Errors with Direct Process -Coupled Digital Sensors

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Abstract. *Today increasingly sensors with direct digital output are used. Because of the direct coupling with the process often an anti-aliasing filtering is not possible. In the paper it is shown in detail that these errors are depending on the signal processing after sampling. As an example with great importance in practice it is shown that in mean-value measurement – for instance in surface measuring – no aliasing errors occur even if the sampling theorem is violated. On the other hand processing with a differential algorithm leads to increasing errors.*

Keywords: Sampling Theorem; Signal Processing; Aliasing Errors

1. Introduction

Today and the more in future because of the advantages of direct computer-coupling sensors with direct digital output will be used. In these cases an anti-aliasing filtering often is not possible: The sensors are direct-connected to the signal source of the process to be measured and if the signal is e.g. a nonelectric quantity as for instance a force an anti-aliasing filtering is not possible.

The paper therefore deals with the problem of both cut-off and aliasing errors with and especially without anti-aliasing filtering. Of special interest with respect to the field of intelligent measuring instruments is the fact, that these errors both are depending on the algorithms of signal processing after sampling.

2. Errors without Signal Processing after Sampling

Using the mean-square error definition often closed mathematical solutions are gained because of the validity of Euclidean geometry [1]. As well-known the cut-off-error is with $S_{xx}(\omega)$ = spectral power density of the signal [2]

$$\overline{\varepsilon_1^{\,2}} = 2\int_{\omega_{s/2}}^{\infty} S_{XX}(\omega)\,d\omega. \tag{1}$$

The same value has the aliasing-error if only the side-band of first order has to be taken into consideration. The general case of a non-ideal anti-aliasing filtering $G_a(j\omega)$ as shown in Fig. 1 yields [3]

$$\overline{e^2} \approx \overline{e_1^2} + \overline{e_2^2} + \overline{e_3^2} =$$

$$2\left[\int_0^{\omega_S/2} S_{XX}(\omega)\left|1-G_a(j\omega)\right|^2 d\omega + \int_{\omega_S/2}^{\infty} S_{XX}(\omega)\,d\omega + \int_{-\omega_S/2}^{\omega_S/2} S_{XX}(\omega+\omega_s)\left|G_a\left[j(\omega+\omega_s)\right]\right|^2 d\omega\right] \tag{2}$$

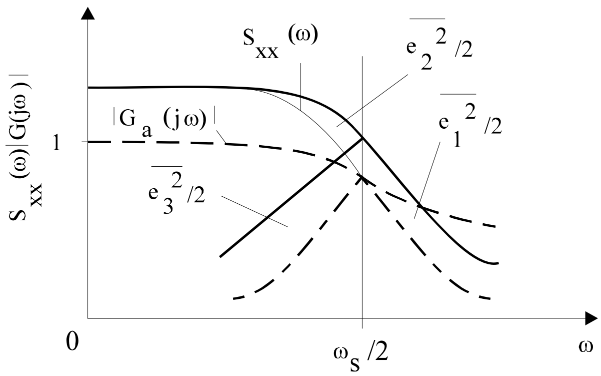

Fig. 1. $\overline{e_1^2}$ = damping error; $\overline{e_2^2}$ = cut-off error; $\overline{e_3^2}$ = aliasing error

If mirrored side-bands of higher orders fall into the range $-\omega_S/2<\omega<\omega_S/2$ the last term in (2) is

$$\overline{e_3^2}=\sum_{r=1}^{\infty}\int_{-\omega_S/2}^{+\omega_S/2} S_{XX}(\omega+r\omega_S)\left|G_a(j[\omega+r\omega_S])\right|^2 d\omega \tag{3}$$

3. Errors with Signal Processing after Sampling

$G_{id}(j\omega) = F\{O_{p_{id}}\}$ may be the ideal and $G_{real}(j\omega) = F\{O_{p_{real}}\}$ the real linear processing procedure. If the sampling Theorem is fulfilled linear system theory yields the mean-square error

$$\overline{e^2(t)}=2\left[\int_{\omega_S/2}^{\infty} S_{XX}(\omega)\left|G_{id}(j\omega)\right|^2 d\omega+\int_{0}^{\omega_S/2} S_{XX}(\omega)\left|G_{id}(j\omega)-G_{real}(j\omega)\right|^2 d\omega\right]. \tag{4}$$

If the sampling Theorem is not fulfilled, e. g. including cut-off and aliasing errors one obtains taking into consideration also side-bands of higher order as with equ. (3)

$$\overline{e^2(t)}=2\left[\begin{array}{l}\int_{\omega_s/2}^{\infty} S_{XX}(\omega)\left|G_{id}(j\omega)\right|^2 d\omega+\int_{0}^{\omega_s/2} S_{XX}(\omega)\left|G_{id}(j\omega)-G_{real}(j\omega)\right|^2 d\omega\\ +\sum_{r=1}^{\infty}\int_{-\omega_s/2}^{+\omega_s/2} S_{XX}(\omega+r\omega)\left|G_{real}(j\omega)\right|^2 d\omega\end{array}\right]. \tag{5}$$

The application of this theory to typical model signals leads to solutions with great importance to problems of intelligent measurement. In most cases the power density of the signal is not known because of missing a-priori-information. Then $S_{XX}(\omega)$ is to be estimated - e. g. caused from a band-limited white noise by a low-pass of first order –

$$S_{XX}(\omega)=\frac{S_O}{1+(\omega/\omega_O)^2}; \qquad S_O=0 \;\; for \;\; \omega\geq\omega_{XO}. \tag{6}$$

For this example it follows

a) without signal processing
   - with anti-aliasing filtering

$$\overline{\varepsilon^2} = 2 \int_{\omega_s/2}^{\omega_{x0}} \frac{S_0}{1+(\omega/\omega_0)^2} d\omega = 2S_0\omega_0(arc\tan\omega_{x0}/\omega_0 - arc\tan\omega_s/2\omega_0); \quad (7a)$$

   - without anti-aliasing filtering

$$\overline{\varepsilon^2} = 4 \int_{\omega_s/2}^{\omega_{x0}} \frac{S_0}{1+(\omega/\omega_0)^2} d\omega = 4S_0\omega_0(arc\tan\omega_{x0}/\omega_0 - arc\tan\omega_s/2\omega_0); \quad (7b)$$

b) with signal processing $G(j\omega) = 1 + j\omega/\omega_0$
   (PD-algorithm)
   - with anti-aliasing filtering

$$\overline{\varepsilon^2} = 2S_0(\omega_{x0} - \omega_s/2) \quad (8a)$$

   - without anti-aliasing filtering

$$\overline{\varepsilon^2} = 4S_0(\omega_{x0} - \omega_s/2) \quad (8b)$$

c) with low-pass filtering ω_{LP} after sampling
   (I-algorithm)
   - with anti-aliasing filtering

$$\overline{\varepsilon^2} = 0 if\, \omega_{LP} < \omega_s/2; \quad (9a)$$

   - without anti-aliasing filtering aliasing errors only appear if $\omega_{xo} > \omega_{S} + \omega_{LP}$

$$\overline{\varepsilon_{al}{}^2} = 2 \int_{\omega_s - \omega_{LP}}^{\omega_{x0}} \frac{S_0}{1+(\omega/\omega_0)^2} d\omega = 2S_0\omega_0\left(arc\tan\omega_{x0}/\omega_0 - arc\tan\frac{\omega_s - \omega_{LP}}{\omega_0}\right). \quad (9b)$$

Of special interest in practice is the case of mean operation after sampling, as for instance used in surface measurement. As Fig. 2 shows that means a low-pass-filtering ω_{LP}.

In this case no aliasing error occurs as long as the limiting frequency of the signal ω_C is less than $(\omega_S - \omega_{LP})$. That means in this case the sampling frequency has to be only

$$\omega_S \geq \omega_C + \omega_{LP} \quad (10)$$

Instead of the value due to the sampling theorem now the sampling frequency is allowed to be only half of this value for the extreme case $\omega_{LP} \rightarrow 0$ without errors!

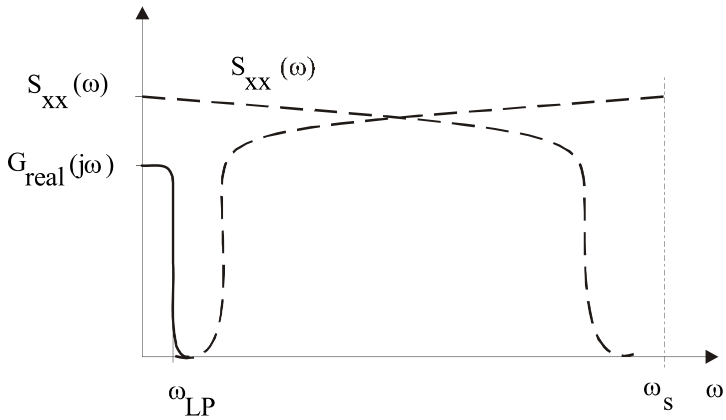

Fig. 2. Mean-operation after sampling

In general we gain the statement

- an algorithm with an integral (I-)part leads to decreasing errors,
- an algorithm with a differential (D-)part leads to increasing errors

compared with the case of not a signal processing after sampling, as normally taken into consideration when dealing with these problems.

4. Conclusions

Today and the more in future in connection with "embedded sensors" sensors with digital output will be used. In these cases an anti-aliasing filtering often is not possible: The sensors are direct-connected to the signal source of the process to be measured and if the signal is e.g. a nonelectric quantity as for instance a force an anti-aliasing filtering (low-pass- filtering) is not possible.

The paper therefore deals with the problem of both cut-off and aliasing errors with and especially without anti-aliasing filtering. Of special interest with respect to the field of intelligent measuring instruments is the fact, that these errors both are depending on the algorithms of signal processing after sampling. It is shown, that an algorithm with an integral (I-)part leads to decreasing errors, while an algorithm with a differential (D-)part leads to increasing errors. In the extreme case $\omega_{LP} \rightarrow 0$ instead of the value due to the sampling theorem the sampling frequency is allowed to be only half of this value.

In general the solution to avoid aliasing errors is oversampling. Due to the generation of micro-electronics it is well-known, that every 7 years the sampling frequencies will increase one order in magnitude [4], [5] and [6], so oversampling will be easily possible in future.

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