MEASUREMENT SCIENCE REVIEW, **20**, (2020), No. 4, 187-195

sciendo

ISSN 1335-8871

MEASUREMENT SCIENCE REVIEW

Journal homepage: https://content.sciendo.com

Detecting an Unknown Abrupt Change in the Band Center of the Fast-Fluctuating Gaussian Random Process

Oleg Chernoyarov[1,2,3], Tatiana Demina[2], Yuri Kabanov[4], Alexander Makarov[1,3]

[1]*International Laboratory of Statistics of Stochastic Processes and Quantitative Finance, National Research Tomsk State University, Lenin Avenue, 36, 634050, Tomsk, Russia*
[2]*Department of Mathematics, Physics and System Analysis, Faculty of Engineering, Maikop State Technological University, Pervomayskaya st., 191, 385000, Maikop, Russia*
[3]*Department of Electronics and Nanoelectronics, Faculty of Electrical Engineering, National Research University "MPEI", Krasnokazarmennaya st., 14, 111250, Moscow, Russia, chernoyarovov@mpei.ru*
[4]*Department of Probability Theory, Faculty of Mechanics and Mathematics, Moscow State University, Leninskiye Gory, 1, 119991, Moscow, Russia*

The generalized maximum likelihood algorithm is introduced for detecting the abrupt change in the band center of a fast-fluctuating Gaussian random process with the uniform spectral density. This algorithm has a simpler structure than the ones obtained by means of common approaches and it can be effectively implemented on the base of both modern digital signal processors and field-programmable gate arrays. By applying the multiplicative and additive local Markov approximation of the decision statistics and its increments, the analytical expressions are calculated for the false alarm and missing probabilities. And with the help of statistical simulation it is confirmed that the proposed detector is operable, while the theoretical formulas describing its quality and efficiency approximate satisfactorily the corresponding experimental data in a wide range of parameters of the observable data realization.

Keywords: Fast-fluctuating random process, abrupt change, maximum likelihood method, discontinuous parameter, unknown band center, local Markov approximation method, false alarm probability, missing probability, statistical simulation.

1. INTRODUCTION

The problem of detecting the moments of changes in the properties of random processes arises when the specific tasks are to be solved including control and monitoring, technical and medical diagnostics, measurement data processing, etc. [1]-[5]. Often enough, the observed random process is the fast-fluctuating one and has the same intensity within the working frequency band [6]-[9]. As it is shown in [8], [9], the introduction of the conditions of the fast fluctuations and the uniform spectral density of the random process allows us to significantly simplify the structure of the synthesized processing algorithms, especially in the presence of the parametric a priori uncertainty.

In [10]-[12], the algorithms for detecting the abrupt change in the energy parameters of band fast-fluctuating Gaussian random processes are studied and tested. At the same time, in certain practically important applications, it is necessary to determine the presence of abrupt changes in the frequency parameters of the observable data realization. Below, the technically simple algorithm is considered for detecting the abrupt change in the Gaussian random process band center. Its operability and efficiency are established both theoretically and experimentally.

2. THE PROBLEM STATEMENT

Let us presuppose that at the receiver input the fast-fluctuating Gaussian random process of the form

$$\xi(t)=\begin{cases}\nu_1(t),\ t\le\lambda_0,\\ \nu_2(t),\ t>\lambda_0\end{cases} \tag{1}$$

arrives over the time interval $[0,T]$ and it is observed against Gaussian white noise $n(t)$ with one-sided spectral density N_0. In (1), the notations are: λ_0 is some unknown point in time, and $\nu_i(t)$, $i=1,2$ are statistically independent centered stationary Gaussian random processes with the spectral densities [7]-[9]

$$G_i(\omega,\vartheta_{0i})=\frac{d}{2}\left[I\left(\frac{\vartheta_{0i}-\omega}{\Omega}\right)+I\left(\frac{\vartheta_{0i}+\omega}{\Omega}\right)\right],\ I(x)=\begin{cases}1,\ |x|\le 1/2,\\ 0,\ |x|>1/2.\end{cases} \tag{2}$$

DOI: 10.2478/msr-2020-0023

Here ϑ_{0i} is the band center, Ω is the bandwidth, and d is the magnitude of the spectral density (intensity) of the process $\nu_i(t)$ determining its dispersion (mean power) $D = d\Omega/2\pi$, while ϑ_{02} is unknown and, in general case, it is not equal to ϑ_{01}.

The value λ_0 can be considered as the moment of the abrupt change in the band center of the fast-fluctuating process $\xi(t)$. The condition of the fast fluctuations can be stated as follows [8], [9]

$$\mu_{\min} = T_{\min}\Omega/2\pi >> 1, \tag{3}$$

where $T_{\min} = \min(\lambda_0, T - \lambda_0)$.

According to the observable realization

$$x(t) = \xi(t) + n(t), \qquad t \in [0, T], \tag{4}$$

it is necessary to make a decision on the presence or absence of the abrupt change in the band center of the process $\xi(t)$, while unknown parameters λ_0, ϑ_{02} can take the values from a priori intervals $0 < \Lambda_1 \le \lambda_0 \le \Lambda_2 < T$, $\Theta_1 \le \vartheta_{02} \le \Theta_2$.

3. THE SYNTHESIS OF THE DETECTION ALGORITHM

In order to synthesize the algorithm for detecting the abrupt change in the band center of the process $\xi(t)$, let us single out two possible hypotheses [5], [12]: 1) hypothesis H_0 stating that $\vartheta_{01} = \vartheta_{02}$, i.e. the abrupt change is absent; 2) hypothesis H_1 stating that $\vartheta_{01} \ne \vartheta_{02}$. For the specified alternatives the analytical expressions should be found for the decision statistics (logarithms of the functionals of the likelihood ratio).

If the received process $\xi(t)$ is the fast-fluctuating one so that the condition (3) holds, then, by applying the results of [8], [9], one gets

$$H_0: \quad L_0 = \frac{d}{N_0(N_0 + d)} \int_0^T y^2(t, \vartheta_{01})\,\mathrm{d}t - \frac{\Omega T}{2\pi} \ln\left(1 + \frac{d}{N_0}\right),$$

$$H_1: \quad L_1(\lambda, \vartheta) = L_0 + \frac{d}{N_0(N_0 + d)} \left[\int_\lambda^T y^2(t, \vartheta)\,\mathrm{d}t - \int_\lambda^T y^2(t, \vartheta_{01})\,\mathrm{d}t\right]. \tag{5}$$

Here λ, ϑ are the current values of the unknown parameters λ_0, ϑ_{02}, respectively, and $y(t, \vartheta) = \int_{-\infty}^{\infty} x(t') h(t - t', \vartheta)\,\mathrm{d}t'$ is the response of the filter to the observable realization (4) while the transfer function $H(\omega, \vartheta)$ of this filter satisfies the condition

$$|H(\omega, \vartheta)|^2 = I[(\vartheta - \omega)/\Omega] + I[(\vartheta + \omega)/\Omega]. \tag{6}$$

The expression (6) does not uniquely determine the function $H(\omega, \vartheta)$, and therefore the function $h(t, \vartheta)$. In particular, the simplest ideal filter satisfying the relation (6) is a bandpass filter with the pulse response $h(t, \vartheta) = 2[\sin(\Omega t/2)/\pi t]\cos(\vartheta t)$. And it should be noted that the physically realizable filter corresponding to it can always be implemented with the required accuracy [7]. The design of bandpass filters with the specified characteristics is considered, for example, in [13].

