Two Decades of Search for Chaos in Brain.

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Abstract. *A short review of applications of methods of chaos theory to investigation of brain dynamics represented by EEG is given.*

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1. Introduction to measures of chaos

Advances in computer technology allowed studying the behaviour of nonlinear systems of differential equations, when there are no solutions for the equations. In 1963 the meteorologist Edward Lorenz, using numerical integration of a simple nonlinear model of convection in the atmosphere called attention to the unpredictable complicated behaviour of the system and published the first picture of a strange attractor (attracting subset of non-integer dimension), the so called Lorenz attractor [1].

The three non-linear Lorenz equations belong to the chaotic systems. Chaos is a non-linear deterministic process, which looks random. The distinguishing feature of chaos is sensitive dependence on initial conditions, meaning that nearby states will rapidly evolve toward very different positions. This makes long-term prediction of the course of chaotic orbits impossible, except in the short run. Moreover, change of system parameters may dramatically change the nature of the system behaviour.

In 1980 Packard et al. showed how a real time series could be represented in a multi-dimensional state space [2]. The process of reconstruction is called phase space embedding. Then Takens mathematically established that, if we can measure any single variable of a dynamical system with sufficient accuracy, then it is possible to reconstruct a state portrait, topologically equivalent to the attractor of the original system [3]. Complexity of the reconstructed attractor may provide important information about the system. The most popular tool to assess the complexity is the correlation dimension (D_2), computed by algorithm of Grassberger and Procaccia (GP) published in 1983 [4]. The correlation dimension is an example of measures that can capture the fractal character of the strange attractors. Moreover, the dimension is related to the minimum number of variables needed to model the system behaviour.

Algorithm of Grassberger and Procaccia made it possible to apply the element of chaos theory to various observations, and led in 1985 to the first applications to electroencephalograms (EEG), when Rapp et al. described their results regarding chaotic analysis of neural activity in the motor cortex of a monkey [5].

At the same time Babloyantz and her co-workers reported the observations on the correlation dimension of human sleep EEG [6]. This and some following studies have concluded that the deeper the sleep, the lower the brain dynamics complexity. Dimension has been reported to be the highest in REM sleep, the smallest in slow wave sleep, and significantly higher in the second half of the night than in the first half.

The early years of nonlinear analysis of brain, roughly between 1985 and 1990, were characterized by enthusiastic search for low-dimensional chaos in EEG signals. Around 1990 some of the limitations of algorithms for chaos analysis became clear, and the previous findings of chaos in the brain were critically reexamined.

First, the understanding of non-integer values of the correlation dimension as a sign of deterministic chaos has been questioned. Initially, fractality was considered to be a manifestation of chaos, but in fact we have chaotic attractors that are not fractal and strange attractors that are not chaotic.

Moreover, Theiler has shown that for data sets with long autocorrelation time the application of GP algorithm leads to spuriously low estimates of dimension due to an anomalous shoulder in the graph of correlation integral [7]. To reduce the effects of linear correlations, the author recommended taking much more data than the characteristic autocorrelation time □, and omitting pairs of points closer in time than □. In 1996, after application of the suggested correction to brain signals, a set of EEGs, previously reported to exhibit low dimensions, was reexamined [8]. The scaling regions disappeared and the authors switched to position that in the case of EEG there is no convincing argument for preference low-dimensional representation over modelling by linearly filtered noise.

Another remarkable paper questioning the view that stochastic time series lead to a non-convergence of the correlation dimension is that of Osborne and Provenzale [9]. They showed that noise exhibiting power-law spectra, may result in low values of the correlation dimension. These noises in fact are self-affine signals, generating a fractal curves whose noninteger dimensions are educible from their power spectra. The GP-method cannot distinguish between fractal attractor of deterministic system and fractal random curve if their dimensions equal.

The above findings have also been tested in the case of EEG [10, 11]. The authors have found negative linear correlation between D_2 estimation and the decay of EEG spectra. As a consequence, low values of D_2 computed for EEG must be reinterpreted and the hypothesis of presence of scale-invariant fractal like structures, rather than the suggestion of deterministic dynamics, is to be preferred.

