

Detection of Gearbox lubrication Using PSO-Based WKNN

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This paper proposes an optimization classification model, which combines particle swarm optimization (PSO) with weighted k -nearest neighbors (WKNN), namely PWKNN. The model optimizes the weight and k parameter of WKNN to improve the detection accuracy of gearbox lubrication levels. In the experiment, the current signals of the generator are measured, and the relative frequency spectrum of the measured signals is illustrated by using fast Fourier transform (FFT). The features from the spectrum are extracted, and then the optimal weight and k parameter of WKNN are obtained by using PSO. The average detection accuracy of gearbox lubrication levels is 96% by using PWKNN, which the result shows that the proposed PWKNN can efficiently detect the lubrication level of gearboxes. The experiment also shows that the performance of the proposed PWKNN by using the current signals of the generator is superior to that by using typical vibration signals of a gearbox. In addition, the accuracy can reach 95.4% even in environments with 20 dB noise interference.

Keywords: Particle swarm optimization, weighted k -nearest neighbors, fast Fourier transform, current signals

1. INTRODUCTION

WIND TURBINES are assembled with complicated components. Any minute component malfunction leads to serious damage to the entire wind turbine. In many components, gearbox faults seriously affect generator efficiency, necessitating gearbox condition monitoring. Planetary gearboxes are often used in wind turbines because its cost is inexpensive and the structure is robust. As lubrication of the gearboxes is inadequate, it causes broken gear teeth or to gear displacement during operations. Moreover, inadequate lubrication leads to gearbox vibrations even cause entire wind turbine damages. These situations can be avoided by observing lubrication levels of gearboxes. Most level detection methods adopt lubrication sight glasses and float balls to monitor system dynamics. These methods need extra measurement apparatus, such as float balls, which methods may not be accurate because of aging or displacement of the apparatus. This paper adopts an alternate approach using current signals, differ from vibration signals, to detect lubrication levels of gearboxes.

Literatures focus the mechanical faults of gearboxes, including gear tooth breaks and gear displacements. The research proposes methods to detect mechanical faults, but they seldom investigate lubrication level detection. Literatures mention the measurement of the vibration signals of gearbox and induction motor by using accelerometers to determine whether the gearbox and motor are healthy [1]-[5]. Recently, both current and voltage signals of a motor are used to detect rotating machinery faults. The research detects induction motor faults by using current signals [7]. The research uses generator output current to monitor generator condition [8]. References [9]-[10] use current signals to detect motor faults. The references show that the use of current signals for rotating machinery faults diagnosis is available.

Signal analysis is important to effectively detect the lubrication levels of gearboxes. Typical methods for signal analysis, such as Fourier transform (FT) [9], waveform

transform (WT) [10]-[14], S transform (ST) [15] and Hilbert Huang transform (HHT) [16], are applied to analyze signals. WT is commonly used in signals analysis, but accurate conclusions depend on the researcher ability to choose preliminary basis functions, as different basis functions lead to different analysis results. ST is commonly used in power-quality analysis, but the analysis is time consuming. HHT is commonly used in transient and non-linear signals analysis. However, the method must address the issue of envelope selection and end effects [17]. This study applies fast Fourier transform (FFT) to shorten computational time [10] and to obtain better analysis results. Meanwhile, using FFT is easier to implement online detection when compared to the other methods.

Many automatic detection systems are based on classification algorithms, such as back propagation neural network (BPNN), probability neural network (PNN) and k -nearest neighbor (KNN) [18]-[20]. The BPNN classification results are affected by various neuron numbers and learning rates. The PNN classification results are affected by various smoothing parameters as well [21]. Typical KNN is a fast and non-training classification algorithm that computes correlations of each known vector and uses nearest k -samples to classify unknown vectors. Each feature of KNN is equal importance which means each feature equally affect classification results [22],[23]. For example, an unknown vector is in a feature space, which contains three types of vectors, Types A, B and C, in Fig.1. The weight of each feature is equal, i.e., $w=[1, 1]$, the unknown type is classified as Type C when k is set as 6, as shown in Fig.1(a). If the weights are changed to $w=[1, 0.6]$, the unknown type is classified as Type B when k is set as 6, as shown in Fig.1(b).

