Biophysical Model for Beat-to-Beat Variations of Vectorcardiogram

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Abstract. *For a realistic biophysical model, a linear deformation of the myocardium lead to a linear transformation of the orthogonal electrocardiosignals under the condition of topological invariability of the depolarization and repolarization processes in the myocardium. Certain relationships have been found between the linear transformations in the physical and functional spaces. Several practically important conclusions follow as a logical consequence of these statements.*

Keywords: Vectorcardiogram, Biophysical Model, Beat-to-Beat Alteration

1. Introduction

Comparison of single beats within one ECG recording or consecutive ECGs from the same patient during cardiologic tests and in long term monitoring is an essential part of the contemporary noninvasive electrocardiologic diagnosis.

Variations of the ECG signals from one heartbeat to another are affected by a number of factors, which do not reflect the electrophysiological state of the myocardium itself and complicate the analysis of the ECG signals. These factors are usually treated as distortion or extracardiac factors [1]. Considering comparison of consecutive ECG cycles from the same person, the main distortion factors are the patient's breathing movements, possible changes in position, and haemodynamic alterations resulting in rotation and deformation of the miocardium.

A simple biophysical model of excitable media is proposed to describe the dipole moment changes after deformation of the excitable media (myocardium). The first approximation of the deformation is linear transformation.

2. Subject and Model

Suppose, that the excitable media (region M, Fig. 1) was deformed by linear transformation **T** (region M_T, Fig. 1):

$$\rho = \mathbf{T}r, \tag{1}$$

where $\rho = (\xi, \eta, \zeta)' \in \mathsf{M}_\mathrm{T}$ is the new position of the myocardium point $r = (x, y, z)' \in \mathsf{M}$ after transformation **T** ($'$, transpose symbol). Let $d, \mathrm{J}(r)$ and $d_\mathrm{T}, \mathrm{J}_\mathrm{T}(r)$ be the dipole moments and current density before and after transformation **T**, respectively. Dipole moment vectors are expressed as integrals of current density over the excitable media regions:

$$d = \int_{\mathsf{M}} \mathrm{J}(\rho)\, \mathrm{d}v_\rho\,,\; d_\mathrm{T} = \int_{\mathsf{M}_\mathrm{T}} \mathrm{J}_\mathrm{T}(\rho)\, \mathrm{d}v_\rho\,. \tag{2}$$

For bidomain model of the excitable media, the current density is determined by gradient of the transmembrane potential [2], then before and after transformation **T** the current densities are:

$$\mathrm{J}(r) = -\sigma_\mathrm{i} \nabla_r \mathrm{U}(r), \quad \mathrm{J_T}(\rho) = -\sigma_\mathrm{i} \nabla_\rho \mathrm{U_T}(\rho), \tag{3}$$

where σ_i – intracellular conductivity, $\mathrm{U}(r)$, $\mathrm{U_T}(\rho)$ – transmembrane potentials before and after transformation **T**, $\nabla_r = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$; $\nabla_\rho = \left(\frac{\partial}{\partial \xi}, \frac{\partial}{\partial \eta}, \frac{\partial}{\partial \zeta}\right)$.

