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# Correction of ADC Errors by Additive Iterative Method with Dithering

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**The iterative method could be used for automatic accuracy improvement of a measurement system. In its application for analog-to-digital converter (ADC) a quantization error represents a limitation for the correction process. Therefore, combination of correction methods is common for ADC error correction. Combination of the additive iterative method (AIM) with nonsubtractive dithering (ND) has been proposed for slow measurement based on ADC where errors could change in time. The principle of combination of both techniques is described in the paper. AIM is based on precise inverse element (IE). In the designed system the IE output signal is created by pulse width modulation and low-pass filtering. A technique similar to deterministic dithering is employed to achieve precise processing of signal from IE. Analysis of influence of stochastic dither upon the results of correction is performed with the aim to find optimal parameters of ND. Finally, dependency of the root mean squared error and error dispersion on the measured value is drawn to show how AIM corrects the nonlinear deterministic error but slightly increases system noise.**

**Keywords: Iterative Correction, Dithering, ADC error**

## 1. INTRODUCTION

FOR PRECISE measuring through the whole life-cycle of a measurement device many special error correction methods have been developed.

In the connection with this trend self-correction functions are becoming very important in modern equipments. Very often an analog-to-digital converter (ADC) integrated within a monolithic microcontroller is used for signal level measurements. Then ADC characteristics determine the overall metrology properties of a measuring channel. Generally it is not difficult to make a correction of offset and gain error of ADC. More sophisticated methods based on look-up table [1] offer fast correction of ADC error nonlinearities. But if an error could change in time, methods with self-correction features should be applied.

If a microcontroller or other universal system building component is used in a measurement channel, usually several hardware elements stay redundant. Frequently, those are DAC or PWM outputs. At such situation the additive iterative method (AIM) of correction could be employed with advantages. This correction naturally suppresses the linear error. But it is suitable also for ADC with nonlinearities. AIM requires precise inverse element (IE) for correction of both nonlinear and linear error of the general analog measurement transducer (MT). However, quantization limits the efficiency of AIM, therefore in the designed measurement system AIM is combined with dithering. We prefer nonsubtractive dithering (ND) for this application where low price is an important aspect. Both stochastic and deterministic dithering techniques are used in the designed measurement system as discussed bellow.

## 2. THE PRINCIPLE OF ITERATIVE CORRECTION

One of the ways for measurement accuracy improvement is the use of the so-called structural-algorithmic methods [2], where the measurement errors are diminished with the help of auxiliary means. Generally, for the AIM three new blocks should be added to MT and then four main blocks of the system are distinguished (Fig.1). In our case the designed measurement system consists mainly of integrated components of a single-chip microcomputer: MT – in our system is represented by ADC; block of processing (BP) – CPU (processor) with memory; inverse element (IE) – pulse width modulation (PWM) circuits with RC-filter; switch – multiplexer (MUX).

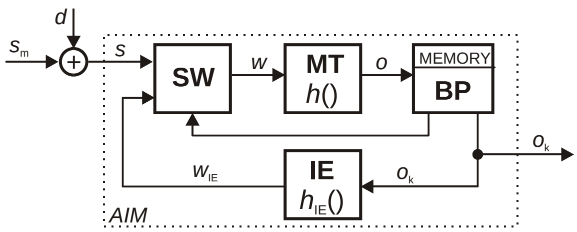


Fig.1. Block diagram of our measuring system, the part in frame corresponds to AIM.

