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A Parameter Estimation Algorithm for Damped Real-value Sinusoid in Noise

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Abstract: To improve the parameter estimation performance of damped real-value sinusoid in noise, a novel algorithm with high accuracy and computational efficiency is proposed that combines the characteristics of good anti-interference, small computation of frequency-domain methods, and high parameter estimation accuracy of time-domain methods. First, the Discrete Fourier Transform (DFT) algorithm and the two-point spectrum interpolation algorithm of the frequency-domain methods are used to improve the noise immunity. Then, the linear prediction property and the enhancement filter of the time-domain methods are used to improve the parameter estimation accuracy. In addition, the parameter estimation performance of the proposed algorithm is verified by computational complexity analysis and test experiments, and the practical application effectiveness of the proposed algorithm is demonstrated on the Coriolis Mass Flowmeter (CMF) experimental platform. The experimental results show that the proposed algorithm effectively improves the real-time performance and the parameter estimation accuracy is better than that of the existing excellent algorithms.

Keywords: Parameter estimation, spectrum interpolation, linear prediction, damped real-value sinusoid.

1. INTRODUCTION

Parameter estimation of damped real-value sinusoidal signal in noise is a basic but significant problem in signal processing. It is used in many areas, such as signal spectrum analysis, power quality detection systems, instrument measurement devices, and others [1]-[4]. For instance, the free damped vibration signal of the flow tube of a digital Coriolis Mass Flowmeter (CMF) can be used to track the natural frequency change of the flow tube, and the natural frequency is used to drive the vibration of the flow tube [5]-[7].

The signal model of damped real-value sinusoid in noise is as follows [8], [9].

$$x(n) = ae^{-\eta n} cos(\omega n + \theta) + z(n) \quad n = 0,1,\cdots,N-1 \tag{1}$$

where $a > 1$, $0 < \omega < \pi$, $-\pi < \theta < \pi$, and $\eta > 0$ denote the amplitude, frequency, initial phase, and damping factor of the sampled signal, respectively. n is a sampling index and N stands for signal length. z(n) is additive white Gaussian noise with a mean value of 0 and a variance of σ^2, and the Signal-to-Noise Ratio (SNR) is defined as:

$$SNR = 10lg\frac{a^2}{2\sigma^2} \tag{2}$$

In recent decades, parameter estimation algorithms for damped real-value sinusoids have been extensively researched by scientists, which can be classified into frequency- and time-domain methods [10],[11].

Frequency-domain methods convert sampled signals from the time domain to the frequency domain for spectrum analysis and mainly include iterative interpolation methods and leakage correction methods [12]. In spectrum analysis, the damped real-value sinusoid can be considered as a superposition signal of positive- and negative-frequency components. To suppress the spectrum leakage influence of the negative frequency component, the sampled signal was processed by windowing and spectrum interpolation in [13]. To improve the parameter estimation accuracy at low frequencies, a novel spectrum interpolation algorithm was proposed in [14]. First, the sampled signal is analyzed by the Discrete Fourier Transform (DFT) algorithm and the signal spectrum is interpolated by two points. Then, spectrum values of the interpolation points are corrected by a subtraction

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strategy. This algorithm effectively reduces the spectrum leakage influence, but the parameter estimation accuracy still decreases with increasing SNRs at low signal frequencies. Based on this, a Spectrum Leakage Correction (SLC) algorithm was proposed in [15]. The authors changed the interpolation interval and added iterations to further improve the parameter estimation accuracy. The parameter estimation accuracy of SLC is higher than that of other frequency-domain algorithms. However, there is still an estimation deviation when the signal frequency is low and SNR is high.

Time-domain methods process the sampled signal directly in the time domain using the linear prediction property, Least Squares method (LS), prediction matrix, etc., and have high parameter estimation accuracy under medium and high SNR conditions [16]-[18]. However, the anti-interference is poor, which means that the estimation accuracy is greatly affected by noise. Moreover, the computation of the time-domain methods is extensive due to the iterative matrix solution [19]. Therefore, due to the poor anti-interference and real-time performance, the time-domain methods are not suitable for practical application [20].