The structure of the algorithm for detecting the abrupt change in the band center of the random process can be determined on the basis of a generalized maximum likelihood approach [7], [8]. Then, in general terms, the decision detection rule is written as

$$\max_{\lambda \in [\Lambda_1, \Lambda_2], \vartheta \in [\Theta_1, \Theta_2]} L_1(\lambda, \vartheta) - L_0 > c, \tag{7}$$

where c is the threshold calculated according to the chosen optimality criterion. Taking into account (5), the expression (7) is transformed to the form

$$\max_{\lambda \in [\Lambda_1, \Lambda_2], \vartheta \in [\Theta_1, \Theta_2]} \int_\lambda^T \left[y^2(t, \vartheta) - y^2(t, \vartheta_{01})\right]\mathrm{d}t > c', \tag{8}$$

where $c' = cN_0(N_0 + d)/d$.

The algorithm (8) for detecting the unknown abrupt change in the band center of the fast-fluctuating random process can be technically implemented in the form of an N-channel device, each channel of which is tuned to the frequency band $[\vartheta_i - \Omega/2, \vartheta_i + \Omega/2]$, where $\vartheta_i = \Theta_1 + (i - 1/2)\Delta\vartheta$, $i = \overline{1, N}$, $\Delta\vartheta = (\Theta_2 - \Theta_1)/N$ (in parallel processing), or based on a serial spectrum analyzer [7], [8].

One of the possible block diagrams of such a device is shown in Fig.1. Here the notations are: 1 is the switch that is open for time $[0, T]$; 2^0 is a filter with transfer function $H(\omega, \vartheta_{01})$ (6); 2^i is a filter with transfer function $H(\omega, \vartheta_i)$ (6); 3 is the squarer; 4 is the subtractor; 5 is an integrator over the time interval $[0, T]$; 6 is the delay line for time T; 7 is an integrator, 8 is a device that generates the greatest of the absolute maxima of N input signals within the interval $[\Lambda_1, \Lambda_2]$ at its output; 9 is the threshold device that compares the input signal with the set threshold c' and fixes the presence of the abrupt change in the band center of the analyzed process, if this threshold is exceeded, or the absence of the abrupt change is real. Obviously, the greater the number N of channels, the more accurately the detector shown in Fig.1. reproduces the algorithm (8).

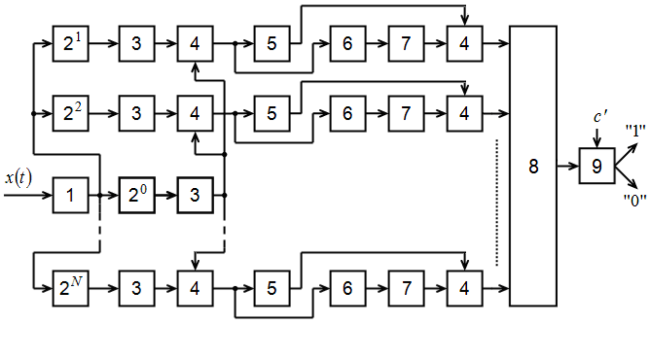

Fig.1. The block diagram of the detector of the abrupt change in the band center of the fast-fluctuating Gaussian random process.

4. THE CHARACTERISTICS OF THE DETECTION ALGORITHM

In order to evaluate the performance of the detector (8) analytically, the expressions for the probabilities of the type I (false alarm) and type II (abrupt change missing) errors α and β [7], [8] should be found. For this purpose, the decision rule (8) is represented in the form

$$\max_{l\in[\tilde{\Lambda}_1,\tilde{\Lambda}_2],\nu\in[\tilde{\Theta}_1,\tilde{\Theta}_2]} M(l,\nu)>\tilde{c}\,,\ \ M(l,\nu)=M_1(l,\nu)-M_2(l), \quad (9)$$

where $\tilde{c}=c(1+q)/\mu q$ is the normalized threshold, $l=\lambda/T$, $\nu=\vartheta/\Omega$, $\tilde{\Lambda}_{1,2}=\Lambda_{1,2}/T$, $\tilde{\Theta}_{1,2}=\Theta_{1,2}/\Omega$,

$$M_1(l,\nu)=\frac{T}{\mu N_0}\int_l^1 y^2(\tilde{t},\Omega\nu)\mathrm{d}\tilde{t}\,,$$

$$M_2(l)=\frac{T}{\mu N_0}\int_l^1 y^2(\tilde{t},\vartheta_0)\mathrm{d}\tilde{t}\,, \quad (10)$$

$\tilde{t}=t/T$, $\mu=\Omega T/2\pi$, $q=d/N_0$.

When the condition (3) is satisfied, the functionals $M_1(l,\nu)$, $M_2(l)$ (and, therefore, the functional $M(l,\nu)$) are the Gaussian ones approximately [8], [9]. The specified property allows for their full statistical description by the moment or correlation functions of the first two orders. And according to this, one represents (10) as the sum of signal and noise functions:

$$M_1(l,\nu)=S_1(l,\nu)+N_1(l,\nu), \qquad M_2(l)=S_2(l)+N_2(l).$$

Here $S_1(l,\nu)=\langle M_1(l,\nu)\rangle$, $S_2(l)=\langle M_2(l)\rangle$ are the signal functions (mathematical expectations) and $N_1(l,\nu)=M_1(l,\nu)-\langle M_1(l,\nu)\rangle$, $N_2(l)=M_2(l)-\langle M_2(l)\rangle$ are the noise functions while averaging $\langle\cdot\rangle$ is carried out over all the possible realizations of $x(t)$ (4) under the fixed values of λ_0, ϑ_{02}. By direct averaging of (10), one can find that

$$\begin{gathered}S_1(l,\nu)=\max(0,l-l_0)\left[1+qC_1(\nu-\nu_{01})\right]+\\+\left[1-\max(l_0,l)\right]\left[1+qC_1(\nu-\nu_{02})\right],\\S_2(l)=(1+q)\max(0,l-l_0)+\\+\left[1-\max(l_0,l)\right]\left[1+qC_1(\Delta\nu)\right],\end{gathered} \quad (11)$$