While in some specific cases, e.g., epileptic seizure, the EEG does appear to exhibit low complexity, in general the brain is continually interacting with many other complex systems and EEG seems to be a mixture of noise, certain cyclic processes and possibly some random fractal signals. Each part of such a composition itself has been frequently reported to fool the algorithms used to detect chaotic dynamics. Therefore, the estimates of correlation dimension and other chaotic measures of brain activity should be interpreted with extreme caution.

2. Self-organized criticality

Scaling behaviour (or scale-free behaviour) means that no characteristic scales dominate the dynamics of the underlying process. The long-range correlations build up until they extend throughout the entire system. Then the dynamics of the system exhibit power-law scaling behaviour, and the underlying process works in a so called critical state.

This alternative type of nonlinear process, discovered in 1987 by Bak et al. [12], is known as self-organized criticality (SOC). Unlike chaos, however, this is a probabilistic process. Bak et al. have shown that, under some very general conditions, systems that consist of a large number of interacting non-linear elements self-organize as energy and matter flow through the system and evolve to a critical state with fluctuations at different spatial and temporal scales. Critical states are characterized by power law (fractal) event size distributions and 1/ *f* noise.

Noise with a 1/ *f* power spectrum is emitted from a huge variety of sources, including electric current passing through a vacuum tube, quasars, sunspots, economic and communication systems, annual amount of rainfall, or rate of traffic flow.

Interestingly, 1/*f* noise and power law scaling have also been found in electroencephalographic recordings. This proposal is consistent with the observation of the background activity pattern - neurons constantly emit pulses. A large cluster of synchronized neurons seems to attract further neurons and causes the oscillation amplitude to increase. Thus, smaller concentration of neurons gives rise to low amplitude signals of higher frequency while large clusters are associated with slow, high amplitude oscillations. As a result, most of the power of the EEG signals is concentrated in the low frequency spectrum. Such a non-trivial long-range correlation within the signal is often present in fractal or scale-invariant processes.

Advocates of the above hypothesis argue that, in a critical state, the brain could be both stable and variable, and may be optimally suited for information processing. Some findings suggest that power law scaling EEG is characteristic of healthy cerebral activity and the breakdown of the scaling may lead to incapability of quick reorganization during processing demands and may be useful in identification of some brain diseases.

3. Discrimination ability of nonlinear measures

In [11] we looked for the presence of exponential or power-law decay in the power spectra of EEG as the next statements are generally accepted: While chaotic behaviour has power spectrum that falls exponentially at high frequencies, stochastic behaviour has power spectrum that decreases as $1/f^{\gamma}$ with increasing frequency. γ came to be called spectral decay, fractal exponent, or power-law exponent. Thus, examination of the power spectrum can help us to answer the question, whether the observed erratic behaviour is essentially deterministic or stochastic. Our result was clear: Power-law model proved to be preferable over exponential model in 99% of frequency ranges both before alpha activity and following alpha activity.

Our next goal was to verify declarations about the relation of correlation dimension to power-law decay. The average value of γ established from the whole EEG spectrum (in our case from 5 to 250 Hz) was about 2.28. As regards correlation dimension, relatively low values of estimates (between 3 and 6) with the mean of 4.35 were found. We found strong negative correlation between the evolutions of the two measures indicating that, in this case they reflect the same information - the dimension estimate by GP-algorithm only mirrors the spectral features of signal.

While it seems that chaotic brain in general sense is no longer an issue, measures used for identifying low dimensional chaotic systems, such as the correlation dimension, continue to be used for studying the EEG signals. There is a range of new EEG measures successful in the monitoring of sleep, anesthesia and seizures and in distinguishing between normal and pathological or otherwise differing states.

Beside correlation dimension, let us mention Lyapunov exponent (reflects the exponential rate of divergence of nearby orbits) and Higuchi's fractal dimension. Higuchi's dimension does not refer to the reconstructed attractor but to the EEG signal itself, which is considered a geometric figure. This dimension yields values between 1 and 2, since a simple curve has dimension 1 and a plane has dimension of 2.