That means that only choosing features cannot obtain an optimal result, and using the feature weights to adjust the importance of each feature of KNN is necessary. Thus, this study applies weighted k -nearest neighbors (WKNN) and particle swarm optimization (PSO) to optimize the weights and k parameter to improve classification accuracy [24].

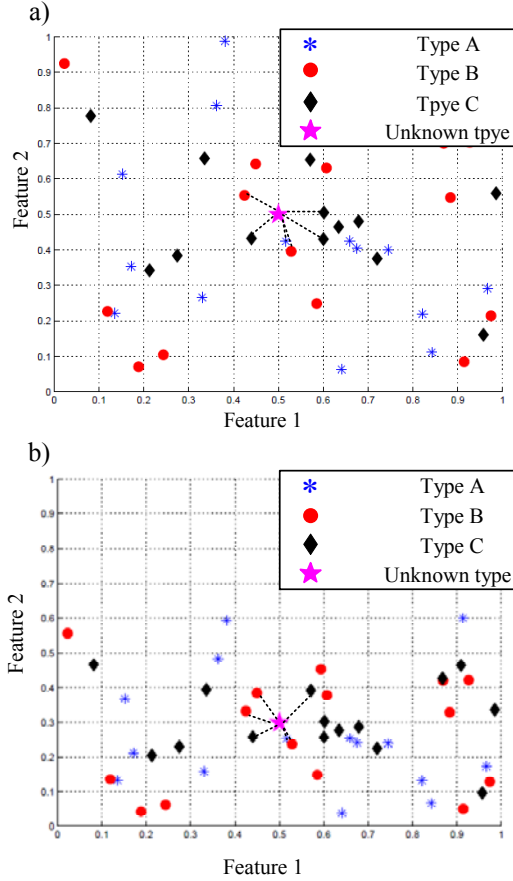


Fig.1. Vector space (a) $w=[1,1]$, (b) $w=[1,0.6]$.

This paper proposes an optimization classification model, which combines PSO with WKNN, namely PWKNN, to detect lubrication levels of a gearbox. The experiment includes three steps: 1) measure the current signals of the generator and vibration signals of gearboxes; 2) use FFT analysis model to obtain frequency spectrum; 3) extract any 10 features from the spectrum, including maximum, minimum, average, mean squared error, standard deviation, kurtosis, skewness, variance, sum and fundamental frequency. The study focuses on the proposed PWKNN manner that can optimize feature weights of WKNN, even some of these features are dependent.

2. PARTICLE SWARM OPTIMIZATION WITH WEIGHTED k -NEAREST NEIGHBORS (PWKNN)

A. Weighted k -nearest neighbors (WKNN)

The typical KNN is a common and simple classification algorithm, which includes three steps: 1) calculate correlations of known feature vectors and unknown feature vectors; 2) consider k nearest known feature vectors in a feature vector; 3) determine the unknown vector by the largest number of known feature vectors in k samples. Inaccuracies arise because KNN weights all feature vectors equally. WKNN addresses this problem by modifying feature vector weights $[w_1, w_2, \dots, w_n]$ of KNN for different features. This can increase correlations of useful features and improve the results of classification; as shown in (1).

$$dist^i = \sqrt{\sum_{m=1}^n w_m (p_m - x_m)^2} \quad (1)$$

where w is weighted feature, n is dimension number of feature vector, $dist$ is Euclidean distance of unknown x and identified sample p , i is between 1 and k .

B. The Proposed PWKNN

Recently, the PSO is commonly be used for searching the optimal solution [23], [25]. The best position of each particle and best position of the group are estimated by the fitness function $Fit(\square)$. The steps of PSO are listed as follows:

Step 1. Random initial $X_i^{t_0}$ and $v_i^{t_0}$, and then let $X_i^{t_0} = w$.

Step 2. Calculate initial Fit^{t_0} , Pb^{t_0} and Gb^{t_0} .

Step 3. The iteration start, let $t=1$.

Step 4. Update velocity v_i^{t+1} , where $c_1=c_2=2.05$ 、 $\varphi=4.1$ and $\kappa=1$, so $\chi \approx 0.72984$, as shown in (2) [26][27].

Step 5. Update position X_i^{t+1} , as shown in (3).

Step 6. Calculate Fit^{t+1} , Pb^{t+1} and Gb^{t+1} , as shown in (4)-(5).

Step 7. $t=t+1$.

Step 8. If do not reach terminal, as shown in (7) go back to *Step 4*.