$$\begin{gathered}\langle N_1(l_1,\nu_1)N_1(l_2,\nu_2)\rangle=\{\left[1-\max(l_1,l_2)\right]C_1(\nu_2-\nu_1)+\\+q(2+q)\left[\max(0,l_0-\max(l_1,l_2))C_3(\nu_{01},\nu_1,\nu_2)+\right.\\\left.+(1-\max(l_0,l_1,l_2))C_3(\nu_{02},\nu_1,\nu_2)\right]\}/\mu_1\,,\\\langle N_2(l_1)N_2(l_2)\rangle=\{(2+q)^2\max(0,\min(l_1,l_2)-l_0)+\\+\left[1-\max(l_0,l_1,l_2)\right]\left[1+q(2+q)C_1(\Delta\nu)\right]\}/\mu_1\,,\end{gathered}$$

where $l_0=\lambda_0/T$, $\nu_{0i}=\vartheta_{0i}/\Omega$, $i=1,2$, $\Delta\nu=\nu_{01}-\nu_{02}$,

$$C_1(x)=\max(0,1-|x|),$$

$$C_3(x,y,z)=\max(0,1+\min(x,y,z)-\max(x,y,z)).$$

Let now the abrupt change in the band center of the process $\xi(t)$ (1) be absent, that is $\vartheta_{01}=\vartheta_{02}$. Then, after using (9)-(11), the false alarm probability can be represented as follows

$$\alpha=P\Big[\max_{l\in[\tilde{\Lambda}_1,\tilde{\Lambda}_2],\nu\in[\tilde{\Theta}_1,\tilde{\Theta}_2]} M(l,\nu)>\tilde{c}\,\Big|\,\vartheta_{01}=\vartheta_{02}\Big]. \quad (12)$$

Here $M(l,\nu)$ is the asymptotically (under $\mu_{\min}$ (3) increasing) Gaussian random field with the signal function

$$S(l,\nu)=\langle M(l,\nu)\rangle=q(1-l)\left[C_1(\nu_1-\nu_{01})-1\right] \quad (13)$$

and the correlation function of the noise function $N(l,\nu)=M(l,\nu)-\langle M(l,\nu)\rangle$ of the form

$$\begin{gathered}\langle N(l_1,\nu_1)N(l_2,\nu_2)\rangle=\min(1-l_1,1-l_2)\times\\\times\{C_1(\nu_1-\nu_2)-C_3(\nu_{01},\nu_1,\nu_2)+(1+q)^2\times\\\times\left[1+C_3(\nu_{01},\nu_1,\nu_2)-C_1(\nu_1-\nu_{01})-C_1(\nu_2-\nu_{01})\right]\}/\mu\,.\end{gathered} \quad (14)$$

Let us denote

$$l'=1-l\,, \qquad \nu'=\nu-\nu_{01}\,.$$

Then, from (13), (14), it follows that the Gaussian random field $M(l,\nu)=M(l',\nu')$ is statistically equivalent to the product

$$U(l')V(\nu'),\ \ l'\in\left[1-\tilde{\Lambda}_2,1-\tilde{\Lambda}_1\right],\ \ \nu'\in\left[\tilde{\Theta}_1-\nu_{01},\tilde{\Theta}_2-\nu_{01}\right]$$

of the independent Gaussian random processes $U(l')$ and $V(\nu')$ with the characteristics

$$S_U(l') = \langle U(l') \rangle = l' ,$$
$$B_U(l'_1, l'_2) = \left\langle \left[U(l'_1) - \langle U(l'_1) \rangle \right] \left[U(l'_2) - \langle U(l'_2) \rangle \right] \right\rangle = \min(l'_1, l'_2) ,$$
$$S_V(v') = \langle V(v') \rangle = -q \min(1, |v'|),$$
$$B_V(v'_1, v'_2) = \left\langle \left[V(v'_1) - \langle V(v'_1) \rangle \right] \left[V(v'_2) - \langle V(v'_2) \rangle \right] \right\rangle = \{ C_1(v'_1 - v'_2) -$$
$$- C_3(0, v'_1, v'_2) + (1+q)^2 [1 + C_3(0, v'_1, v'_2) - C_1(v'_1) - C_1(v'_2)] \} / \mu . \quad (15)$$

Taking into account the last statement, one can represent the probability (12) in the form

$$\alpha = P[\max_{l' \in [1-\tilde{\Lambda}_2, 1-\tilde{\Lambda}_1]} U(l') \max_{v' \in [\tilde{\Theta}_1 - v_{01}, \tilde{\Theta}_2 - v_{01}]} V(v') > \tilde{c}] . \quad (16)$$

For the random processes $U(l')$ and $V(v')$, one introduces the distribution functions of their greatest maxima

$$F_U(x) = P[\max_{l' \in [1-\tilde{\Lambda}_2, 1-\tilde{\Lambda}_1]} U(l') < x],$$
$$F_V(y) = P[\max_{v' \in [\tilde{\Theta}_1 - v_{01}, \tilde{\Theta}_2 - v_{01}]} V(v') < y] \quad (17)$$

within the intervals $[1-\tilde{\Lambda}_2, 1-\tilde{\Lambda}_1]$ and $[\tilde{\Theta}_1 - v_{01}, \tilde{\Theta}_2 - v_{01}]$, respectively. Further, using the property of the statistical independence of the processes $U(l')$ and $V(v')$, one can define the probability α (16) as

$$\alpha = 1 - \int_0^{\infty} F_V(\tilde{c}/x) \mathrm{d}F_U(x) - \int_{-\infty}^{0} [1 - F_V(\tilde{c}/x)] \mathrm{d}F_U(x). \quad (18)$$

Let us determine the functions $F_U(x)$, $F_V(y)$ (17). According to (15), the process $U(l')$ is the Gaussian Markov (Wiener) random one [14]. Its drift $K_{U1}(y, l')$ and diffusion $K_{U2}(y, l')$ coefficients are determined by the expressions

$$K_{U1} = \left. \frac{\mathrm{d}S_U(l'_1)}{\mathrm{d}l'_1} \right|_{l_1 = l+0} + \left. \frac{y - S_U(l')}{B_U(l', l')} \frac{\partial B_U(l', l'_1)}{\partial l'_1} \right|_{l'_1 = l' + 0'} ,$$
$$K_{U2} = \left. \frac{\mathrm{d}B_{\varsigma}(l_1, l_1)}{\mathrm{d}l_1} \right|_{l_1 = l+0} - 2 \left. \frac{\partial B_{\varsigma}(l, l_1)}{\partial l_1} \right|_{l_1 = l+0} . \quad (19)$$

Here $S_U(l')$, $B_U(l', l'_1)$ are the mathematical expectation and the correlation function of the process $U(l')$ (15). Differentiating in (19) taking into account (15), one can be obtained as $K_{U1} = 1$, $K_{U2} = 1$.