For our application only slow changes of measured value $s_m$ are assumed. This value will be considered constant during one correction process and it represents the mean of actual system input $s_m=E[s]$. Time duration of the correction depends on the response of IE discussed later, on error value and on the number of steps. For precise evaluation of $s_m$, dither $d$ is added before AIM. So in the initial step of AIM, signal $s=s_m+d$ is connected to the input of MT. Corresponding MT output $h(s_m)$ is then sent from BP to IE $o_{k,0}=h(s_m)$. At the same time BP changes the state of SW. Therefore in the next steps signal from IE $w_{IE}= h_{IE}(o_k)$ is being sent to the input of MT. Every next input to IE is solved in BP from the actual MT output $o_i$ and previous IE input $o_{k,i-1}$ according to the following iterative formula

$$o_{k,i} = o_{k,i-1} + o_{k,0} - o_i \tag{1}$$

If the characteristic of IE is ideal, i.e. it is equal to inverse of ideal MT characteristic $h_{IE}^{-1}= h_I$, the described algorithm decreases the static error of MT $\Delta h(s_m)=h(s_m)-h_I(s_m)$ according to geometric series $\Delta h_{k,i} = (-C)^i\Delta h$. Factor $C$ is determined by ratio of derivatives (sensitivities) of error and ideal characteristics [3] and if the condition of convergence is satisfied $|C|<1$, then AIM suppresses MT error.

A suitable ending condition for iterative correction should be used for stopping the process after sufficient number of steps, e.g. small difference between last two corrected values $|o_{k,i}-o_{k,i-1}|<\varepsilon$. Generally, convergent iterative process tends to values given by characteristics of IE $o_k=h_{IE}^{-1}(s_m)$. If it is not possible to have an ideal IE, the error of IE will influence the results of correction. Deeper investigation of IE properties will help us to obtain negligible IE error.

If IE has no deterministic error it does not mean that AIM will give precise measurement results if noise of this element $g_{IE}$ is too big. Similarly, noise of MT $g_{MT}$ will influence results of correction. It would be a significant disadvantage of iterative correction if with every step of iteration random error increases. Lets neglect other possible sources of noise except noise $d$ present at the input of the measurement channel. If then quantization error is modeled as random error and included in $g_{MT}$, it could be shown by reasoning similar to [4] that in the step $i$ of AIM random error is

$$g_{AIM} = d + g_{MT,0} - g_{MT,i} - g_{IE,i} \tag{2}$$

As members of (2) could be taken as independent noise sources it could be said that all considered random error sources will add to the final error once but only $g_{MT}$ twice. And, fortunately, this implication is independent from the number of steps of AIM. Random error of the corrected measurement result is larger than that of the uncorrected measurement but for usual cases suppression of the deterministic error should be a more significant contribution of correction. However, by suitable digital signal processing, the random error could be suppressed too.

### A. Inverse element operation

The transfer characteristic of IE determines the theoretically reachable accuracy of correction result if MT sensitivity is sufficiently high. Inverse element for ADC is digital-to-analog converter (DAC) and is built by means of pulse width modulation output of a microcontroller. PWM circuits are naturally precise but to get the mean of IE output $w_{IE,m}=E[w_{IE}]$ corresponding to a precise DAC result, low-pass filter should be added. A simple RC-filter (RCF) is used. The frequency characteristic of this filter is determined by time constant $\tau_{RC}$, which has to be large enough to get small amplitude oscillation of signal from IE. Large $\tau_{RC}$ slows down the measurement process because after every step the process should wait until the settling of the filter output. The block diagram of IE is depicted in Fig.2. It includes also processing of the signal from IE. To speed up the process we proposed using the combination of analog and digital filter. Time constant $\tau_{RC}$ is selected such that the output of the analog RCF oscillates in the range of several LSB. Simple averaging of $N$ samples (block AVE in the Fig.2) in each step of the iterative correction is used as digital filter.

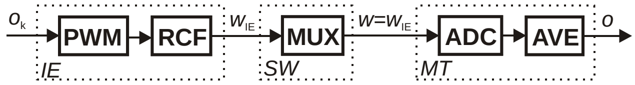


Fig.2. Block diagram of creating and processing of signal from IE. Dotted frames indicate blocks from Fig.1