Step 9. $Gb^{t+1} = w_{Best}$.

$$v_i^{t+1} = \chi \left(v_i^t + c_1 r_1 (Pb_i^t - X_i^t) + c_2 r_2 (Gb^t - X_i^t) \right) \quad (2)$$

$$X_i^{t+1} = X_i^t + v_i^{t+1} \quad (3)$$

$$Pb_i^t = \begin{cases} X_i^t & , Fit(X_i^t) > Fit(Pb_i^{t-1}) \\ Pb_i^{t-1} & , Fit(X_i^t) \leq Fit(Pb_i^{t-1}) \end{cases} \quad (4)$$

$$Gb = Pb_j, \text{ where } Fit(Pb_j)_{best} \quad (5)$$

$$\chi = \frac{2\kappa}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\kappa} \right|} \text{ where } \varphi = c_1 + c_2, \varphi > 4, \kappa \in [0, 1] \quad (6)$$

$$\begin{cases} Fit(Gb^{t-1}) - Fit(Gb^t) < 0.5\%, t = 1, 2, 3, \dots \\ t \leq 1000 \end{cases} \quad (7)$$

where t is the iteration time, \mathbf{X} is the particle i , v is the velocity of the particle i , Pb is the current best particle (position), Gb is the global best for any particles. r_1, r_2 is random number between 0 and 1. c_1 and c_2 are the self confidence factor and swarm confidence factor, respectively. χ is a constriction factor.

A new vector can be written in (8), in which the vector is assembled by weight of KNN and a k -parameter. This study uses PSO to optimize the new vector w , and the best vector w_{best} (group best position) is obtained, as shown in (9). The PSO flowchart is shown in Fig.2.

$$w = [w_1, w_2, w_3, \dots, w_n, k]$$

$$w_{\text{best}} = [w_1, w_2, w_3, \dots, w_n, k]_{\text{best}}$$

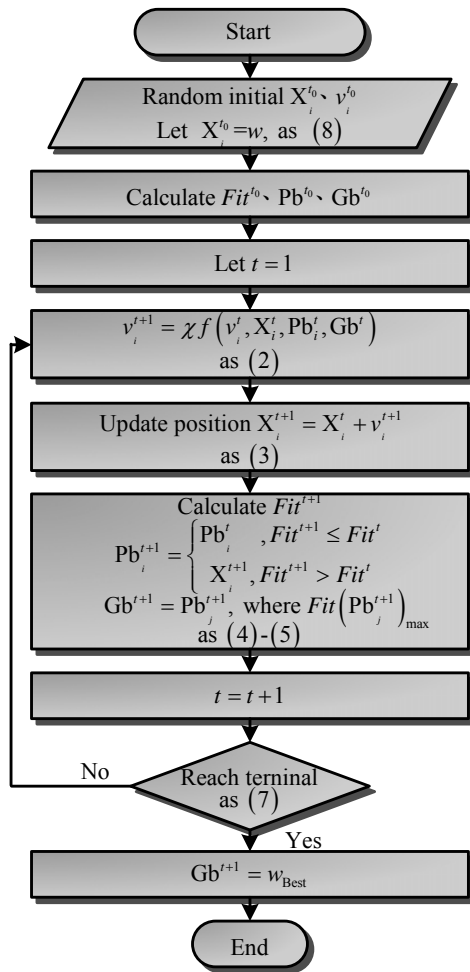


Fig.2. PWKNN flowchart.

3. RESULTS OF SIGNALS ANALYSIS AND LUBRICATION LEVEL DETECTION

This study applies a dynamometer test bed composed to a 11 kW/4,000 rpm induction motor to drive a 10:1 planetary gearbox and three-phase permanent magnet generator. The generator is connected to a DC electronic load through a bridge rectifier. A National Instruments PIX-1033 signal acquisition and accelerometer are adopted to measure simultaneously the current and vibration signals.

The experiment structure and test bed apparatus are shown in Figs.3. and 4., respectively. Three processes of experiment are measurement, analysis and recognition. First, in the measurement process, the both signals of various lubrication levels, the current and vibration signals, are acquired when the gearbox is operating. Second, in the analysis process, the spectrums of the both signals are obtained, and then the features are extracted from the two spectrums. Finally, in the recognition process, the recognition rates by using the typical classifiers and the proposed PWKNN are calculated.