The distribution function of the greatest maximum of a Gaussian Markov random process of the diffusion type with the constant drift and diffusion coefficients has been found, for example, in [15]. Using the results of [15], for the function $F_U(x)$ (17) one gets

$$F_U(x) = \frac{1}{\sqrt{2\pi(1-\tilde{\Lambda}_2)}} \int_0^{\infty} \exp\left[-\frac{(\zeta - x + 1 - \tilde{\Lambda}_2)^2}{2(1-\tilde{\Lambda}_2)} \right] \times$$
$$\times \left[\Phi\left(\frac{\zeta - 1 + \tilde{\Lambda}_1}{\sqrt{1-\tilde{\Lambda}_1}} \right) - \exp(2\zeta) \left(1 - \Phi\left(\frac{\zeta + 1 - \tilde{\Lambda}_1}{\sqrt{1-\tilde{\Lambda}_1}} \right) \right) \right] \mathrm{d}\zeta . \quad (20)$$

Let us now turn to the definition of the function $F_V(y)$ (17). From (15), it follows that $\max\langle V(v') \rangle = \langle V(0) \rangle$. Then, according to [8], [16], when $\mu \to \infty$, the position of the maximum of the process $V(v')$ converges to the value $v' = 0$ in mean square. Thus, if the condition (3) is satisfied, then the position of the maximum of the process $V(v')$ is located in a small δ-neighborhood of the point $v' = 0$ with the probability close to 1. With that in mind, the definitional domain of the process $V(v')$ can be limited to the interval $[-\delta, \delta]$, $\delta << 1$. In this case, one gets that $\max(0, 1 - f(v'_1, v'_2)) = 1 - f(v'_1, v'_2)$ where $f(v'_1, v'_2)$ is any of the functions $|v'_2 - v'_1|$, $|v'_1|$, $|v'_2|$, $\max(0, v'_1, v'_2) - \min(0, v'_1, v'_2)$, and for the correlation function (15) of the process $V(v')$, the following representation is valid:

$$B_V(v'_1, v'_2) = \frac{1 + (1+q)^2}{\mu} \begin{cases} \min(|v'_1|, |v'_2|), & v'_1 v'_2 \geq 0, \\ 0, & v'_1 v'_2 < 0. \end{cases} \quad (21)$$

From (21), it can be seen that at the intervals $[-\delta, 0]$ and $(0, \delta]$ the realizations of the process $V(v')$ are not correlated, and, therefore, they are asymptotically statistically independent as being the asymptotically Gaussian ones. Besides, within each of the specified intervals, the correlation function of the process satisfies the conditions of the Doob's theorem [17], according to which the random process $V(v')$ is the Markov random process of the diffusion type. By applying (19) and taking into account (15), the drift K_{V1} and diffusion K_{V2} coefficients of the process $V(v')$ can be determined as

$$K_{V1} = \begin{cases} q , & v' < 0 , \\ -q , & v' \geq 0 , \end{cases} \qquad K_{V2} = \left[1 + (1+q)^2 \right] / \mu .$$

The general expression for the distribution function of the absolute maximum of the Gaussian Markov random process with the piecewise constant drift coefficient and the constant diffusion coefficient has been obtained, for example, in [18]. Using the results of [18], for the function $F_V(y)$ (17) one gets

$$F_V(y)=\left[1-\exp\left(-\frac{2|K_{V1}|y}{K_{V2}}\right)\right]^2=\left[1-\exp\left(-\frac{2\mu q y}{1+(1+q)^2}\right)\right]^2, \quad (22)$$

if $y \geq 0$, and $F_V(y)=0$, if $y<0$. The error in calculating the values of the false alarm probability (12) using the formulas (18), (20), (22) decreases with μ (10) increasing.

Let us assume now that $\vartheta_{01} \neq \vartheta_{02}$ and it represents the missing probability as follows

$$\beta = P[\sup_{l\in[\tilde\Lambda_1,\tilde\Lambda_2],\, v\in[\tilde\Theta_1,\tilde\Theta_2]} M(l,v)<\tilde c \,|\, \vartheta_{01}\neq\vartheta_{02}]. \quad (23)$$

One makes the change of variables:

$$l \to l', \qquad v \to \tilde v' = v - v_{02}.$$

Then, taking into account (11), the signal function $S(l,v)=\langle M(l,v)\rangle$ and the correlation function of the noise function $N(l,v)=M(l,v)-\langle M(l,v)\rangle$ of the decision statistics $M(l,v)$ (9) can be represented in the form

$$\begin{gathered} S(l',\tilde v')=q\min(0,l_0'-l')\left[1-C_1(\tilde v'-\Delta v)\right]+ \\ +q\min(l_0',l')\left[C_1(\tilde v')-C_1(\Delta v)\right], \\ \langle N(l_1',\tilde v_1')N(l_2',\tilde v_2')\rangle=\{\min(l_1',l_2')[1+C_1(\tilde v_1'-\tilde v_2')- \\ -C_1(\tilde v_1'-\Delta v)-C_1(\tilde v_2'-\Delta v)]+q(2+q)[\max(0,\min(l_1',l_2')-l_0')\times \\ \times(1+C_3(\Delta v,\tilde v_1',\tilde v_2')-C_2(\tilde v_1'-\Delta v)-C_2(\tilde v_2'-\Delta v))+\min(l_0',l_1',l_2')\times \\ \times(C_1(\Delta v)+C_3(0,\tilde v_1',\tilde v_2')-C_3(0,\Delta v,\tilde v_1')-C_3(0,\Delta v,\tilde v_2'))]\}/\mu, \end{gathered} \quad (24)$$

where $l_0'=1-l_0$, while Δv, $C_1(x)$, and $C_3(x,y,z)$ are defined as the same as in (11).