The best way would be synchronous sampling of periodic signal from RCF but there could be no possibility to synchronize ADC and PWM circuits. The mathematical model of error caused by non-synchronous sampling was created. In this model quantization is not considered. The part of IE error represented by this model is dominant for many values of $N$ and, therefore, it could be still used as a model of IE error $\Delta h_{IE}$. Maximum is achieved by the theoretical model in the middle of ADC range. In Fig.3, theoretical results (black lines) obtained in the middle of the scale for two selected numbers of $N$ are shown in the area under 1 LSB (0.098 %). For appropriate number of samples $N$ special case of quasi-synchronous sampling is achieved with very low error $\Delta h_{IE}$ (solid lines, $N$=59). Then, if the number of samples is even increased (dashed lines, $N$=60), IE accuracy drops as this is a poor case of non-synchronous sampling. In Fig.3 theoretical results are compared to simulation with quantization (gray lines). In the simulation, maximal error was evaluated from 100 LSB range around the middle of ADC range. Entering 1 LSB region the results of simulation start to deviate from the mathematical model but the error is still decreasing because deterministic dithering comes to effect. Only close to both ends of the scale, where signal oscillation are tiny to toggle ADC outputs, real error will start to increase with $\tau_{RC}$. Therefore, time constant of RC-filter should be selected from the displayed region ($\tau_{RC}$=0,1 s) near to the edge between the steep and flat parts of curves.

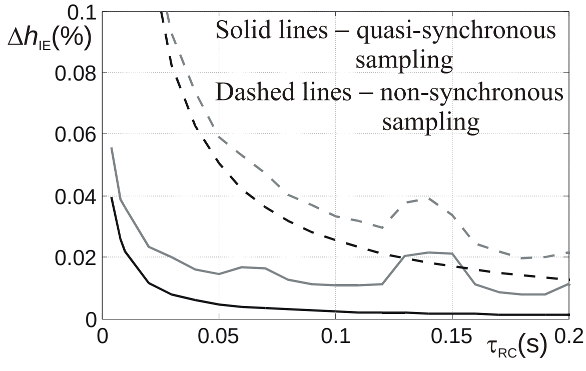


Fig. 3. Error of IE in the middle of range (theory – black, simulation – gray).

## 3. STOCHASTIC DITHER AND AVERAGING

Quantization error limits accuracy of the correction process. To overcome this limitation in evaluation of measured value $s_m$, intentionally added noise $d$ (dither) prior to quantization (prior to SW in Fig.1) can help. Then, averaging of samples of signal $s = s_m + d$ can lead to resolution improvement for the measurement of mean $s_m = E[s]$. This technique is called nonsubtractive dithering (ND), as the noise is not subtracted from the signal after quantization.

Influence of dither on quantizer could be generally investigated from the theory of quantization described in [5] or [6] where the technique of creating the mathematical model is called area sampling. However, other sources like [7] avoid area sampling while evaluating a particular accuracy parameter. In [7], mean error is derived from known deterministic behavior of quantization error and known probability density function (PDF) $f_d$ of dither. The same result for mean error could be obtained according to the theory of area sampling. Assuming dither with zero mean and symmetrical (and real) PDF it holds

$$ME(s_m, \sigma_d) = \sum_{k=1}^{\infty} \frac{q(-1)^k}{\pi k} \Phi_d\left(k\frac{2\pi}{q}\right) \sin\left(k\frac{2\pi}{q}s_m\right) \quad (3)$$

where $q$ is quantization step, $\Phi_d$ is characteristic function of dither with standard deviation $\sigma_d$ and $s_m$ is measured input of quantizer with ND. Disadvantage of (3) is dependency on input $s_m$. To evaluate the single parameter describing the error of the whole range measurement it is better to solve mean squared error (MSE) [8] within one quantization step

$$MSE(\sigma_d) = \frac{1}{q}\int_{-\frac{q}{2}}^{\frac{q}{2}} ME^2(s_m, \sigma_d) ds_m = \frac{q^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\Phi_d^2\left(k\frac{2\pi}{q}\right) \quad (4)$$