(8) A. Experiment explanation

Spectrums of vibration signals analyzed by FFT are shown in Fig.5(a). However, observing the spectrums of vibration signals cannot directly perceive discrepancy to various lubrication levels. Thus, current signals are adopted and analyzed by FFT. The fundamental frequency of current signals are decreased about 5 Hz as the lubrication levels of the gearbox change from high to low, as shown in Fig.5(b). The decreased lubricity affects the friction and rotation speed of the gearbox, and consequently the current frequency of the generator is affected.

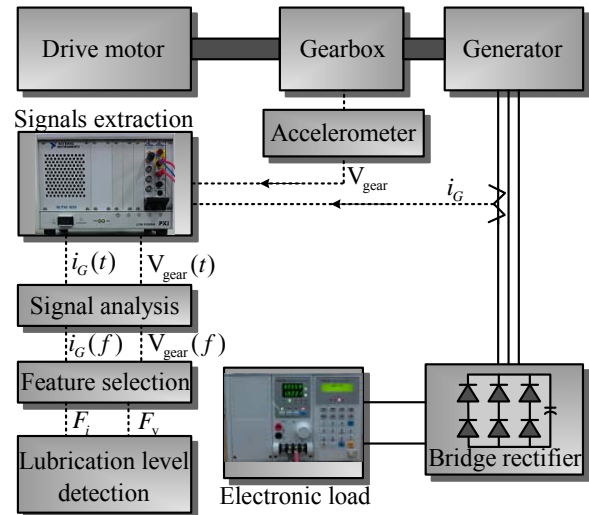


Fig.3. Experiment structure chart.

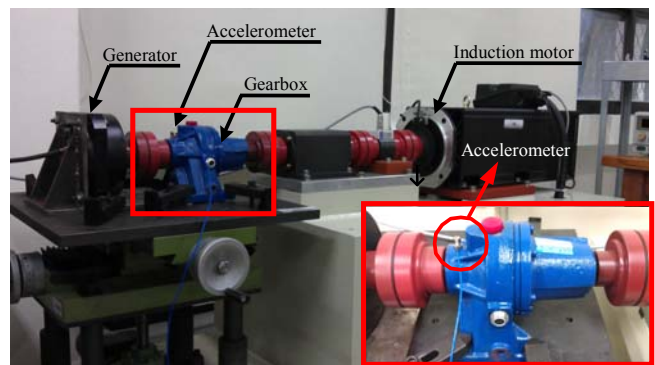


Fig.4. Experimental test bed.

The frequency spectrum spam of vibration signals is wider because of the environment noise and over sensitivity of vibration sensor. That is the reason the spectrum cannot be obviously detect the lubrication level. The benefit of using current signals is that the fundamental frequency is certain, and the variation of the environment noise and over sensitivity of vibration sensor does not affect seriously the fundamental frequency. Thus, the changes of the current caused by the inadequate lubrication of gearboxes can be obviously illustrated.

In this paper, we use the common analysis method, extracting features from the frequency spectrum, to obtain two feature distributions of current and vibration signals, as shown in Fig.6(a). and 6(b). respectively. In which the

extracted features are maximum, minimum, average, mean squared error, standard deviation, kurtosis, skewness, variance, sum and fundamental frequency. In Fig.6., we can be easier to determine the level by using the current signals, as shown in Fig.6(b)., but not to vibration signals, as shown in Fig.6(a). The two feature distributions show that the feature extraction method cannot improve the performance of lubrication level detection.