One takes into account that the signal function $S(l',\tilde v')$ reaches the absolute maximum at the point $(l_0',0)$, while the realizations of the noise function are continuous with the probability 1. Then the output signal-to-noise ratio (SNR) for the algorithm (9) can be determined as [7], [8]

$$z^2=\frac{S^2(l_0',0)}{\langle N^2(l_0',0)\rangle}=\mu\frac{l_0'q^2\min(1,|\Delta v|)}{1+(1+q)^2}. \quad (25)$$

From (25), it follows that the SNR $z^2 >> 1$, if the inequality (3) is satisfied and the value of q is not too small. In this case, the coordinates of the position of the absolute maximum of the functional $M(l',\tilde v')$ are located in a small δ-neighborhood of the point $(l_0',0)$. With z^2 increasing ($z^2 \to \infty$), the size of this neighborhood $\delta=\max(|l'-l_0'|,|\tilde v'|)\to 0$ [8], [16], and for the signal function and the correlation function of the noise function (24), the following asymptotic representations can be applied:

$$\begin{gathered} S(\tilde l',\tilde v')=S_0+S_1(\tilde l')+S_2(\tilde v')+o(\delta), \\ \langle N(\tilde l_1',\tilde v_1')N(\tilde l_2',\tilde v_2')\rangle=\left[R_1(\tilde l_1',\tilde l_2')+R_2(\tilde v_1',\tilde v_2')\right]/\mu+o(\delta), \end{gathered} \quad (26)$$

where $\tilde l'=l'-l_0'$,

$$\begin{gathered} S_0=l_0'q\min(1,|\Delta v|), \quad S_1(\tilde l')=-q\min(1,|\Delta v|)|\tilde l'|, \\ S_2(\tilde v')=-ql_0'|\tilde v'|, \\ R_1(\tilde l_1',\tilde l_2')=\left[1+(1+q)^2\right]\min(1,|\Delta v|)\min(\tilde l_1',\tilde l_2'), \\ R_2(\tilde v_1',\tilde v_2')=l_0'\left[\min(1,|\tilde v_1'-\Delta v|)+\min(1,|\tilde v_2'-\Delta v|)-|\tilde v_2'-\tilde v_1'|\right]+ \\ +l_0'q(2+q)\left[1+\min(0,\tilde v_1',\tilde v_2')-\max(0,\tilde v_1',\tilde v_2')+\right. \\ \left.+C_1(\Delta v)-C_3(0,\Delta v,\tilde v_1')-C_3(0,\Delta v,\tilde v_2')\right] \end{gathered} \quad (27)$$

and $o(\delta)$ denotes the higher-order infinitesimal terms compared with δ.

As it is described in [18], one moves from (9) to the difference functional

$$\Delta(\tilde l',\tilde v')=\left[M(\tilde l',\tilde v')-M_0\right]/\sigma,$$

where $M_0=M(0,0)$ is the asymptotically (with $\mu_{\min}$ (3) increasing) Gaussian random variable with the characteristics $\langle M_0\rangle=S_0$, $\langle M_0^2\rangle=\sigma^2$, $\sigma^2=l_0'\left[1+(1+q)^2\right]\min(1,|\Delta v|)/\mu$, so the missing probability (23) can be written in the form

$$\beta=P[\sup_{\tilde l'\in[l_0-\tilde\Lambda_2,\,l_0-\tilde\Lambda_1],\ \tilde v'\in[\tilde\Theta_1-v_{02},\,\tilde\Theta_2-v_{02}]}\Delta(\tilde l',\tilde v')<(\tilde c-M_0)/\sigma\,|\,v_{01}\neq v_{02}]. \quad (28)$$

From (26), (27), it follows that under $\delta\to 0$ the first two moments of the functional $\Delta(\tilde l',\tilde v')$ are determined as

$$\begin{gathered} \langle\Delta(\tilde l',\tilde v')\rangle=S_{\Delta 1}(\tilde l')+S_{\Delta 2}(\tilde v')+o(\delta), \\ \left\langle\left[\Delta(\tilde l_1',\tilde v_1')-\langle\Delta(\tilde l_1',\tilde v_1')\rangle\right]\left[\Delta(\tilde l_2',\tilde v_2')-\langle\Delta(\tilde l_2',\tilde v_2')\rangle\right]\right\rangle= \\ =B_{\Delta 1}(\tilde l_1',\tilde l_2')+B_{\Delta 2}(\tilde v_1',\tilde v_2')+o(\delta), \end{gathered} \quad (29)$$

where

$$\begin{gathered} S_{\Delta 1}(\tilde l')=-z|\tilde l'|/l_0', \qquad S_{\Delta 2}(\tilde v')=-z|\tilde v'|/\min(1,|\Delta v|), \\ B_{\Delta 1}(\tilde l_1',\tilde l_2')=\frac{1}{l_0'}\begin{cases}\min(|\tilde l_1'|,|\tilde l_2'|), & \tilde l_1'\tilde l_2'\geq 0, \\ 0, & \tilde l_1'\tilde l_2'<0,\end{cases} \\ B_{\Delta 2}(\tilde v_1',\tilde v_2')=\frac{1}{\min(1,|\Delta v|)}\begin{cases}\min(|\tilde v_1'|,|\tilde v_2'|), & \tilde v_1'\tilde v_2'\geq 0, \\ 0, & \tilde v_1'\tilde v_2'<0,\end{cases} \end{gathered} \quad (30)$$

and z is the SNR (25).

One should take into consideration the statistically independent Gaussian random processes $r_1(\tilde{l}')$, $r_2(\tilde{v}')$ with mathematical expectations (30) $\langle r_1(\tilde{l}')\rangle = S_{\Delta 1}(\tilde{l}')$, $\langle r_2(\tilde{v}')\rangle = S_{\Delta 2}(\tilde{v}')$ and correlation functions (30)

$$\left\langle \left[r_1(\tilde{l}'_1) - \langle r_1(\tilde{l}'_1)\rangle \right]\left[r_1(\tilde{l}'_2) - \langle r_1(\tilde{l}'_2)\rangle \right]\right\rangle = B_{\Delta 1}(\tilde{l}'_1, \tilde{l}'_2),$$
$$\left\langle \left[r_2(\tilde{v}'_1) - \langle r_2(\tilde{v}'_1)\rangle \right]\left[r_2(\tilde{v}'_2) - \langle r_2(\tilde{v}'_2)\rangle \right]\right\rangle = B_{\Delta 2}(\tilde{v}'_1, \tilde{v}'_2).$$

If $\delta \to 0$, then the characteristics (29) of the functional $\Delta(\tilde{l}', \tilde{v}')$ coincide with the corresponding characteristics of the sum of $r_1(\tilde{l}') + r_2(\tilde{v}')$. Therefore, under conditions of high a posteriori accuracy, when $z^2 >> 1$ (25), the probability (28) can be approximately represented as follows [19]

$$\beta \approx P[\sup_{\tilde{l}' \in [-\delta,\delta]} r_1(\tilde{l}') + \sup_{v' \in [-\delta,\delta]} r_2(\tilde{v}') < (\tilde{c} - M_0)/\sigma] = \int_{-\infty}^{\tilde{c}/\sigma} w_0(u) \left[\int_0^{\tilde{c}/\sigma - u} F_1(\tilde{c}/\sigma - u - x) w_2(x) \mathrm{d}x \right] \mathrm{d}u. \quad (31)$$

Here $w_2(x) = \mathrm{d}F_2(x)/\mathrm{d}x$, $F_1(x) = P[\sup_{l' \in [-\delta,\delta]} r_1(\tilde{l}') < x]$, $F_2(x) = P[\sup_{v' \in [-\delta,\delta]} r_2(\tilde{v}') < x]$ are the distribution functions of the absolute maxima of the random processes $r_1(\tilde{l}')$ and $r_2(\tilde{v}')$, while

$$w_0(u) = \exp\left[-(u-z)^2/2\right] / \sqrt{2\pi} \quad (32)$$

is the probability density of the random variable M_0/σ. In (31), one takes into account that if $x < 0$, then $w_2(x) = 0$ and $F_1(x) = 0$ as $r_1(0) = 0$ and $r_2(0) = 0$.

