterize the ADCs for frequencies from 400 Hz to 2500 Hz. Fig.6 shows the error e_{AC} of the DVM (relative to the voltage reference value measured by AC-DC

transfer technique) according to the frequency f_1 of the signal. The aperture time selected is 60 µs. The error varies from – 3 µV/V at 40 Hz to +48 µV/V at 2500 Hz. This error is taken into account in the estimation of the measurement uncertainty.

B. Measurements of the RMS value of distorted waveforms

For digitizing the signals, 1024 samples are taken over 5 periods and the sampling frequency is 10.24 kHz to satisfy the Shannon condition. The aperture time is 60 µs. Twenty voltage waveforms are available, one sinusoidal waveform and nineteen distorted waveforms (table 1). The distorted waveforms have a dominant fundamental component at 50 Hz and one or several stationary harmonics. The *THD* varies from 0 % (sine wave) to 84 %. Their amplitudes do not exceed 0.8 V and the DVM is used in the 1 V range.

Fig.7 represents the amplitude spectrum of a distorted signal containing a fundamental component at 50 Hz and two harmonics ($k = 3$ and $k = 10$). The uncertainty introduced by rounding the sample amplitudes to discrete levels can be viewed as adding quantization noise to the signal. The amount of this 'noise' decreases with increasing amplitude resolution. It can be expressed by the Signal-to-noise ratio (SNR) :

$$(SNR)_{dB} \approx 6Q$$

where Q is the number of bits used to represent the signal.

In our experimental configuration, 18 bits are used to represent the signal. It means that when measuring a 1V input signal, the noise level is around 10^{-5} V according to the theory. As shown in Fig.7, experimental results are in good agreement with the theoretical calculation.
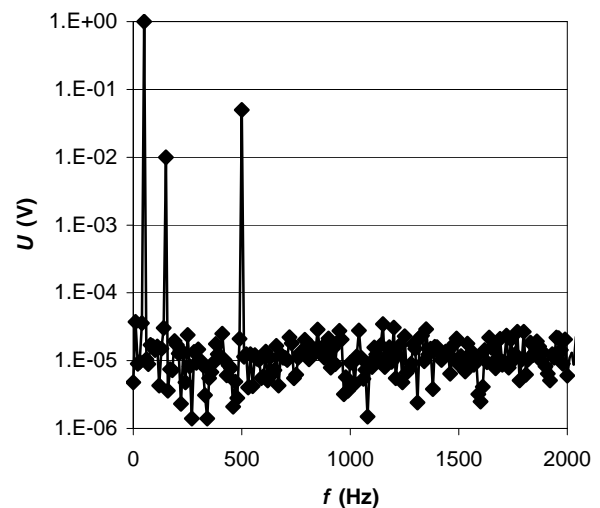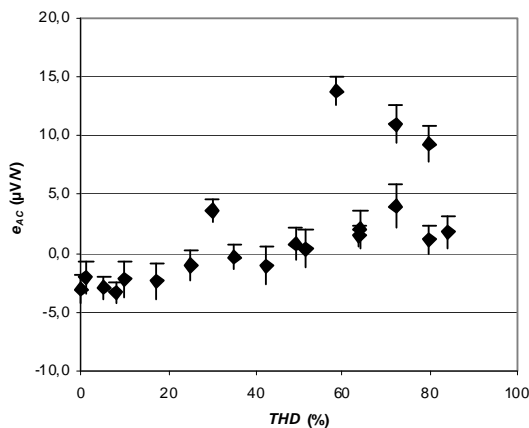

Fig.7. Example of amplitude spectrum for a distorted signal.

Fig.8 shows the error e_{AC} of the DVM as a function of the *THD* of the distorted waveforms. It appears that the difference between the two techniques is always below 15 parts in 10^6. Most of the measurements were repeatable to within 2 parts in 10^6.