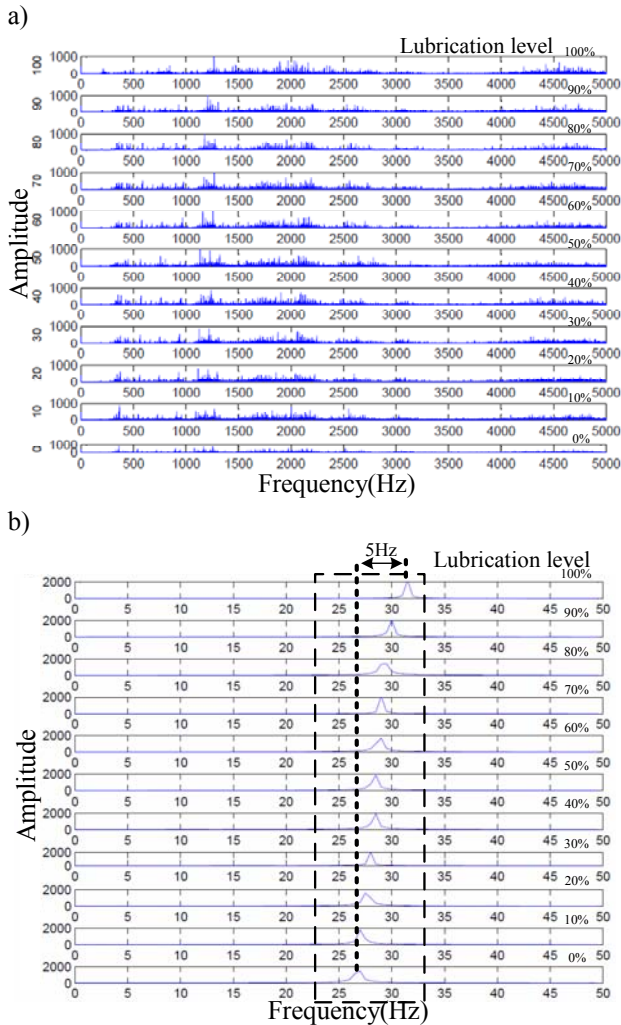


Fig.5. All lubrication level of spectrum
a) vibration signals, b) current signals.

The sampling rate of current signals in the experiment is 1,000 Hz by using typical acquisition, and the spectrum shows the resolution is enough to recognize the gearbox lubrication status. However, the resolution is inappropriate to recognize the lubrication situation by using the same sampling rate, 1,000 Hz. Furthermore, we enlarge the sampling rate of vibration signals to 10,000 Hz. Even the sampling rate, 10,000 Hz, of vibration signal is much higher than that of current signals, 1,000 Hz, the spectrum obtained by using current signals is clearer and superior to that using vibration signals to recognize the lubrication status in this situation. The results show that the current signals can recognize the lubrication status instead of the function of typical vibration signals.

B. Lubrication level detection results

B.1. Various lubrication levels

The experiment measures the generator current to obtain the 100 samples for each lubrication level of gearboxes, and the total 1,100 samples (100-samples and 11-layers) are divided into the 990 training and 110 test data, where the parameter of *k* of typical KNN is 3. The hidden layer number of back propagation neural network is set as 1, and the neural number is set as 10.

We discuss the three facets, which are (1) the comparison of the typical KNN, BPNN and the proposed PWKNN and (2) the comparison of the classification accuracies by using the current signals and the typical vibration signals. The results of the two facets are described as below:

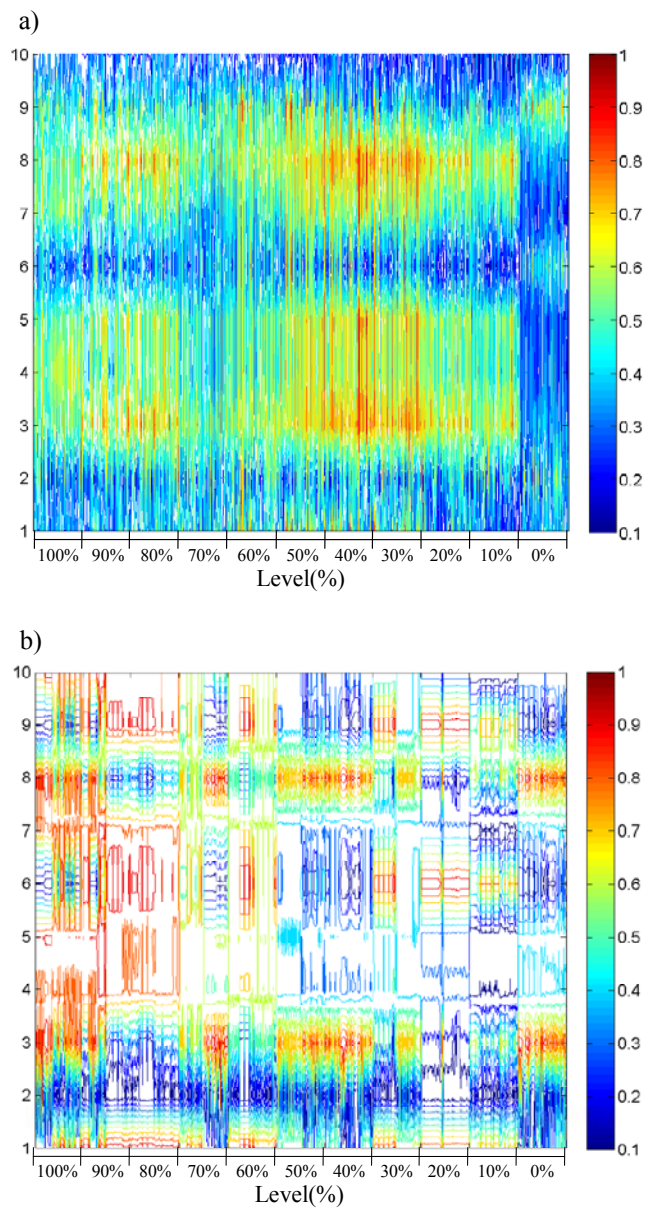


Fig.6. All lubrication level of feature distribution
a) vibration signals, b) current signals.

(1). The classification accuracy obtained by using the proposed PWKNN model is 94.1%, and accuracies by using the KNN and BPNN model are 93.5% and 92.4% respectively. The result shows that the proposed PWKNN method is superior to that by using typical KNN and BPNN models based on the measured current signals, as shown in Table 1, in which the proposed PWKNN method is superior to that by using typical KNN and BPNN models based on the measured vibration signals.

(2). Based on the current signals, the classification accuracies obtained by using the proposed PWKNN, KNN and BPNN models are 94.1%, 93.5% and 92.4% respectively. Based on the vibration signals, the accuracies obtained by using the proposed PWKNN, KNN and BPNN models are 87.0%, 85.5% and 77.5% respectively. The result shows that the accuracy using the current signals is superior to that using the vibration signals.

B.2. Lubrication levels with various loads

The motor current in this experiment commonly operates at 1.1A, which is referred by the guideline of the motor. Three loads, which are 1.5A, 1.1A and 0.7A as the light, regular and heavy loads respectively, are designed to realize the performance of various loads application. The classification accuracies of 1.5A, 1.1A and 0.7A cases

obtained by using the proposed PWKNN are 98.0%, 94.1% and 96.0%, respectively. Comparing to the classification accuracies obtained by using KNN and BPNN models, the classification accuracy of the proposed PWKNN model is superior in each load. The availability and superiority of the proposed PWKNN can be validates even using in the vibration signals based experiment, as shown in Table 2.

B.3. Lubrication levels with noise interference

To verify the robustness of the proposed PWKNN model, 20dB of white noise is added into the original current and vibration signals. The classification accuracy with 20dB white noise, obtained by using the proposed PWKNN model, is 90.1%, and accuracies by using the KNN and BPNN model are 88.0 and 77.2% respectively, as shown in Table 3. The result shows that the proposed PWKNN method is superior to that by using typical KNN and BPNN models based on the measured *current signals*, even in the environments of interference. Observing the results of using vibration signals with 20 dB noise in Table 3., it is noted that the all classification models cannot be available to detect the lubrication level of gearboxes. Thus, the proposed PWKNN with the current signals are more appropriate to solve the level detection problem.

Table 1. Detection accuracy of various lubrication levels based on regular load.

Level (%)	Current signals			Vibration signals		
	PWKNN(%)	KNN(%)	BPNN(%)	PWKNN(%)	KNN(%)	BPNN(%)
100	98	98	100	99	97	98
90	88	87	96	97	99	66
80	86	84	97	87	87	89
70	90	92	90	84	80	45
60	99	100	98	75	78	67
50	100	100	93	99	97	100
40	91	87	93	79	67	51
30	100	100	100	98	97	81
20	100	100	87	100	99	100
10	100	100	100	72	70	91
0	83	81	62	67	69	64
Average accuracy	94.1	93.5	92.4	87.0	85.5	77.5

Table 2. Detection accuracy in various loads.

Load (A)	Current signals			Vibration signals		
	PWKNN(%)	KNN(%)	BPNN(%)	PWKNN(%)	KNN(%)	BPNN(%)
Heavy 1.5	98.0	96.0	91.5	82.7	78.9	77.1
Regular 1.1	94.1	93.5	92.4	87.0	85.5	77.5
Light 0.7	96.0	93.2	93.0	88.1	86.1	74.7

Table 3. Detection result with 20dB noise.