There are several types of dither analyzed in [7]. Usually uniform noise leads to the best results, if peak-to-peak value $D_d$ of noise equal to one quantization step $q$ could be provided. However, it is often assumed in theory, that the number of processed samples $N$ is large. For that case (3) or (4) give good error estimations. In a microcomputer application like this $N$ is small and therefore dispersion of measurement output should be included in the model of error like in [8] or [9]. The mean squared error is a suitable parameter for rating of dithering and averaging performance if the stochastic part of error after averaging is included into the total MSE [8]

$$MSE_T(\sigma_d, N) = \frac{\frac{q^2}{12} + \sigma_d^2}{N} + \left(1 - \frac{1}{N}\right)\frac{q^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\Phi_d^2\left(k\frac{2\pi}{q}\right) \quad (5)$$

Such total MSE ($MSE_T$) was theoretically evaluated in [8] for Gaussian dither. But it is difficult to generate artificial Gaussian noise. Other usual dither types were analyzed in [10]. Uniform dither generally leads to good accuracy and could be generated easily, e.g. by asynchronous sampling of triangular signal. Therefore, we designed uniform dither for our application and derived the theoretical model of total RMSE (Root MSE) for uniform dither ($RMSE^2 = MSE$)

$$RMSE_T^2(D_d, N) = \frac{q^2 + D_d^2}{12N} + \left(1 - \frac{1}{N}\right)\frac{q^2}{2\pi^2}\operatorname{sinc}^2\left(\frac{\pi D_d}{q}\right) \quad (6)$$

This formula embodies both the mean error and dispersion of measurement results. The second part of (6) reflects the deterministic part of error which is zero for $D_d = l.q$ ($l$ is positive integer) and negligible for large $D_d$. However, the first part reflects dispersion of error and rises with dispersion of added noise. This part is meaningful for small number $N$ of averaged samples, when optimal dispersion of dither is dependent on $N$. Optimal $D_d$ is then less than $q$.

## 3. EXPERIMENTAL RESULTS AND DISCUSION

Experiments were performed with the designed measurement system [3], where AIM and ND with averaging was implemented for error correction of 10-bit ADC. Curves of total RMSE ($RMSE_T$) are depicted in Fig.4. Theoretical dependency of RMSE solved from (6) is proved by simulation results seeing that light and dark gray lines are almost identical for major part of the graph. The implemented method significantly corrected gain error and offset in experiments. Main contribution of the designed correction lies in suppression of the nonlinear error component. Therefore, the linear error part had been subtracted from measurement results before evaluation of experimental RMSE. If only natural noise is used ($\sigma_d$=0), the reduction of nonlinear error by AIM is negligible. After increasing dither the error reduction is significantly enhanced (*-curve and o-curve) but without iterative correction still notably shifted against the theory (x-curve) due to INL of ADC. AIM corrects INL and therefore it moves the curve closer to theoretical values. Fig.4 shows that the quasi-optimal standard deviation (STD) of dither is found $\sigma_d = 0.2333q$ (0.0228 %) as the final parameter of our design. Because of natural noise present in the input signal, resulting optimal dither dispersion is lower than theoretical optimum.

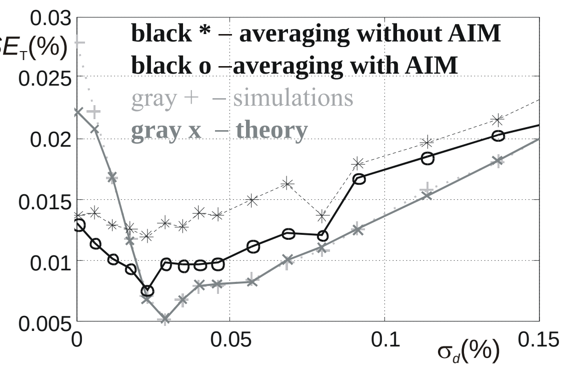


Fig.4. Total RMSE of ADC with averaging before and after correction with AIM.

