Evaluation of Calibration of a Thermocouple

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Abstract. *This contribution describes the procedure of evaluating the calibration of thermocouple by means of its comparison with the thermocouple standard. In the process of thermocouple calibration by means of comparison, the resulting uncertainty specified by applying the generalized procedure for evaluating the calibration of measuring devices with continuous scale.*

Keywords: Calibration, Thermocouple, Estimation of Unknown Parameters, Uncertainties, Covariance

1. Introduction

For measuring instrument with continuous scale a generalized procedure for evaluating the calibration uncertainties and covariances has been developed by Palenčár, Wimmer [1,2] and Kubáček [4]. In this paper authors are presenting these procedures for evaluating uncertainties of the calibration of a thermocouple (hereafter TC only) type S by means of comparison.

2. Calibration procedure

Calibration is carried out by comparison of the unit under test TC type S against standard TC type S calibrated in defined fixed points according to ITS-90 (Fig.2.1). The calibration is represented as a curve fitted to the measured values of the deviation E-E_{ref} and generally given as a function of temperature t. This curve is representing deviation function.

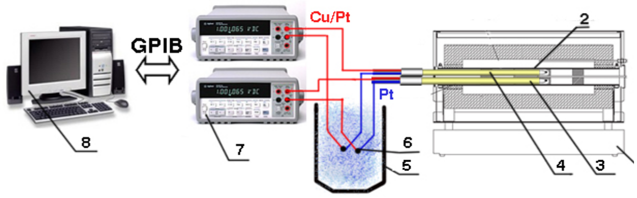


Fig. 2.1: Scheme of calibration

1- Calibration furnace, 2- Isothermal block, 3- Standard TC, 4- Unit under test, 5- Dewar flask, 6- Reference junction of TC's, 7- Voltmeters, 8- Computer with GPIB port

Table 2.1: Measured and computed values
i- calibration points (nominal values), $t(\bar{E}_{\mathrm{S}_i})$(°C)- values measured by standard TC, $\bar{E}_{\mathrm{K}_i}$ (°C)- values measured by unit under test TC

Cal. Points i	$t(\bar{E}_{\mathrm{S}_i})$ (°C)	$\bar{E}_{\mathrm{K}_i}$ (°C)
100	99,8188	643,85
200	199,7252	1437,82
300	299,7120	2319,79
400	399,7737	3255,32
500	499,8191	4228,52
600	599,6689	5230,63
700	699,7653	6265,91
800	799,8341	7334,86
900	899,7653	8437,38
1000	999,5335	9569,7
1100	1099,4	10736,91

3. Methodology

We consider the case, when number of calibration points r is higher than number of unknown parameters p, $r>p$ the model is overdetermined. Calibration model should be established using following relations

$$W_i = a_0 + a_1 \cdot t_i + a_2 \cdot t^2{}_i + a_3 \cdot t^3{}_i + a_4 \cdot t^4{}_i \qquad i=1, ..., n \qquad (3.1)$$

in matrix notation

$$\boldsymbol{W} = \boldsymbol{Ta} \qquad (3.2)$$

where $\boldsymbol{T}$ is a matrix, which contains values, arithmetical means of series of measurements in each calibration points measured by standard TC.

Left side of the model (3.1) or (3.2), the observation vector $\boldsymbol{W}$ is presenting the measurement model of unit under test TC

$$\boldsymbol{W} = \Delta\boldsymbol{E} + \boldsymbol{C}_\mathrm{K}\boldsymbol{\Lambda} \qquad (3.3)$$

where $\Delta\boldsymbol{E}$ is the vector of deviations from the reference function. Reference function is given by IEC 584.2 standard [3].

$$\Delta\boldsymbol{E} = \overline{\boldsymbol{E}} - \boldsymbol{E}_\mathrm{ref} \qquad (3.4)$$

in product of $\boldsymbol{C}_\mathrm{K}\boldsymbol{\Lambda}$ fills every influences of measurement.

Vector of correction $\boldsymbol{\Lambda}$ is given by and matrix $\boldsymbol{C}_\mathrm{K}$ is the known matrix, usually its elements are sensitivity coefficients.