To overcome the spectrum leakage influence, poor anti-interference and mass calculations, a novel algorithm combining the advantages of frequency- and time-domain methods is proposed. In this paper, the classical Steiglitz and McBride (STMB) algorithm of time-domain methods is introduced in section 2, and the proposed algorithm is presented in section 3. The algorithm performance analysis is demonstrated in section 4, and measurement experiments are performed in section 5. Finally, the entire text is summarized.

2. THE STMB ALGORITHM

STMB is a classical time-domain algorithm with a very high parameter estimation accuracy, and the computational process is described as follows [21].

First, according to the prediction property of the sinusoidal signal, the prediction relation is constructed.

$$x(n) = c(1)x(n-1) + c(2)x(n-2) + c(3)\delta(n) + c(4)\delta(n-1) \tag{3}$$

where $\delta(n) = [1,0,0,...,0]_{N*1}$ is the unit impulse function, and the prediction coefficients are:

$$\begin{cases} c(1) = 2\,cos(\omega)\,e^{-\eta} \\ c(2) = -e^{-2\eta} \\ c(3) = a\,cos(\theta) \\ c(4) = -ae^{-\eta}\,cos(\omega-\theta) \end{cases} \tag{4}$$

Second, a filter with transfer function H(z) is designed to suppress noise influence.

$$H(z) = \frac{1}{1-c(1)z^{-1}-c(2)z^{-2}} \tag{5}$$

The initial values of the filter parameters are obtained by calculating with (3) using a LS method when δ(n) is not considered.

The prediction matrix is structured by filtering the sampled signal.

$$\begin{bmatrix} V(2) \\ V(3) \\ \vdots \\ V(N-1) \end{bmatrix} = \begin{bmatrix} V(1) & V(0) & U(2) & U(1) \\ V(2) & V(1) & U(3) & U(2) \\ \vdots & \vdots & \vdots & \vdots \\ V(N-2) & V(N-3) & U(N-1) & U(N-2) \end{bmatrix} \begin{bmatrix} c(1) \\ c(2) \\ c(3) \\ c(4) \end{bmatrix} \tag{6}$$

where V(n) and U(n) denote the filtered signal of x(n) and δ(n), respectively.

Then, the accurate prediction coefficients are obtained by calculating with (5) and (6) using a LS method and an iteration procedure. In [21], the number of iterations is recommended to be 4.

By solving the polynomials:

$$1 - C(1)z^{-1} - C(2)z^{-2} = 0 \tag{7}$$

The complex root is obtained.

$$\lambda = e^{-\eta + i\omega} \tag{8}$$

Finally, signal frequency and damping factor are calculated.

$$\begin{cases} \hat{\eta} = -\,Re[ln(\lambda)] \\ \hat{\omega} = Im[ln(\lambda)] \end{cases} \tag{9}$$

where the hat above the parameter stands for the estimated value, and Re[ ] and Im[ ] denote a real and a complex part of a complex number, respectively.

Furthermore, we can obtain the amplitude and phase according to (4).

$$\begin{cases} \hat{\theta} = \tan^{-1} \frac{-(2C(4)+C(1)C(3))}{2C(3)\sin\hat{\omega}e^{-\hat{\eta}}} \\ \hat{a} = \frac{C(3)}{\cos\hat{\theta}} \end{cases} \tag{10}$$

The STMB algorithm has the highest parameter estimation accuracy among the existing algorithms. However, the matrix calculation is performed many times, which causes high computational cost. Therefore, the STMB algorithm is not suitable for practical application.

3. THE PROPOSED ALGORITHM

To reduce computational cost and improve parameter estimation accuracy, a novel algorithm is proposed that can be divided into two steps.

Step 1: The DFT and spectrum interpolation algorithm are used to estimate the coarse values of frequency and damping factor.

First, DFT is used to analyze the sampled signal and obtain the spectrum X(k) and spectrum index k_0.