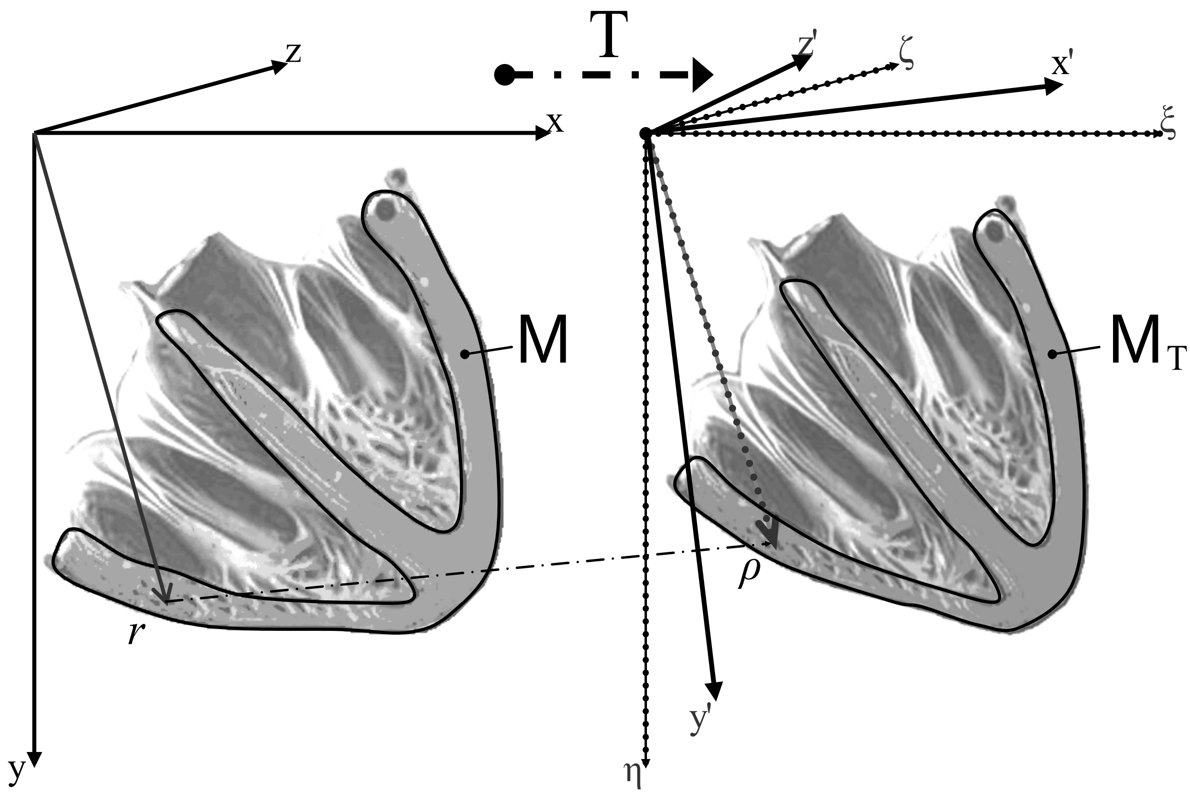

Fig. 1. Deformation of myocardium. The point $\rho = (\xi, \eta, \zeta)' \in \mathsf{M_T}$ is the new position of the myocardium point $r = (x, y, z)' \in \mathsf{M}$ after transformation **T**.

Assume further that the state of myocardial cells does not change after transformation **T**, and transmembrane potential remains the same, so

$$\mathrm{U_T}(\rho) = \mathrm{U_T}(\mathbf{T}r) = \mathrm{U}(r). \tag{4}$$

The main aim of the present reasoning is to obtain the relation between dipole moments before and after the deformation of the excitable media. Using Eqs. 1 – 4, taking into account that gradient of a function is a contravariant tensor of rank 1, after several simple manipulations and change of variables $r = \mathbf{T}^{-1}\rho$, the relation is obtained:

$$\begin{aligned} d_\mathrm{T} &= \int_{\mathsf{M_T}} \mathrm{J_T}(\rho)\,\mathrm{d}v_\rho = -\sigma_\mathrm{i} \int_{\mathsf{M_T}} \nabla_\rho \mathrm{U_T}(\rho)\,\mathrm{d}v_\rho = -\sigma_\mathrm{i} \int_{\mathsf{M}} (\mathbf{T}')^{-1} \nabla_r \mathrm{U}(r) \frac{\partial v_\rho}{\partial v_r}\,\mathrm{d}v_r \\ &= -\sigma_\mathrm{i} (\mathbf{T}')^{-1} |\mathbf{T}| \int_{\mathsf{M}} \nabla_r \mathrm{U}(r)\,\mathrm{d}v_r = (\mathbf{T}')^{-1} |\mathbf{T}|\, d, \quad |\mathbf{T}| = \det(\mathbf{T}). \end{aligned} \tag{5}$$

3. Results and Corollaries

For the bidomain model, a linear deformation **T** of the excitable media leads to linear transformation **G** of the dipole moment under the condition of the topological invariability of the activation propagation. The relation between these two linear transformations is:

$$d_{\mathrm{T}} = \mathbf{G}\, d\,, \quad \mathbf{G} = (\mathbf{T}')^{-1}|\mathbf{T}|. \tag{6}$$