In [13, 14] we tested a lot of traditional and novel scoring measures on the same data to produce a quite systematic overview, missing in the literature, of how varying audio–visual input influences the brain signals and of how single measures allow the sleep stages classification. We confirmed the remarkable efficiency of some novel measures. Our concurrent testing of 73 measures showed that a lot of traditionally used characteristics had poor classification ability as compared with the small number of the best spectral and nonlinear measures. For instance, fractal exponent and the closely related fractal dimension were overcoming the most of traditional spectral measures in discrimination between the individual states of sleep. Presumably the new measures capture the fundamental properties of the underlying system. This indicates potential advantage of nonlinear methods over standard spectral methods in any task associated with classification, modelling or prediction of the discussed systems.

4. Conclusion

The complex system of the brain ranges from neurons to large neuronal networks. It seems to evolve continuously and in real time in conjunction with changes in the surrounding world. Although a convincing demonstration of chaos has only been obtained at the level of neurons, acting as coupled oscillators, some scientists still believe that there could be considerable benefits for the brain to operate in chaotic regimes due to rich range of behaviours. Similar ideas were presented by Skarda and Freeman in 1987 already [15]. They hypothesized that brain self-organizes to generate relevant (possibly chaotic) activity patterns that serve as the essential ground states of the brain activity. Even today, the discussion of this topic is far from finished.

On the other hand, regardless of the presence of chaos in brain activity, it became more and more obvious that the neuroscience should benefit from methods developed for the analysis of nonlinear and chaotic behaviour. So far, nonlinear methods are used mainly in research as they are not yet rooted in everyday clinical practice. But this will certainly change in the near future.

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References

[1] Lorenz EN. Deterministic Nonperiodic Flow. *Journal of the Atmospheric Sciences,* 20 (2): 130 - 141, 1963.

[2] Packard NH et al. Geometry from a Time Series. *Phys. Rev. Lett.* 45, 712 - 716, 1980.

[3] Takens F. Detecting Strange Attractors in Turbulence. *Lecture Notes in Mathematics,* 898, 366-381, 1981.

[4] Grassberger P, Procaccia I. Measuring the strangeness of strange attractors. *Phys. Rev. Lett.* 50 (5), 346 - 349, 1983.

[5] Rapp PE et al. Dynamics of spontaneous neural activity in the Simian motor cortex: the dimension of chaotic neurons. *Physics Letters A*, 110 (6), 334-338, 1985.

[6] Babloyantz A, Salazar JM, Nicolis C. Evidence of chaotic dynamics of brain activity during the sleep cycle. *Physics Letters A*, 111 (3), 152 - 156, 1985.

[7] Theiler J. Spurious dimension from correlation algorithms applied to limited time-series data. *Phys. Rev. A,* 34, 2427 – 2432, 1986.

[8] Theiler J, Rapp PE. Re-examination of the evidence for lowdimensional, nonlinear structure in the human electroencephalogram, *Electroencephalogr.Clin. Neurophysiol,* 98, 213 - 222, 1996.

[9] Osborne AR, Provenzale A. Finite correlation dimension for stochastic systems with power-law spectra, *Physica D*, 35, 357–381, 1989.

[10] Pereda E, Gamundi A, Rial R, González J. Nonlinear behaviour of human EEG: fractal exponent versus correlation dimension in awake and sleep stages. *Neuroscience Letters,* 250(2), 91 - 94, 1998.

[11] Krakovská A, Štolc S. Spectral decay vs. correlation dimension of EEG. *Neurocomputing*, 71, 2978 - 2985, 2008.

[12] Bak P, Tang C, Wiesenfeld K. Self-organized criticality: an explanation of 1/*f* noise. *Phys Rev Lett,* 59, 381 – 384, 1987.

[13] Teplan M, Krakovská A, Štolc S. EEG responses to long-term audio–visual stimulation. *International Journal of Psychophysiology,* 59, 81 – 90, 2006.

[14] Šušmáková K, Krakovská A. Discrimination ability of individual measures used in sleep stages classification. *Artificial Intelligence in Medicine,* 44, 261 – 277, 2008.

[15] Skarda C, Freeman WJ. How brain make chaos in order to make sense of the world. *Behav. Brain Sci.,* 10, 161 – 195, 1987.