Level (%)	Current signals			Vibration signals		
	PWKNN(%)	KNN(%)	BPNN(%)	PWKNN(%)	KNN(%)	BPNN(%)
Without noise	94.1	93.5	92.4	87.0	85.5	77.5
With 20dB noise	90.1	88.0	77.2	28.1	22.5	37.7

4. RESULT DISCUSSION.

This study extracts the features of the spectrum of the measured signal and proposes a new PWKNN model to improve the performance detection accuracy of lubrication level of gearboxes. In the study, the parameter w and k of WKNN is optimized by PSO, and the effectiveness of this PWKNN model is verified by the experiment. Three advantages of PWKNN are summarized as follow:

- 1). *Adoptive for generator current* –The study uses current signals of generator to effectively improving the inaccuracy of using typical method, vibration signals, to detect the statuses of the lubrication level. The study results verify that the detection accuracy of using current signals is superior to that of using vibration signals to detect lubrication level of gearboxes.
- 2). *Optimize the w & k of WKNN* - The proposed PWKNN improves the equal weight problems of typical KNN. The weights and k parameter are optimized by PSO. The optimized weights and k parameter can improve the classification accuracy. And the proposed model can be available even in various loads.
- 3). *Improve robustness* - The detection accuracies of lubrication level of gearboxes between with and without the noise are 90.1% and 94.1%, respectively, That similar accuracies means that the proposed PWKNN can efficiently detect the lubrication level of interference even in environments of noise interference.

5. CONCLUSION

The study proposes the PSO-based PWKNN model to improve the typical WKNN model, and the weights and k parameter of WKNN are optimized by PSO in the proposed PWKNN model. We also adopt the current signals to detect the lubrication levels of a gearbox. Comparing to the. typical vibration signals method to detect the status of gearboxes, the results of the experiment show that the proposed PWKNN and the adopted current signals method can improve the detection accuracy, and the availability and superiority of the proposed PWKNN model is verified. Furthermore, the proposed PWKNN is superior even in environments of noise interferences.

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REFERENCES

- [1] Randall, R.B. (2011). *Vibration-based Condition Monitoring: Industrial, Aerospace and Automotive Applications*. Wiley.
- [2] He, P., Li, Q., Kong, F. (2010). A new method of feature extraction for gearbox vibration signals. In *Image and Signal Processing (CISP) : 3rd International Congress*, 16-18 October 2010. IEEE, Vol. 8, 3980-3984.
- [3] Yang, Z., Hoi, W.I., Zhong, J. (2011). Gearbox fault diagnosis based on artificial neural network and genetic algorithms. In *International Conference on System Science and Engineering (ICSSE)*, 8-10 June 2011. IEEE, 37-42.
- [4] Yang, S., Li, W., Wang, C. (2008). The intelligent fault diagnosis of wind turbine gearbox based on artificial neural network. In *International Conference on Condition Monitoring and Diagnosis (CMD 2008)*, 21-24 April 2008. IEEE, 1327-1330.
- [5] Mohanty, R., Kar, C., (2006). Fault detection in a multistage gearbox by demodulation of motor current waveform. *IEEE Transactions on Industrial Electronics*, 53 (4), 1285-1297.
- [6] Rajagopalan, S., Habetler, T.G., Harley, R.G., Sebastian, T., Lequesne, B. (2007). Current/voltage-based detection of faults in gears coupled to electric motors. *IEEE Transactions on Industry Applications*, 42 (6), 1414-1420.
- [7] Huang, X., Habetler, T.G., Harley, R.G. (2007). Detection of rotor eccentricity faults in closed-loop drive-connected induction motors using an artificial neural network. *IEEE Transactions on Power Electronics*, 22 (4), 1552-1559.
- [8] Wan, S.T., Li, H.M., Li, Y.G, Meng, F.C. (2005). Analysis of stator winding parallel-connected branches circulating current and its application in generator fault diagnosis. In *Conference Record of the Industry Applications Conference : Fourtieth IAS Annual Meeting*, 2-6 October 2005. IEEE, Vol. 1, 42-45.
- [9] Blodt, M., Chabert, M., Regnier, J., Faucher, J. (2006) Mechanical load fault detection in induction motors by stator current time-frequency analysis. *IEEE Transactions on Industry Applications*, 42 (6), 1454-1463.
- [10] Thomas, V.V., Vasudevan, K., Kumar, V.J. (2003). Online cage rotor fault detection using air-gap torque spectra. *IEEE Transactions on Energy Conversion*, 18 (2), 265-270.
- [11] Gaouda, A.M., Salama, M.M.A., Sultan, M.R., Chikhani, A.Y. (1999). Power quality diction and classification using wavelet-multiresolution signal decomposition. *IEEE Transactions on Power Delivery*, 14 (4), 1469-1476.
- [12] Jena, D.P., Kumar, R. (2011). Implementation of wavelet denoising and image morphology on welding image for estimating HAZ and welding defect. *Measurement Science Review*, 11 (4), 108-111.
- [13] Wei, L., Wang, H., Li, F. (2009). Fault diagnosis of turbine generator vibration based on wavelet packet and data-driven. In *ISECS International Colloquium on Computing, Communication, Control, and Management (CCCM 2009)*, 8-9 August 2009. IEEE, Vol. 2, 29-32.
- [14] Fan, X., Zuo, M.J. (2006). Gearbox fault detection using Hilbert and wavelet packet transform. *Mechanical Systems and Signal Processing*, 20 (4), 966-982.