According to (29), (30), the values of the Gaussian process $r_1(\tilde{l}')$ ($r_2(\tilde{v}')$) at the intervals $[-\delta, 0]$ and $(0, \delta]$ are not correlated and, therefore, they are statistically independent. Then

$$F_1(x) = F_{11}(x) F_{12}(x), \quad F_2(x) = F_{21}(x) F_{22}(x), \quad (33)$$

where

$$F_{11}(x) = P[\sup_{\tilde{l}' \in [-\delta,0]} r_1(\tilde{l}') < x], \quad F_{12}(x) = P[\sup_{\tilde{l}' \in (0,\delta]} r_1(\tilde{l}') < x],$$
$$F_{21}(x) = P[\sup_{\tilde{v}' \in [-\delta,0]} r_2(\tilde{v}') < x], \quad F_{22}(x) = P[\sup_{\tilde{v}' \in (0,\delta]} r_2(\tilde{v}') < x] \quad (34)$$

Now it is time to introduce the random processes $\Delta_{11}(\tilde{l}') = x - r_1(-\tilde{l}')$, $\Delta_{21}(v') = x - r_2(-v')$, $\Delta_{12}(l') = x - r_1(\tilde{l}')$, $\Delta_{22}(\tilde{v}') = x - r_2(\tilde{v}')$, $\tilde{l}' \in [0, \delta]$, $\tilde{v}' \in [0, \delta]$. From (29), (30), it follows that the processes $\Delta_{1i}(\tilde{l}')$, $\Delta_{2i}(\tilde{v}')$, $i = 1,2$, while $\tilde{l}' \geq 0$, $\tilde{v}' \geq 0$, satisfy the conditions of the Doob's theorem [17] and they are Gaussian Markov processes with the drift coefficients $a_{li} = z/l'_0$, $a_{vi} = z/\min(1, |\Delta v|)$ and the diffusion coefficients $b_{li} = 1/l'_0$, $b_{vi} = 1/\min(1, |\Delta v|)$, $i = 1,2$. Moreover, according to [18], [19], the desired distribution functions (34) can be represented in the form

$$F_{1i}(x) = P[\sup_{\tilde{l}' \in [0,\delta]} \Delta_{1i}(\tilde{l}') > 0] = \int_0^\infty w_{1i}(y, \delta) \mathrm{d}y,$$
$$F_{2i}(x) = P[\sup_{\tilde{v}' \in [0,\delta]} \Delta_{2i}(\tilde{v}') > 0] = \int_0^\infty w_{2i}(y, \delta) \mathrm{d}y,$$

where the probability densities $w_{1i}(y, \tilde{l}')$, $w_{2i}(y, \tilde{v}')$ are determined from the solution of the direct Fokker-Planck-Kolmogorov equation [14] with the drift coefficients a_{li}, a_{vi} and the diffusion coefficients b_{li}, b_{vi}, while the starting conditions are $w_{1i}(y,0) = w_{2i}(y,0) = \delta(x-y)$ and the boundary conditions are $w_{1i}(0, \tilde{l}') = w_{1i}(\infty, \tilde{l}') = w_{2i}(0, \tilde{v}') = w_{2i}(\infty, \tilde{v}') = 0$. After solving these equations and integrating the found solutions as described, for example, in [18], taking into account (33), one obtains

$$\begin{gathered} F_1(x) = f(x, a_{l1}, b_{l1}) f(x, a_{l2}, b_{l2}), \\ F_2(x) = f(x, a_{v1}, b_{v1}) f(x, a_{v2}, b_{v2}), \\ w_2(x) = g(x, a_{v1}, b_{v1}) f(x, a_{v2}, b_{v2}) + \\ + g(x, a_{v2}, b_{v2}) f(x, a_{v1}, b_{v1}), \end{gathered} \quad (35)$$

if $x \geq 0$, and $F_1(x) = F_2(x) = 0$, $w_2(x) = 0$, if $x < 0$. Here

$$f(x,a,b) = \Phi\left(\frac{a\delta + x}{\sqrt{b\delta}}\right) - \exp\left(-\frac{2ax}{b}\right)\Phi\left(\frac{a\delta - x}{\sqrt{b\delta}}\right),$$
$$g(x,a,b) = \frac{2a}{b}\exp\left(-\frac{2ax}{b}\right)\Phi\left(\frac{a\delta - x}{\sqrt{b\delta}}\right) + \sqrt{\frac{2}{\pi b\delta}}\exp\left[-\frac{(a\delta + x)^2}{2b\delta}\right]. \quad (36)$$

In practice, the calculation of the probability (23) by the formula (31) using (35), (36) is very difficult (as the value of δ cannot be exactly determined). However, if condition $z^2 >> 1$ (25) holds, then in (35), without any significant loss in accuracy, one can use the asymptotic approximations of the functions (36) of the form [18]

$$f(x,a,b) = 1 - \exp(-2ax/b),$$
$$g(x,a,b) = (2a/b)\exp(-2ax/b). \quad (37)$$

By substituting (37) into (35) and then (32), (35) into (31) with subsequent integration by the variables *x*, *u*, after all the corresponding transformations, one gets

$$\beta \approx \Phi\left(\frac{\tilde{c}}{\sigma} - z\right) + 4\left(6z^2 - \frac{2\tilde{c}z}{\sigma} + 1\right)\exp\left[2z\left(2z - \frac{\tilde{c}}{\sigma}\right)\right] \times$$
$$\times \Phi\left(\frac{\tilde{c}}{\sigma} - 3z\right) + \left(20z^2 - \frac{4\tilde{c}z}{\sigma} - 5\right)\exp\left[4z\left(3z - \frac{\tilde{c}}{\sigma}\right)\right] \times \quad (38)$$
$$\times \Phi\left(\frac{\tilde{c}}{\sigma} - 5z\right) - \frac{12z}{\sqrt{2\pi}}\exp\left[-\frac{(\tilde{c} - z\sigma)^2}{2\sigma^2}\right].$$

The accuracy of the formula (38) increases with $\mu_{\min}$ (3) and *z* (25).