Our aim is to get estimation for unknown parameters of deviation function. This aim could be reached by using least-square method [1,2,3]. Uncertainties are taken into account as well. We apply following expression iteratively because of stochastic character of quantity t [1].

$$\hat{\boldsymbol{a}} = \left(\boldsymbol{T}^\mathrm{T}\boldsymbol{U_W}^{-1}\boldsymbol{T}\right)^{-1}\boldsymbol{T}^\mathrm{T}\boldsymbol{U_W}^{-1}\boldsymbol{W} \qquad (3.5)$$

Initial values of unknown parameters $\hat{\boldsymbol{a}}$ of deviation function are determined by zero estimation. Then covariance matrix of input quantities $\boldsymbol{U_W}$ is

$$\boldsymbol{U_W} = \boldsymbol{U}_{\Delta\boldsymbol{E}} + \boldsymbol{C}_\mathrm{K}\boldsymbol{U}_{\boldsymbol{\Lambda}}\boldsymbol{C}_\mathrm{K}{}^\mathrm{T} \qquad (3.6)$$

where

$\boldsymbol{U}_{\Delta\boldsymbol{E}}$ - covariance matrix of the vector $\Delta\boldsymbol{E}$ is diagonal matrix, principal-diagonal elements present square of uncertainties estimated by type A method

$\boldsymbol{C}_\mathrm{K}\boldsymbol{U}_{\boldsymbol{\Lambda}}\boldsymbol{C}_\mathrm{K}{}^\mathrm{T}$ - product of these matrix gives diagonal covariance matrix, principal-diagonal elements present square of uncertainties estimated by type B method

$\boldsymbol{U}_{\boldsymbol{\Lambda}}$ - uncertainties of correction measurement by unit under test TC are included in this covariance matrix

Covariance matrix $\boldsymbol{U}_{\hat{\boldsymbol{a}}}$ is represented by matrix of the uncertainties of the estimates

$$\boldsymbol{U}_{\hat{\boldsymbol{a}}} = \left(\boldsymbol{T}^\mathrm{T}\boldsymbol{U_W}^{-1}\boldsymbol{T}\right)^{-1} \qquad (3.7)$$

Deviation associated with the reference function is solved by

$$\Delta\hat{\boldsymbol{E}} = \boldsymbol{T}\hat{\boldsymbol{a}} \qquad (3.8)$$

uncertainty of the deviation can be achieved by application of law of propagations of uncertainties

$$u_{\Delta \boldsymbol{E}}^2 = \boldsymbol{T} \cdot \boldsymbol{U}_{\hat{\boldsymbol{a}}} \cdot \boldsymbol{T}^{\mathrm{T}} \tag{3.9}$$

Zero estimation of vector $\hat{\boldsymbol{a}}$ is biased (see Fig.4.1(a)). It is caused by stochastic characters of the quantity *t*. Therefore the model is nonlinear and requires a solution procedure. It is linearized by application of Taylor series and higher elements of estimated values are neglected. After linearization left side of model vector $\boldsymbol{W}$ will be

$$\boldsymbol{W} = \Delta \boldsymbol{E} + \boldsymbol{C}_{\mathrm{K}} \boldsymbol{\Lambda} + \boldsymbol{D}(\boldsymbol{\delta t}_1 + \boldsymbol{C}_{\mathrm{S}} \boldsymbol{\delta t}_2) \tag{3.10}$$

$\boldsymbol{D} = \mathrm{diag}(d_{100} \quad d_{200} \quad \ldots \quad d_{1100})$ - is the known matrix, obtained by application of expansion of Taylor series

After linearization covariance matrix $\boldsymbol{U}_{\boldsymbol{W}}$ has the form

$$\boldsymbol{U}_{\boldsymbol{W}} = \boldsymbol{U}_{\Delta \boldsymbol{E}} + \boldsymbol{C}_{\mathrm{K}} \boldsymbol{U}_{\Lambda} \boldsymbol{C}_{\mathrm{K}}^{\mathrm{T}} + \boldsymbol{D}\left(\boldsymbol{U}_{\boldsymbol{\delta t}_1} + \boldsymbol{C}_{\mathrm{S}} \boldsymbol{U}_{\boldsymbol{\delta} t_2} \boldsymbol{C}_{\mathrm{S}}^{\mathrm{T}}\right)\boldsymbol{D}^{\mathrm{T}} \tag{3.11}$$

where

$\boldsymbol{U}_{\boldsymbol{\delta t}_1}$ - covariance matrix of the vector $\boldsymbol{\delta t}_1$ is diagonal matrix, principal-diagonal elements present square of uncertainties estimated by type A method