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-i\frac{2\pi}{N}kn} \quad k = 0,1,\cdots,N/2-1 \tag{11}$$

$$k_0 = argmax\{|X(k)|\} \tag{12}$$

where *argmax*{ } is an index for the maximum value of a sequence.

Then the signal spectrum is interpolated with an interval of 0.5 on both sides of k_0.

$$X(k_0+\Delta k)=\sum_{n=0}^{N-1} x(n)e^{-i\frac{2\pi}{N}(k_0+\Delta k)n} \quad \Delta k=\pm 0.5 \tag{13}$$

So, the coarse values of frequency offset and damping factor are estimated.

$$\begin{cases}\hat{\delta}_c=\frac{1}{2}Re\left[\frac{X(k_0+0.5)+X(k_0-0.5)}{X(k_0+0.5)-X(k_0-0.5)}\right]\\ \hat{\eta}_c=\frac{\pi}{N}Im\left[\frac{X(k_0+0.5)+X(k_0-0.5)}{X(k_0+0.5)-X(k_0-0.5)}\right]\end{cases} \tag{14}$$

Hence, the coarse value of signal frequency is calculated.

$$\widehat{\omega}_c=\frac{2\pi}{N}\left(k_0+\hat{\delta}_c\right) \tag{15}$$

Step 2: The idea of the STMB algorithm is used to estimate the fine signal parameter values.

First, using the coarse values of frequency and damping factor, the initial values of the prediction coefficients in (4) are obtained.

$$\begin{cases}c(1)=2\cos\widehat{\omega}_c\\ c(2)=-e^{-2\hat{\eta}_c}\end{cases} \tag{16}$$

Second, the transfer function H(z) of the enhancement filter is calculated by (5).

Then, the prediction matrix is constructed according to (6) and the matrix is solved by a LS method to obtain accurate prediction coefficients c(1) and c(2). Thus, the fine estimation values for the frequency and damping factor are obtained.

$$\begin{cases}\widehat{\omega}=\cos^{-1}\frac{c(1)}{2\sqrt{-c(2)}}\\ \hat{\eta}=-\frac{\ln(-c(2))}{2}\end{cases} \tag{17}$$

Finally, the spectrum leakage influence of the negative frequency component is suppressed by a spectrum leakage correction strategy, and the complex amplitude of the sampled signal is calculated.

$$A=\frac{1-e^{-\hat{\eta}}}{1-e^{-\hat{\eta}N}}\left(\left(\sum_{n=0}^{N-1} x(n)e^{-i\widehat{\omega}n}\right)-\hat{A}^*\frac{1-e^{-(i2\widehat{\omega}+\hat{\eta})N}}{1-e^{-(i2\widehat{\omega}+\hat{\eta})}}\right) \tag{18}$$

where $A = 0.5ae^{i\theta}$, $A^* = 0.5ae^{i\theta}$.

The estimation accuracy is gradually improved by an iterative procedure and the initial value of A^* is set to 0. Simulation tests show that only four iterations are needed to calculate the complex amplitude due to precise estimation values for the frequency and damping factor. Depending on the estimation value of A, the fine estimation values for amplitude and initial phase are obtained.

$$\begin{cases}\hat{a}=2|A|\\ \hat{\theta}=\angle A\end{cases} \tag{19}$$

where | | and ∠ represent a modulus and an angle of a complex number, respectively.

4. THE ALGORITHM PERFORMANCE ANALYSIS

In this section, the algorithm performance is analyzed. First, the computational complexity is counted to analyze the real-time performance. Then, the parameter estimation accuracy is verified by experiments under different simulation conditions. Since SLC and STMB are representative algorithms for frequency- and time-domain methods, the parameter estimation performance of the proposed algorithm is compared with that of SLC and STMB in this paper.

A. *Computational complexity*

When we analyze the computational complexity, the simple computational steps are omitted and only the steps with large computation are counted. In statistics, all quantities of complex-value computations are converted into quantities of real-value computations. For example, one complex-value multiplication requires four real-value multiplications and two real-value additions, and one complex-value addition requires two real-value additions. To reduce the computational cost, the DFT is calculated using the Fast Fourier Transform (FFT). Moreover, one N-point FFT calculation requires $3N\log_2^N$ real-value additions and $2N\log_2^N$ real-value multiplications. The statistical results of the proposed algorithm are compared with those of SLC and STMB and are shown in Table 1 and Fig. 1.

Table 1. Computational complexity of algorithm.

Algorithm	Multiplication	Addition
SLC	$2N\log_2^N+96N$	$3N\log_2^N+64N$
STMB	$170N$	$152N$
Proposed	$2N\log_2^N+46N$	$3N\log_2^N+42N$

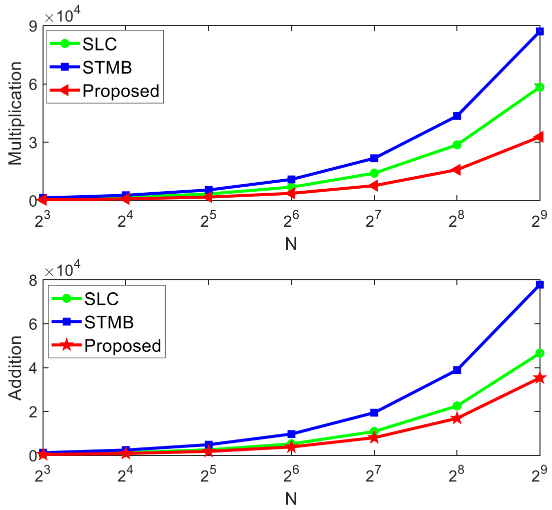


Fig. 1. Computational complexity of algorithms.

The prediction matrix is calculated by a LS method with four iterations in STMB, and the filter parameters are also obtained by one matrix calculation, which significantly increases the computational complexity. Based on the FFT calculation, SLC directly constructs a two-point interpolation spectrum and performs spectrum leakage correction by a subtraction strategy. Therefore, the computational complexity of SLC is lower than that of STMB. However, when the complex amplitude is calculated by SLC, the LS method is used for 4 times, which increases the computational complexity. In the proposed algorithm, DFT, which is calculated by the FFT, is used to estimate the coarse parameter values, and the linear prediction property and filter are used to estimate the fine parameter values. In our algorithm, the LS method is needed only once when we solve the prediction matrix to obtain high-precision parameter estimation values. Therefore, the computational complexity of the proposed algorithm is lower than that of SLC and far lower than that of STMB. Compared with SLC and STMB, the proposed algorithm has good real-time performance.

B. *Test experiments*

In this section, the parameter estimation accuracy of the proposed algorithm is tested, mainly simulating the frequency and the damping factor under different conditions. To reduce random errors caused by the computation, 10000 Monte Carlo experiments are performed for each simulation group. To simplify the analysis, the estimation results are converted into mean square errors (MSEs) and expressed in logarithms.

$$MSEs = 10\,lg\frac{\sum_{l=1}^{L}(\hat{\omega}_l-\omega)^2}{L} \tag{20}$$

where L represents the experiment times, and $\hat{\omega}_l$ is the estimation value of the *l-th* experiment.

At the same time, simulation results are compared with those calculated using SLC, STMB, and Cramer-Rao Lower Bound (CRLB). For the damped real-value sinusoid, the lower bound of the MSEs of each parameter estimation value is defined as (21) in [22].

$$\begin{cases} var(\hat{\omega}) = var(\hat{\eta}) \geq \dfrac{(1-d^2)^3(1-d^{2N})}{\left(-N^2d^{2N}(1-d^2)^2+d^2(1-d^{2N})^2\right)SNR} \\ var(\hat{\theta}) \geq \dfrac{1-d^2}{SNR}\left[1+\dfrac{(1-d^{2N})d^2(d^2+d^{2N})-2(1-d^2)d^2Nd^{2N}}{-N^2d^{2N}(1-d^2)^2+d^2(1-d^{2N})^2}\right] \\ var(\hat{a}) \geq \dfrac{(1-d^2)a^2}{SNR}\left[1+\dfrac{(1-d^{2N})d^2(d^2+d^{2N})-2(1-d^2)d^2Nd^{2N}}{-N^2d^{2N}(1-d^2)^2+d^2(1-d^{2N})^2}\right] \end{cases} \tag{21}$$

where $d = e^{-\eta}$.

Different SNRs: To test the anti-interference performance, we increase the SNR from 0 dB to 50 dB in steps of 1 dB. In the simulation test, the signal length N is 128. The amplitude a is 1, and the damping factor η is 0.005. The spectrum index k_0 is set to 2, and the spectrum offset δ is a random value of (-0.5, 0.5). In addition, the initial phase is set to θ = π/6. The results are shown in Fig. 2.

At low SNRs, the parameter estimation accuracy of SLC is high and close to CRLB. However, since the simulation signal frequency is low, SLC is seriously affected by spectrum leakage of the negative frequency component. At high SNRs, the spectrum leakage influence is greater than the noise influence. The parameter estimation accuracy of SLC gradually decreases, but does not follow the change of CRLB until the estimation saturation occurs, which means the spectrum suppression capability of SLC is limited. The anti-interference performance of STMB is poor, and the parameter estimation accuracy is not high at low SNRs. As SNR increases, the parameter estimation accuracy improves significantly and changes with CRLB. At all SNRs, the proposed algorithm has good parameter estimation accuracy, which is better than the others. The results show that the proposed algorithm effectively suppresses the influence of spectrum leakage and noise.

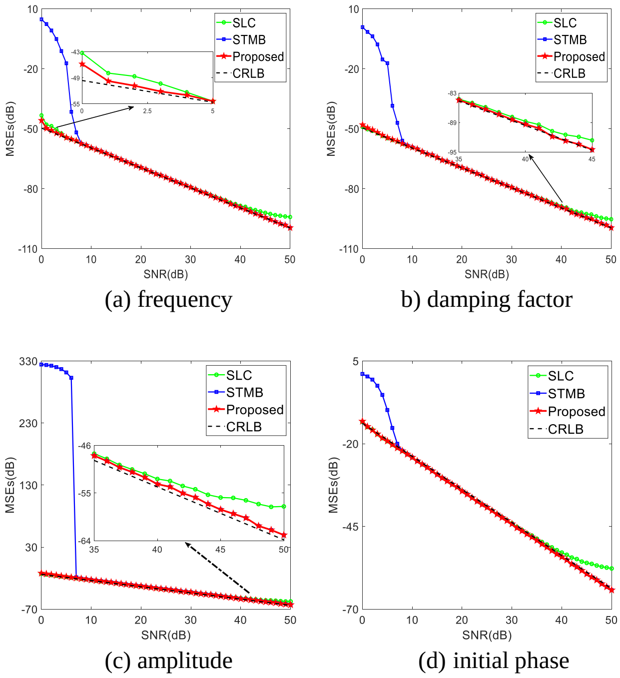


Fig. 2. Parameter estimation accuracy under different SNRs.

Different frequencies: When the frequency ω increases from 0.01 π to 0.5 π in steps of 0.01 π, the frequency and damping factor estimation experiments are performed to test the influence of different frequencies under the conditions of SNR = 10 dB and SNR = 40 dB, respectively. In this test, the signal length N is 128, and the amplitude a is 1. The damping factor η is 0.005, and the initial phase θ is a random value in (-π, π). The results are shown in Fig. 3 and Fig. 4.

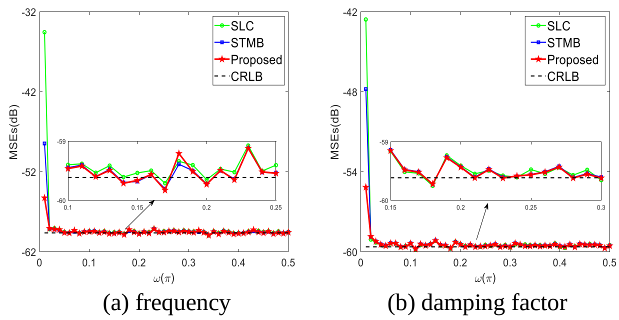
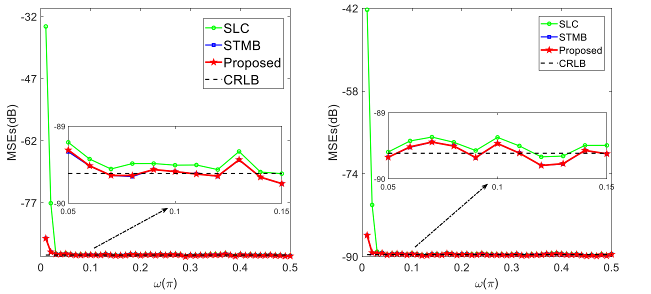


Fig. 3. Estimation results of frequency and damping factor under different frequencies when SNR is 10 dB.



(a) frequency (b) damping factor

Fig. 4. Estimation results of frequency and damping factor under different frequencies when SNR is 40 dB.

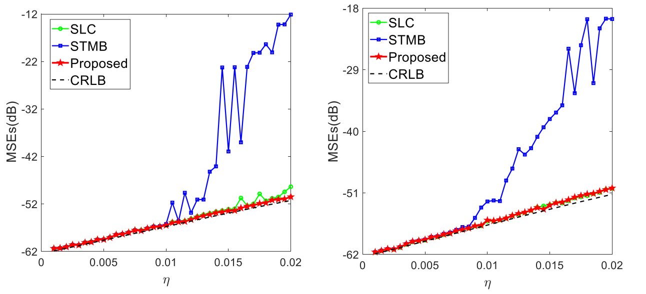
When the SNR is 10 dB and the signal frequency is low, the parameter estimation accuracy of the proposed algorithm is slightly higher than that of SLC and STMB. As the signal frequency increases, the three algorithms have equivalent estimation accuracy, which is close to that of CRLB. When the SNR is 40 dB, the proposed algorithm and STMB have equivalent parameter estimation accuracy in the whole frequency range. The accuracy is always close to that of CRLB, but better than that of SLC, which suffers from the spectrum leakage influence. Moreover, the proposed algorithm and STMB have more obvious advantages at low frequencies.

It is worth noting that CRLB is the Lower Bound of unbiased estimation algorithm, and the proposed algorithm, SLC and STMB are biased estimation algorithms. Therefore, a few estimation results slightly exceed CRLB, but this does not affect the performance comparison of the different algorithms.

Different damping degrees: To test the parameter estimation accuracy of the proposed algorithm under different damping degrees, we increase the damping factor η from 0.001 to 0.02 in steps of 0.0005 and perform experiments to estimate the frequency and damping factor under the conditions of SNR = 10 dB and SNR = 40 dB, respectively. The test results are shown in Fig. 5 and Fig. 6. In this test, the signal length N is 128, and the amplitude a is 1. The spectrum index k_0 is 2, and the spectrum offset is $\delta = -0.2$. Moreover, the initial phase θ is a random value in $(-\pi, \pi)$.

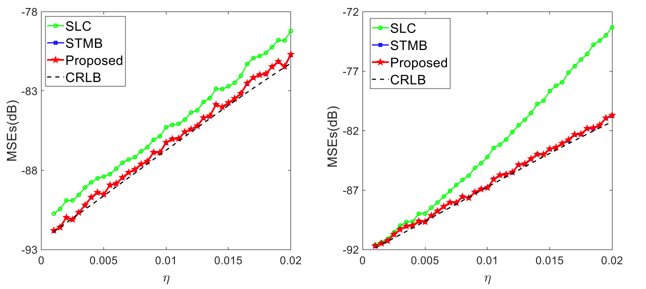
With a SNR of 10 dB and a low damping factor, the proposed algorithm, SLC, and STMB have a considerable parameter estimation accuracy, which is close to that of CRLB. With the increase of the damping factor, the parameter estimation accuracy of STMB gradually decreases and deviates from that of CRLB due to the poor anti-interference of STMB. However, the proposed algorithm and SLC follow CRLB in the whole range of the damping factor, which means that the two algorithms have good parameter estimation accuracy. In addition, the proposed algorithm is slightly better than SLC.