- [15] Stockwell, R.G., Mansinha, L., Lowe, R.P. (1996). Localization of the complex spectrum: The S-transform. *IEEE Transactions on Signal Processing*, 44 (4), 998-1001.
- [16] Yuping, Z. (2006). Hilbert–Huang transform and marginal spectrum for detection of bearing localized defects. In *The 6th World Congress on Intelligent Control and Automation (WCICA 2006)*. IEEE, Vol. 2, 5457-5461.
- [17] Cheng, J.S., Yu, D.J., Yang, Y. (2007). Application of support vector regression machines to the processing of end effects of Hilbert–Huang transform. *Mechanical Systems and Signal Processing*, 21 (3), 1197-1211.
- [18] Chaovalitwongse, W.A., Fan, Y., Sachdeo, R.C. (2007). On the time series K-nearest neighbor classification of abnormal brain activity. *IEEE Transactions on System, Man and Cybernetics, Part A: System and Humans*, 37 (6), 1005-1016.
- [19] Manoharan, S.C., Veezhinathan, M., Ramakrishnan, S. (2008). Comparison of two ANN methods for classification of spirometer Data. *Measurement Science Review*, 8 (3), 53-57.
- [20] Rumelhart, D.E., Hinton, G.E., Willia, R.J. (1986). Learning representations by back-propagation errors. *Nature*, 323, 533-536.
- [21] Song, T., Jamshidi, M.M., Lee, R.R., Huang, M. (2007). A modified probabilistic neural network for partial volume segmentation in brain MR image. *IEEE Transactions on Neural Network*, 18 (5), 1424-1432.
- [22] Chen, B., Liu, H., Chia, J., Bao, Z. (2009). Large margin feature weighting method via linear programming. *IEEE Transactions on Knowledge and Data Engineering*, 21 (10), 1475-1488.
- [23] Middlemiss, M.J., Dick, G. (2003). Weighted feature extraction using genetic algorithms for intrusion detection. In *The 2003 Congress on Evolutionary Computation (CEC '03)*, 8-12 December 2003. IEEE, Vol. 3, 1669-1675.
- [24] Kennedy, J., Eberhart, R. (1995). Particle swarm optimization. In *IEEE International Conference on Neural Networks*. IEEE, Vol. 4, 1942-1948.
- [25] Biswal, B., Dash, P.K., Panigrahi, B.K. (2009). Power quality disturbance classification using fuzzy C-means algorithm and adaptive particle swarm optimization. *IEEE Transactions on Industrial Electronics*, 56 (1), 212-220.
- [26] Parsopoulos, K.E., Vrahatis, M.N. (2002). Particle swarm optimization method for constrained optimization problems. In *Intelligent Technologies – Theory and Application: New Trends in Intelligent Technologies*. IOS Press, Vol. 76, 214-220.
- [27] Valle, Y.D., Venayagamoorthy, G.K., Mohagheghi, S., Hernandez, J., Harley, R.G. (2008). Particle swarm optimization: Basic concepts, variants and applications in power systems. *IEEE Transactions on Evolutionary Computation*, 12 (2), 171-195.

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