5. THE SIMULATION RESULTS

In order to determine the errors in the approximate formulas obtained for the characteristics of the synthesized detection algorithm, the statistical computer simulation of the detector (9) operation is carried out [20].

In the simulation process, over the interval $\tilde{t} \in [0,1]$ (10) the samples $\tilde{y}_{km} = y(T\tilde{t}_k, \Omega v_m)\sqrt{T/N_0}$ (10) are formed at the discrete points in time $\tilde{t}_k = k\Delta\tilde{t}$, $k = \overline{0, \mathrm{int}\{1/\Delta\tilde{t}\}}$ and for each of the values of $v_m = \tilde{\Theta}_1 + (m + 1/2)\Delta V$, $m = \overline{0, \mathrm{int}\{(\tilde{\Theta}_2 - \tilde{\Theta}_1)/\Delta V - 1\}}$ belonging to the normalized band center *v* (10), as described in [18], [21]. Further, using the generated samples $\tilde{y}_{km}$, within the intervals $[\tilde{\Lambda}_1, \tilde{\Lambda}_2]$, $[\tilde{\Theta}_1, \tilde{\Theta}_2]$, the samples $M_{1nm} = M_1(l_n, v_m)$, $M_{2n} = M_1(l_n, v_{m_0})$ of the random field $M_1(l,v)$ and the random process $M_2(l)$ (10) are calculated with the discretization steps Δl and ΔV by the variables *l* and *v*, respectively. Here $l_n = \tilde{\Lambda}_1 + n\Delta l$, $n = \overline{0, \mathrm{int}\{(\tilde{\Lambda}_2 - \tilde{\Lambda}_1)/\Delta l\}}$, and the value m_0 corresponds to the frequency v_{01}, so $v_{m_0} = \tilde{\Theta}_1 + m_0\Delta V = v_{01}$.

The discretization steps are chosen to be equal to $\Delta\tilde{t} = 0.05/\mu_{\min}$ by the variable $\tilde{t}$ and to be equal to $\Delta l = \Delta v = 0.01$ by the variables *l* and *v*. As a result, the relative standard error of the stepwise approximation of the functional $M(l,v)$ (9) based on the generated samples, calculated by the technique proposed in [19], does not exceed 10 %.

As an empirical estimate of the false alarm probability α (12), there is used the relative frequency of exceeding the threshold $\tilde{c}$ (9) by the greatest samples out of the decision statistic samples $M_{nm} = M(l_n, v_m)$ in the absence of the abrupt change in the random process $\xi(t)$ (1) band center.

As an empirical estimate of the missing probability β (23), there is taken the relative frequency of not exceeding the threshold $\tilde{c}$ (9) by the greatest sample out of the decision statistics samples M_{nm} in the presence of the abrupt change in the random process band center.

In Fig.2. and Fig.3., some simulation results and the corresponding theoretical curves are presented. In order to obtain each experimental value, at least 10^4 realizations of $x(t)$ (4) are processed under $\tilde{\Lambda}_1 = 0.1$, $\tilde{\Lambda}_2 = 0.9$, $\tilde{\Theta}_1 = 8$, $\tilde{\Theta}_2 = 12$, $v_{01} = 9.5$. All of this allows us to provide such deviation of the confidence interval boundaries from the obtained experimental data that is not greater than 15 % with the probability 0.9.

In Fig.2., by solid lines the theoretical dependences are presented of the false alarm probability α upon the threshold $\tilde{c}$ calculated by the formulas (18), (20), (22). Curve 1 is plotted for $\mu = 100$, $q = 0.25$; curve 2 is plotted for $\mu = 200$, $q = 0.25$; curve 3 is plotted for $\mu = 200$, $q = 1$. The corresponding experimental values of the false alarm probability are drawn by squares, crosses, and rhombuses.

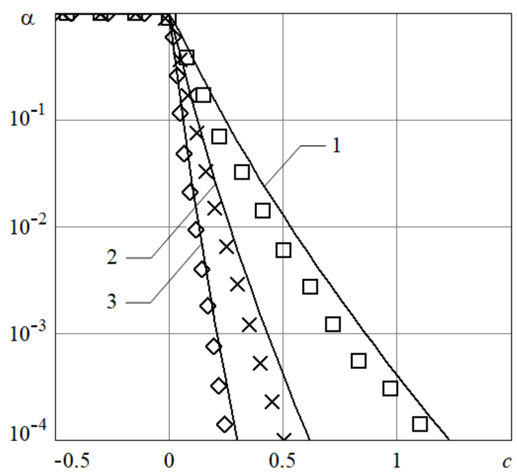

Fig.2. The false alarm probability.

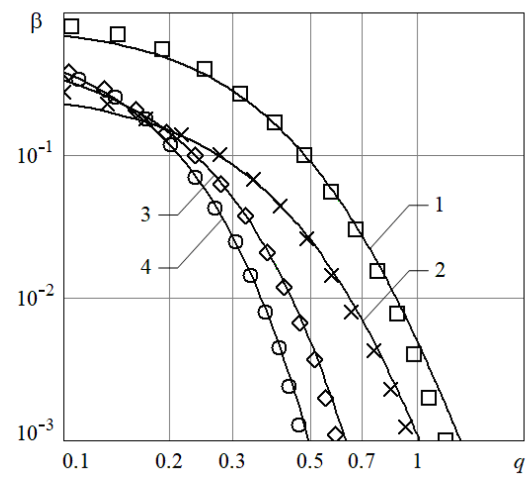

Fig.3. The missing probability.

In Fig.3., by solid lines, the theoretical dependences are presented of the missing probability β upon the value of the parameter q (11) calculated by the formula (38). The threshold value $\widetilde{c}$ is calculated by means of (18), (20), (22) according to the Neumann-Pearson criterion for the chosen level of false alarm probability that is 10^{-6} in this case. Curve 1 is plotted for $\mu = 100$, $l_0 = 0.5$, $|\Delta v| = 1$; curve 2 is plotted for $\mu = 200$, $l_0 = 0.5$, $|\Delta v| = 0.5$; curve 3 is plotted for $\mu = 200$, $l_0 = 0.5$, $|\Delta v| = 1$; curve 4 is plotted for $\mu = 100$, $l_0 = 0.25$, $|\Delta v| = 1$. The corresponding experimental values of the missing probability are drawn by squares, crosses, rhombs, and circles.