For the designed measurement system the RMSE (of nonlinear error part) dependency on measured value calculated from 20 measurements at each input level $s_m$ is depicted in Fig.5. The minimal reachable RMSE is influenced by dispersion of results, which is higher after correction with AIM. The contribution of AIM is suppression of peaks in Fig.5 caused by INL. Finally, error is corrected considerably under the 1/sqrt(12) of LSB (0.,0282 %). In our case appropriate dither enables this suppression of INL with AIM deeply below 1 LSB.

To check how random error of measurement result is affected by the designed correction, standard deviation of error (STDE) is drawn in Fig.6. Theoretically, variance of error (2) should be reduced by factor of $N$ because of averaging of $N$ ADC samples in a microcontroller. Therefore, all values of STDE with or without AIM in the figure are deep below total STD of error obtained from measurement by ADC without any processing or dither addition (0.0266 %). Following (2) STDE after iterative correction is higher compared to the results obtained before correction and the total value was increased from 0.0049% to 0.0064 %. However, more significant reduction of the nonlinear deterministic error part by AIM caused reduction of total RMSE from 0.0120 % to 0.0076% and random error remained as a dominant part of total RMSE after correction.

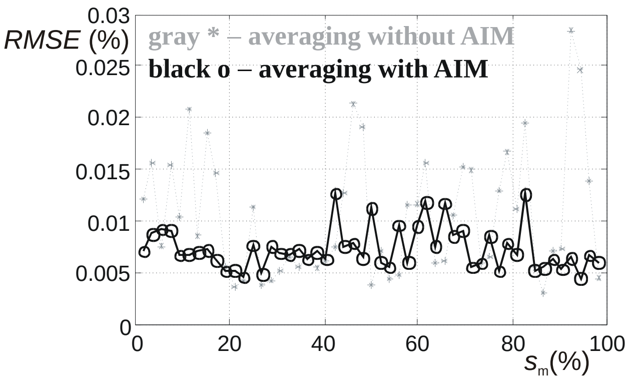


Fig. 5. RMSE of ADC with averaging before and after correction with AIM evaluated in each point of range for quasi-optimal standard deviation of dither.

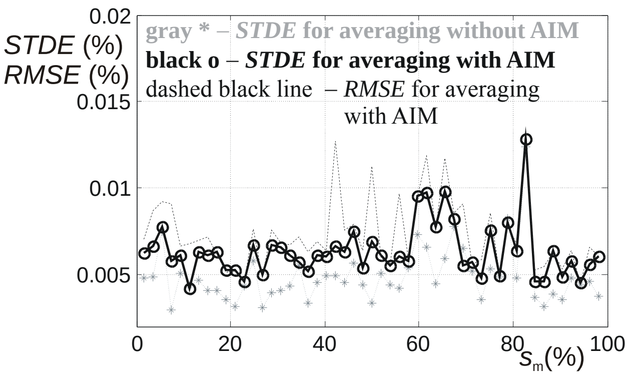


Fig. 6. STDE of ADC with averaging – STDE before and after correction by AIM compared to final RMSE.

## 4. CONCLUSION

Automatic correction of analog-to-digital converter (ADC) errors has been carried out. The designed technique requires minimal additional hardware components, so it may be used in each microcontroller based measuring channel for which high speed is not the necessity. Such microcontroller applications are widely used in industrial practice today. In our case and experiments one measurement lasts several seconds, however, the method enables to obtain several results per second.

The discussed correction uses a combination of two methods. The additive iterative method (AIM) automatically corrects integral nonlinearity. It is based on the precisely designed inverse element. Functionality of AIM is retained even in the case of time-changing errors. Non-subtractive dithering with averaging enables correction below the level of 1 LSB of used 10-bit ADC. For uniform dither theoretical dependence of total root mean squared error (RMSE) upon standard deviation of added noise has been proved through measurements in the whole ADC range. Quasi-optimal value of dither dispersion has been found. Finally, the RMSE has been reduced significantly despite the increase of noise caused by AIM. Accuracy improvement evaluated in ENOB (Effective Number of Bits) is 2.26 bit from 9.64 to final value of 11.90 bit after correction.

## ACKNOWLEDGMENT

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