signal	Amplitude of the different harmonics	THD (%)
1	$U_1 = 1$ V	0
2	$U_1 = 1$ V, $U_6 = 0.01$ V	1
3	$U_1 = 1$ V, $U_3 = 0.01$ V, $U_{10} = 0.05$ V	5.1
4	$U_1 = 1$ V, $U_5 = U_{15} = 0.05$ V $U_{30} = U_{40} = U_{50} = 0.02$ V	7.9
5	$U_1 = 1$ V, $U_{20} = 0.1$ V	10
6	$U_1 = 1$ V, $U_3 = U_{25} = U_{45} = 0.1$ V	17.3
7	$U_1 = 1$ V, $U_2 = U_3 = 0.1$ V $U_4 = 0.05$ V, $U_5 = 0.02$ V, $U_{50} = 0.2$ V	25.1
8	$U_1 = 1$ V, $U_{50} = 0.3$ V	30
9	$U_1 = 0.9$ V, $U_2 = U_{36} = 0.2$ V, $U_{34} = U_{35} = 0.1$ V	35.1
10	$U_1 = 1$ V, $U_3 = U_4 = 0.3$ V	42.4
11	$U_1 = 0.8$ V, $U_2 = 0.25$ V, $U_9 = 0.25$ V $U_{22} = U_{41} = U_{47} = 0.1$ V	49.2
12	$U_1 = 0.8$ V, $U_{20} = 0.3$ V, $U_{36} = U_{50} = 0.2$ V	51.5
13	$U_1 = 0.8$ V, $U_{48} = 0.2$ V, $U_{49} = U_{50} = 0.3$ V	58.6
14	$U_1 = 0.8$ V, $U_2 = U_4 = U_6 = U_{18} = U_{25} = U_{26} = 0.2$ V, $U_{27} = U_{28} = 0.1$ V	63.7
15	$U_1 = 0.7$ V, $U_2 = 0.3$ V, $U_{13} = 0.28$ V $U_{34} = 0.18$ V	64
16	$U_1 = 0.65$ V, $U_{22} = U_{34} = 0.3$ V $U_{40} = 0.2$ V	72.2
17	$U_1 = 0.65$ V, $U_4 = U_5 = U_6 = U_7 = 0.2$ V, $U_{14} = U_{15} = U_{16} = U_{17} = U_{29} = U_{30} = 0.1$ V	72.2
18	$U_1 = 0,65$ V, $U_{22} = 0,4$ V, $U_{34} = 0,3$ V $U_{40} = U_{50} = 0,1$ V	79.9
19	$U_1 = 0.65$ V, $U_2 = 0.3$ V, $U_3 = 0.25$ V $U_4 = 0.2$ V, $U_5 = 0.1$ V, U_6 to $U_{32} = 0.05$ V	79.9
20	$U_1 = 0.65$ V, $U_2 = U_3 = 0.3$ V $U_4 = U_5 = U_6 = 0.2$ V	84.3

Table 1. Harmonic contents of the different signals measured.

Fig.8. Relative error e_{AC} of the DVM according to the *THD* of the distorted waveforms studied.

C. Uncertainty analysis for the effective value.

The uncertainty has been determined for the sampling of a spectrally "pure" (THD<0.001%) sinusoidal signal with the DVM operating in the "direct sampling" mode. Taking into account the different sources of errors [5], the relative uncertainty (1σ) is estimated to be 3 µV/V at 50 Hz.

In our experiment, the signal has a dominant fundamental component at 50 Hz and its harmonic content is limited to

50 stationary harmonics (periodic signals). From (2), the relative uncertainty for the effective value U_{eff} of this signal can be expressed as

$$\left(\frac{\sigma_{U_{eff}}}{U_{eff}}\right)^2 = \sum_{k=1}^{50}\left(\frac{U_k}{U_{eff}}\right)^4 \cdot \left(\frac{\sigma_{U_k}}{U_k}\right)^2 \quad (11)$$

From the results presented in Fig.6, one can assume that the uncertainty of the term $\left(\dfrac{\sigma_{U_k}}{U_k}\right)$ is inferior to 50×10^{-6} whatever the value of k. Besides, the ratio $\left(\dfrac{U_k}{U_{eff}}\right)$ does not exceed 50% (tab.1).

Finally, the relative uncertainty for the effective value U_{eff} is inferior to 25×10^{-6}.

5. CONCLUSION

A standard to measure RMS value of distorted voltage waveforms has been developed at LNE. Measurements have been performed on waveforms with *THD* ranging between 0 and 84 %. Good agreement with thermal converters, always better than 15 parts in 10^6, has been obtained in all cases.

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