From Fig.2. and Fig.3., it follows that the theoretical dependences obtained for the probability α that are (18), (20), (22) and for the probability β that is (38) are in good agreement with the experimental data, at least in the case defined by $\mu_{min} \geq 25$, $q \geq 0.1$, $\widetilde{\Lambda}_1 \geq 0.1$, $\widetilde{\Lambda}_2 \leq 0.9$.

6. Conclusions

Based on the results obtained, the following conclusions can be made.

1. The developed algorithm applying the maximum likelihood method for detecting the unknown abrupt change in the random process band center in the conditions of the fast fluctuations of the observable data realization allows such hardware implementation that is significantly simpler than the ones that are required by the algorithms obtained by means of the common approaches.

2. The characteristics of the algorithm for detecting the unknown abrupt change in the fast-fluctuating random process band center can be analytically calculated with the help of the multiplicative and additive generalizations of the local Markov approximation method adapted for the case of several unknown discontinuous parameters. And it should be noted that the quality of the algorithm for detecting the abrupt change is better, the earlier the abrupt change time occurs and the greater is the difference (within the random process bandwidth) between the center frequencies before and after the abrupt change. If the difference between the center frequencies before and after the abrupt change exceeds the random process bandwidth, then the detection quality does not improve with its further increase.

3. The presented theoretical results deal with a wide range of the random process parameter values and as such are in good agreement with the corresponding experimental data obtained by means of statistical computer simulation.

4. As it can be seen from the additional analysis, the detectors synthesized on the basis of the proposed approach can also be used, and without any significant loss in performance, in case when receiving fast-fluctuating non-Gaussian random processes with the unknown abrupt change in the band center.

The devices specified above can be effectively implemented by means of modern digital signal processors or field-programmable gate arrays [22].

Acknowledgment

This research was financially supported by the Russian Science Foundation (research project No. 20-61-47043).

References

[1] Kligene, N., Tel'ksnis, L. (1983). Methods to determine the times when the properties of random processes change. *Automation and Remote Control*, 41 (11), 1241-1283.

[2] Basseville, M., Nikiforov, I.V. (1993). *Detection of Abrupt Changes: Theory and Application*. Prentice-Hall.

[3] Hanus, R., Kowalczyk, A., Chorzepa, R. (2018). Application of conditional averaging to time delay estimation of random signals. *Measurement Science Review*, 18 (4), 130-137.

[4] Konev, V., Vorobeychikov, S. (2017). Quickest detection of parameter changes in stochastic regression: Nonparametric CUSUM. *IEEE Transactions on Information Theory*, 63 (9), 5588-5602.

[5] Tartakovsky, A.G. (2019). Asymptotic optimality of mixture rules for detecting changes in general stochastic models. *IEEE Transactions on Information Theory*, 65 (3), 1413-1429.

[6] Chernoyarov, O.V., Marcokova, M., Salnikova, A.V., Maksimov, M.I., Makarov, A.A. (2019). Measuring the moment and the magnitude of the abrupt change of the Gaussian process bandwidth. *Measurement Science Review*, 19 (7), 250-256.

[7] Van Trees, H.L., Bell, K.L., Tian, Z. (2013). *Detection, Estimation, and Modulation Theory, Part I: Detection, Estimation, and Filtering Theory*, Second Edition. Wiley.

[8] Trifonov, A.P., Nechaev, E.P., Parfenov, V.I. (1991). *Detection of Stochastic Signals with Unknown Parameters*. Voronezh, USSR: Voronezh State University. (in Russian)

[9] Chernoyarov, O.V., Vaculik, M., Shirikyan, A., Salnikova, A.V. (2015). Statistical analysis of fast fluctuating random signals with arbitrary-function envelope and unknown parameters. *Communications – Scientific Letters of the University of Zilina*, 17 (1A), 35-43.

[10] Chernoyarov, O.V., Sai Si Thu Min, Salnikova, A.V., Kuba, M. (2014). Detection and estimation of abrupt changes in Gaussian random processes with unknown parameters. In *10th International Conference ELEKTRO 2014*. IEEE, 46-51.

[11] Chernoyarov, O.V., Shahmoradian, M.M., Salnikova, A.V., Buravlev, I.A. (2017) Detection and measurement of the abrupt change of the power parameters of the fast-fluctuating Gaussian random process. In *Proceedings of the 2017 2nd International Conference on Modelling, Simulation and Applied Mathematics (MSAM2017)*, AISR 132. Atlantis Press, 164-168.

[12] Chernoyarov, O.V., Dobrucky, B., Salnikova, A.V., Makarov, A.A. (2019). Detection and measurement of the unknown moment and magnitude of the Gaussian random process energy parameters abrupt change. *International Review on Modelling and Simulations*, 12 (5), 264-280.

[13] Kuphaldt, T.R. (2012). *Lessons in Electric Circuits, Vol. 2: Alternate Current*. Koros Press.

[14] Dynkin, E.B. (2006). *Theory of Markov Processes*. Dover Publications.

[15] Chernoyarov, O.V., Salnikova, A.V., Golpaiegani, L.A. (2017). On probability of the Gaussian random processes crossing the barriers. In *2017 3rd International Conference on Frontiers of Signal Processing*. IEEE, 1-7.

[16] Kutoyants, Y.A. (1994). *Identification of Dynamical Systems with Small Noise*. Springer.

[17] Kailath, T. (1966). Some integral equations with nonrational kernels. *IEEE Transactions on Information Theory*, 12 (4), 442-447.

[18] Chernoyarov, O.V., Sai Si Thu Min, Salnikova, A.V., Shakhtarin, B.I., Artemenko, A.A. (2014). Application of the local Markov approximation method for the analysis of information processes processing algorithms with unknown discontinuous parameters. *Applied Mathematical Sciences*, 8 (90), 4469-4496.

[19] Zakharov, A.V., Pronyaev, E.V., Trifonov, A.P. (2001). Detection of step random disturbance. *Izvestiya Akademii Nauk. Teoriya i Sistemy Upravleniya*, (7), 29-37.

[20] Altug, Y., Wagner, A.B. (2012). Source and channel simulation using arbitrary randomness. *IEEE Transactions on Information Theory*, 58 (3), 1345-1360.

[21] Chernoyarov, O.V., Salnikova, A.V., Makarov, A.A. (2019). Simulation of the measurer of the time of appearance and the average power of the random pulse signal. In *2019 5th International Conference on Frontiers of Signal Processing*. IEEE, 105-110.

[22] Garro, U., Muxika, E., Aizpurua, J.I., Mendicute, M. (2020). FPGA-based stochastic activity networks for online reliability monitoring. *IEEE Transactions on Industrial Electronics*, 67 (6), 5000-5011.

Received June 4, 2020
Accepted August 10, 2020