Assume further that vectorcardiogram (VCG) reflects the heart vector evolution during depolarization process in the heart. Let $\{\mathbf{f}_0(t), \mathbf{f}_1(t), \ldots, \mathbf{f}_n(t)\}$ be a sequence of n QRS loops, and $\mathbf{G}_i$ be the linear transformation that maps $\mathbf{f}_0$ (reference loop) to approximate $\mathbf{f}_i$ with the minimum root mean square error ε_i:

$$\mathbf{f}_i = \mathbf{G}_i\,(\mathbf{f}_0)+\mathbf{e}_i\ (i=1,..\mathrm{n}),\ \varepsilon_i = \|\,\mathbf{e}_i\,\| \tag{7}$$

The transformations $\mathbf{G}_i$ of the reference QRS loop are represented by the rotations $\mathbf{O}_i$ and dilatations $\mathbf{S}_i$: $\mathbf{G}_i = \mathbf{O}_i\,\mathbf{S}_i$. Any linear transformation in the Euclidean space may be obtained by sequential executing rotation and dilatation (in any sequence). This provides separate analysis of these two kinds of transformations. The rotation of the reference QRS loop is characterized by the eigenvector of this transformation and the angle of turn around this vector.

The dilatation of the reference QRS loop is characterized by the coefficients of dilatation along three orthogonal directions, or, in other words, by the eigenvalues (σ_{i1} , σ_{i2} , σ_{i3}) and three orthogonal eigenvectors of this transformation. The product of these three eigenvalues is equal to the factor of the volume expansion for any three dimensional body after the transformation.

Looking back to the corresponding myocardium transformations $\mathbf{T}_i$ through Eq. 6, it is possible to calculate the eigenvalues (λ_{i1} , λ_{i2} , λ_{i3}) and eigenvectors that characterize the myocardium expansion (or contraction) for the i-th beat against reference beat. The product of eigenvalues $\lambda_i = |\mathbf{T}_\mathrm{i}| = \lambda_{i1}\,\lambda_{i2}\,\lambda_{i3}$ may be treated as the end-diastolic heart volume dilatation (or contraction) relative to the reference beat.

The simple relations between the eigenvalues of linear transformations in the physical and functional spaces are obtained using Eq. 6:

$$\sigma_{i1} = \lambda_{i2}\,\lambda_{i3}\,,\ \sigma_{i2} = \lambda_{i1}\lambda_{i3}\,,\ \sigma_{i3} = \lambda_{i1}\,\lambda_{i2}\,;\ \sigma_i = \lambda_i^2 \text{ or } |\mathbf{G}_i| = |\mathbf{T}_i|^2. \tag{8}$$

The same results may be obtained while analyzing the double layer activation front deformation (Fig. 2), and for more general cases.

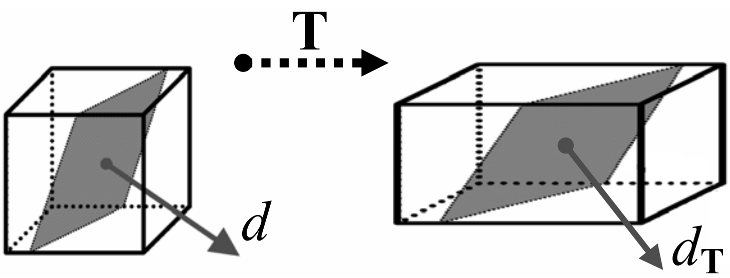

Fig. 2. Elementary cube and double layer dilatation along three eigenvectors of transformation **T**. Calculation of the relation between dipole moments before and after transformation and integration over the whole surface of excitation gives the Eqs. 6, 8.

4. Discussion

The influence of the ventricular volume on the electric heart vector is usually explained by Brody effect [1] without mentioning the changes of myocardium that contain the sources of the electric field. Brody effect means that the intracardiac blood surface plays a role of an imperfectly reflecting curved mirror. Changes in VCG are caused both by myocardial deformation and its reflection on intracardiac blood. It is important to find out their relative contribution. The same changes of VCG after changes of the left ventricular end-diastolic volume may be easily explained by Brody effect [1] and also by linear deformation of myocardium.

The presented model was used in [3], where a set of vectorcardiograms recorded during the parabolic flights of a laboratory aircraft was analyzed. In all normal cases, the QRS loop for each person, as a curve in the three dimensional (3D) vectorcardiographic space, remains virtually unchanged after the proper 3D linear transformation (the relative error of approximation was less than 0.05). The distance between the superimposed QRS loops served as an indicator of changes in the heart depolarization process. Changes of the QRS volume factor $|\mathbf{G}_i|$ were in accordance with haemodynamic changes due to gravitation acceleration.

5. