Self-Correction of ADC Error Using Additive Iterative Method and Averaging of Dithered Samples

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***Abstract**. In the paper the combination of additive iterative algorithm with averaging of dithered samples is designed for self-correction of ADC linearity error. Iterative method is known as the aid for automated error correction and dithering uses to be applied for quantiser performance enhancement. Dither theory for Gaussian noise has been used for exhibition of new method abilities in ADC characteristic improvement. Experimental ENOB value improvement is more than 2 bits.*

Keywords: ADC Error Correction, Iterative Method, Nonsubtractive Dithering

1. Introduction

Self-correction functions become important part of modern measurement devices. Analog-to-digital converter (ADC) could be used for direct voltage measurements and it is also the basic part of a general digital measurement channel. It is often not difficult to make correction of offset and gain error of measurement transducer (MT) such as ADC. But correction of nonlinearities of the static transfer characteristic is problematic especially if they vary in time.

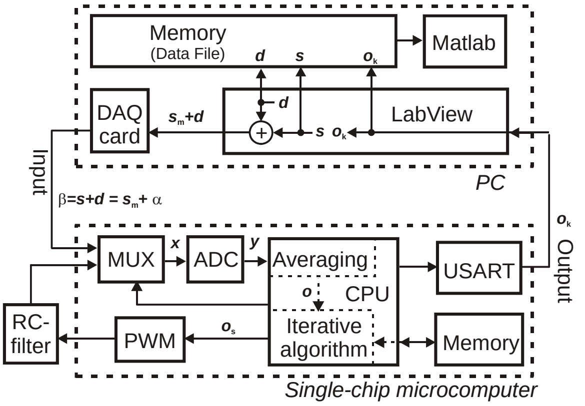

Fig. 1 Block diagram of the workplace.

Methods for automatic correction of ADC have been employed as discussed below. Proposed correction is focused on ADC nonlinearities. Additive iterative method (AIM) is suitable for nonlinear error correction in the case of analog MT. But the ideal characteristic of ADC is fundamentally nonlinear reflecting quantisation error. Quantisation error limits efficiency of AIM, therefore in designed measurement system AIM is combined with nonsubtractive dithering (ND). Block diagram in the Fig.1 shows the experimental system. Diagram of tested measurement unit (MU) is located in the lower part of the figure. Designed measurement system consists only of single-chip microcomputer (with some basic peripherals like power supply etc.) and low-pass RC-filter. The correction method is implemented there. The upper side is devoted to the main PC components used for testing and experiments.

2. Aditive iterative method

For this method four main blocks of the system are needed: MT – in the Fig. 1 it is represented by ADC; block of processing (BP) – CPU (processor) with memory; inverse element (IE) – Pulse Width Modulation (PWM) with RC-filter; switch (SW) – multiplexer (MUX). The correction is performed iteratively in several steps. The BP controls the whole process. It receives input value from the MT and according to the implemented algorithm and data in the memory it calculates next input o_s to the IE. Evaluation of o_s is assigned by correction formula [1]

$$o_{\mathrm{s},i} = o_{\mathrm{s},i-1} + \left(o_{s,0} - h\left[h_{\mathrm{IE}}\left(o_{\mathrm{s},i-1}\right)\right]\right) \qquad (1)$$

Function $h(x)$ is transfer characteristic of MT and it determines initial value $o_{\mathrm{s},0} = h(s_\mathrm{m})$. Argument s_m represents measured value, which is in these measurements the mean value of input β (Fig. 1). After initial step SW controlled by BP switches the input of ADC from measured signal $x = \beta$ to signal from IE $x = h_{\mathrm{IE}}(o_{\mathrm{s},i})$. With each next step of iterations $o_{\mathrm{s},i}$ should become more accurate representation of measured value s_m. After appropriate number of steps actual $o_{\mathrm{s},i}$ could be sent to the output of the MU as a result o_k of correction.

The iterative process is convergent if condition of convergence is satisfied [2]. Then it tends to value given by characteristics of IE $h_{\mathrm{IE}}^{-1}(s_\mathrm{m})$. Therefore the aim is to have an ideal IE with inverse characteristics equal to ideal characteristics of MT $h_\mathrm{I}(x) = h_{\mathrm{IE}}^{-1}(x)$.

Inverse element with averaging

Transfer characteristic of IE determines the best resp. theoretically reachable accuracy of measurement output corrected with the iterative method. Therefore this element must be designed thoroughly. Inverse element for ADC is digital-to-analog converter (DAC) and it has been built by means of pulse width modulation output of microprocessor. PWM circuits are naturally precise but to get the mean $a_{\mathrm{PWM},0}$ ($a_{\mathrm{PWM},k}$ denotes k-th frequency component) of PWM output corresponding to precise DAC result, low-pass filter should be added. Simple RC-filter has been used with frequency characteristics $A_{\mathrm{RCF}}(\omega)$ [2], which is influenced by the time constant τ_{RC}. The filter slows down the correction process because after every step the process should wait until settling of filter output.

To speed up the process we proposed to use combination of analog and digital filter. Output of analog RC-filter oscillates in the range of several LSB. As digital filtering a sampling and averaging of N samples in each step of the iterative correction is used. The best way is to use synchronous sampling here but there could be no possibility to synchronize ADC and PWM circuits. Generally sampling gets samples from rectangular window $T_{\mathrm{RW}}=N.T_\mathrm{s}$ wide (T_s is sampling period) with frequency characteristic $A_{\mathrm{RW}}(\omega)$. Denoting $\omega_{\mathrm{RW}} = 2\pi/T_{\mathrm{RW}}$ and $\omega_{\mathrm{PWM}} = 2\pi/T_{\mathrm{PWM}}$ (T_{PWM} is period of PWM output), mathematical model of IE error caused by non-synchronous sampling is [2]

$$\Delta h_{\mathrm{IE}}(o_\mathrm{s}) = \left[\frac{1}{T_{\mathrm{RW}}}\sum_{k=-\infty}^{\infty} a_{\mathrm{PWM},k}(\{o_\mathrm{s}\}) A(k\omega_{\mathrm{PWM}},\{\tau_{\mathrm{RC}}\}) A_{\mathrm{RW}}(-k\omega_{\mathrm{PWM}},\{t_{s1},T_{RW}\})\right] - a_{\mathrm{PWM},0} \qquad (2)$$

By {} brackets dependency on parameters of system is expressed. According to simulation the RC-filter time constant was set to τ_{RC}=0,1 s. If choosing appropriate number of sample N resp. value $T_{\mathrm{RW}}=N.T_\mathrm{s}$, special case of quasi-synchronous sampling could be achieved. Suitable values are N=20, 39, 59 (etc.) for which IE error dependencies are shown in the Fig. 2. As could be seen, also time shift $t_{\mathrm{s}1}$ – shift from the start of PWM counter to the first sample – influences the accuracy. Expecting error of measurement after correction near to 0,01% of scale, the value N=59

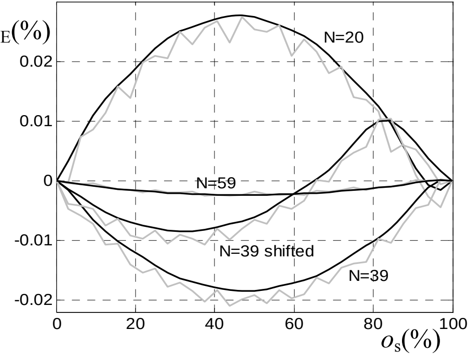

Fig.2 Theoretical error of IE mean evaluated through averaging of samples.

has been chosen.

3. Gaussian noise and averaging

ADC represents real quantiser and its quantisation error limits measurement accuracy of an ideal converter or of real ADC corrected with AIM. Averaging of samples is the way to overcome this limitation employing natural noise present in measured signal. In many cases intentionally added noise (dither) d could help. A process of resolution improvement is called nonsubtractive dithering (ND), if the noise is not subtracted from the signal after quantization.

Noise is present in real applications and usually it is of Gaussian nature, but its dispersion may be too small to get significant resolution improvement. For evaluation of noise influence on accuracy of our system, where dithering with averaging is implemented, the appropriate error parameter must be chosen. Theory [3][4] yields for mean error ε of noisy samples $m_{\varepsilon|s}$

$$m_{\varepsilon|s} = q\sum_{k=1}^{\infty}\frac{(-1)^k}{\pi k}\exp\left[-2\pi^2k^2\left(\frac{\sigma_\alpha}{q}\right)^2\right]\sin\left(\frac{2\pi k s_\mathrm{m}}{q}\right) \tag{3}$$

Fig. 3 exposes dependency of the mean error from measured value s_m depicted in the range of one quantization step q. Theoretical curves show that with increasing standard deviation σ_α of total input noise α – composed from natural noise and dither together – the mean error decreases. Gray lines are obtained from simulation as a mean from P=20 results of averaging of N=59 samples. The mean is estimated through averaging. The bigger the input noise dispersion is the noisier the curves are. It suggests that an optimal noise variance exists in system with ND and averaging. The formula of theoretical mean error (3) does not include this fact.

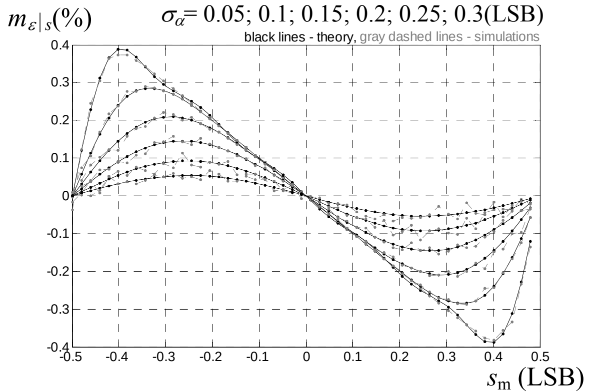

Fig.3 Mean error curves - dependency from standard deviation of Gaussian noise.

The mean-square error (MSE) was chosen as suitable parameter of dithering and averaging performance rating for finding an optimal noise dispersion. It is theoretically evaluated as mean for one whole quantization step [3]. Using (3) and theory from [3] for Gaussian noise it holds

$$\mu_\mathrm{a}^2(\sigma_\alpha, N) \cong \frac{\frac{q^2}{12}+\sigma_\alpha^2}{N} + \left(1-\frac{1}{N}\right)\frac{q^2}{2\pi^2}e^{-4\pi^2\left(\frac{\sigma_\alpha}{q}\right)^2} \tag{4}$$

This formula embodies influence of both the mean error (3) and the occurrence of noise in measurement results. For Gaussian dither and N=59 according to theoretical relation from [3] the optimal σ_d is 0,347q, if natural noise is not present in the input signal.

4. Experimental results and discusion

Experiments were performed with designed measurement system, where AIM and ND with averaging of N=59 samples was implemented. In 51 levels of input voltage P=20 correction processes were accomplished. Fig. 4 shows the mean error obtained from these 20 measurement results. As could be seen the additive iterative method used for averaged samples corrects error considerably under the 1 LSB level. In our case appropriate dither helps to suppress nonlinearities involved by quantization. Although implemented method can significantly correct gain error and offset, only nonlinear error component is investigated.

Therefore linear error component has been subtracted from measurement results. To evaluate also dispersion of results within error analysis, RMSE (Root MSE) resp. μ_a curves have been depicted in the Fig. 5. Dither with standard deviation $\sigma_d < 0.05$ % (0.5 LSB) has improved results, but differential nonlinearity (DNL) of ADC has caused notable shift of experimental curve against the theory. AIM corrects DNL and therefore it shifts the curve closer to theoretical values. Optimal dither dispersion (see fig. 5) is lower than theoretical optimum because natural noise is present in the input signal β.

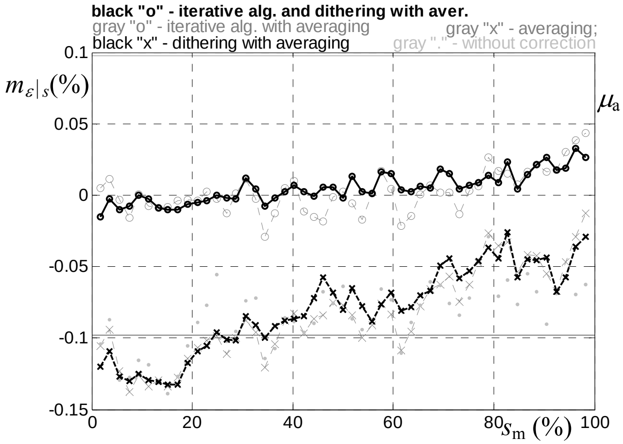

Fig.4 Mean error before and after correction.

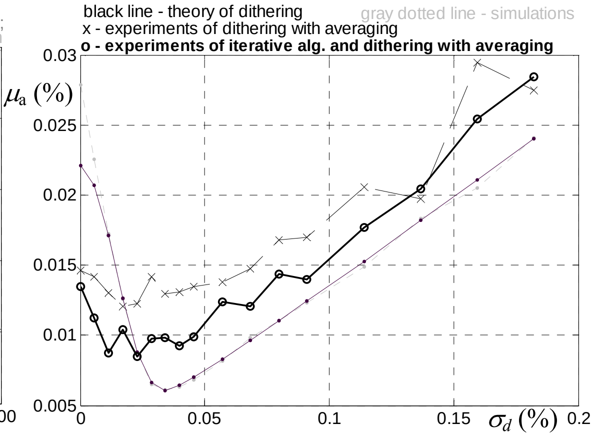

Fig.5 RMSE before and after correction.

5. Conclusions

Combination of additive iterative method and nonsubtractive dithering has been implemented in experimental measuring device. AIM has automatically corrected integral nonlinearity in every process of measurement result evaluation. Averaging has enabled correction under level of 1 LSB of used 10-bit ADC. Analysis of quasi-synchronous sampling of periodic IE output has been performed, leading to negligible error of mean evaluation through averaging. Dispersion of natural noise present in real signal is usually smaller than optimal for dithering. Theoretical dependence of root mean square error (RMSE) upon standard deviation of added noise has been proved through measurements in the whole range and the best dispersion of dither has been found. The RMSE has decreased from 0.036 % to 0.0085 % and adequately ENOB has significantly grown from 9.64 bit to 11.73 bit using proposed correction techniques.

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