When the SNR is 40 dB, the noise influence is small. The proposed algorithm and STMB have equivalent parameter estimation accuracy, which is always close to that of CRLB. However, SLC is affected by spectrum leakage, and the frequency estimation has a deviation. Moreover, the discrepancies of the estimation results of the damping factor between SLC and CRLB gradually increase with the increase of the damping factor.



(a) frequency (b) damping factor

Fig. 5. Estimation results of frequency and damping factor under different damping factors when SNR is 10 dB.



(a) frequency (b) damping factor

Fig. 6. Estimation results of frequency and damping factor under different damping factors when SNR is 40 dB.

5. THE MEASUREMENT EXPERIMENTS

To verify the feasibility of our algorithm in practical application, the RHEONIK CMF experimental platform is used to measure the natural frequency of the flow tube, as is shown in Fig. 7.

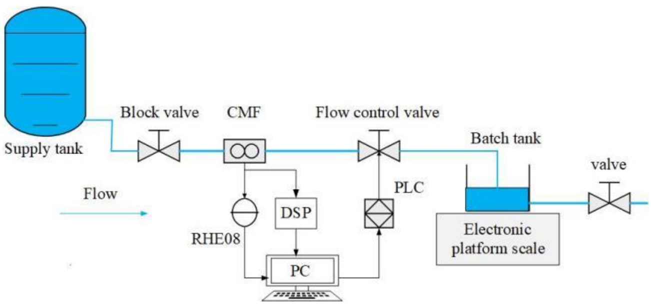


Fig. 7. CMF experimental platform [15].

First, an analog driving scheme is used to drive the flow tube vibration. According to the analog driving principle, the vibration frequency of the flow tube is the natural frequency. Then the flow tube is in the free attenuation state, and the vibration frequency is calculated by the proposed algorithm. By comparing the estimation frequency of the damped signal with the natural frequency, the frequency estimation effectiveness of the proposed algorithm can be tested. SLC and STMB are also tested at the same time.

In this section, the experiments to measure the natural frequency under the conditions of empty tube, air bubbles in the tube, and full water in the tube are demonstrated. The results are shown in Table 2.

The analysis showed that the SNR of the vibration signal is about 26 dB and the frequency is about 0.12 π, which satisfies the application conditions of the proposed algorithm. Under various conditions, the natural frequency of the flow tube can be accurately tracked by SLC, STMB and the proposed algorithm, and the estimation errors are about 0.0012 Hz, 0.0011 Hz and 0.0011 Hz, respectively. STMB and the proposed algorithm have equivalent frequency estimation accuracy and are slightly better than SLC, which is consistent with the test experiment results.

Table 2. Measurement results under different conditions.

condition	Natural frequency (Hz)	Estimation frequency (Hz)			Estimation error (Hz)		
		SLC	STMB	Proposed	SLC	STMB	Proposed
empty	147.1482	147.1495	147.1471	147.1472	0.0013	-0.0011	-0.0010
bubble	146.6946	146.6957	146.6955	146.6956	0.0011	0.0009	0.0010
full	145.5948	145.5935	145.5960	145.5960	-0.0013	0.0012	0.0012

6. CONCLUSION

To suppress the spectrum leakage influence of the frequency-domain methods at medium and high SNR conditions and the disadvantage of large computational cost of time-domain methods, the advantages of anti-interference of frequency-domain methods and high parameter estimation accuracy of time-domain methods are combined to propose a novel parameter estimation algorithm for damped real-value sinusoids in noise. The proposed algorithm significantly reduces the computational cost by computational complexity analysis, which is more effective than STMB and SLC. The test results under different simulation conditions show that the proposed algorithm improves the parameter estimation accuracy, and the MSEs of the parameter estimation values are closer to CRLB than those of the other algorithms. Moreover, the measurement experiments are performed on the CMF experimental platform to verify the effectiveness of the proposed algorithm, which indicates that the proposed algorithm can be applied in engineering practice.

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