$\boldsymbol{C}_{\mathrm{S}} \boldsymbol{U}_{\boldsymbol{\delta} t_2} \boldsymbol{C}_{\mathrm{S}}^{\mathrm{T}}$ - product of this matrix is given by diagonal covariance matrix, principal-diagonal elements present square of uncertainties estimated by type B method

$\boldsymbol{U}_{\boldsymbol{\delta} t_2}$ -uncertainties of correction of measurements by standard are included in this covariance matrix

Now in new iteration we consider the observation vector $\boldsymbol{W}$ (3.10) and covariance matrix $\boldsymbol{U}_{\boldsymbol{W}}$ (3.11) and we use formula for estimation of parameters (3.5).

Numerically, in the most cases design matrix $\boldsymbol{T}$ is badly scaled and its columns are nearly linearly dependent. For this it is reasonable to transform quantities of *t* to interval $-1 \leq t \leq 1$

From the viewpoint of the user relevant results are the temperature values and their uncertainties. Temperature value can be obtained by interpolation table which can be edited from deviation function and its uncertainty is determined by application of theorem for implicit function.

$$f(E, t, a) = E - g(t, \boldsymbol{a}) = 0 \tag{3.12}$$

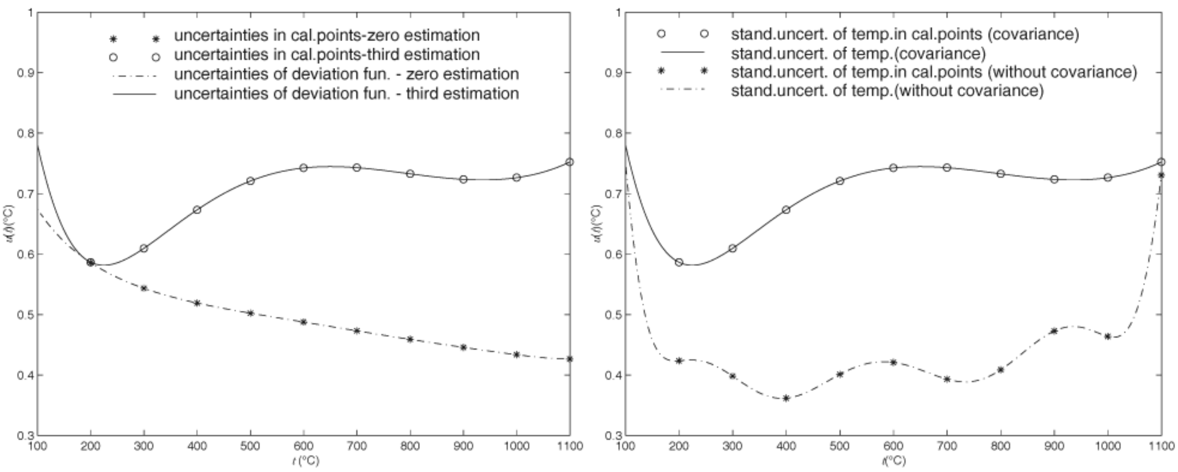
we get it by adding up deviation function and reference function, where variable *E* is representing the current measured value of emf. Now consider function $t=(h,\boldsymbol{a})$ is defined from the implicit function.

Standard uncertainty is then obtained from the (3.13) relation

$$u^2(t) = \boldsymbol{h}^{\mathrm{T}} \cdot \boldsymbol{U}_{\hat{a}} \cdot \boldsymbol{h} \tag{3.13}$$

4. Conclusion

Procedure for evaluating the calibration of TC was applied to demonstrate whether considering the covariances has an impact on final result of standard uncertainty. For this reason was carried out the evaluation twice. The difference is shown in Fig. 4.1(b). As a conclusion we can claim that covariances had significant effect on final result of a calibration.



(a) (b)

Fig 4.1: Standard uncertainties of deviation function: (a) Difference between zero and third estimation of parameters, (b) Standard uncertainties derived from third estimation when consider covariance and not

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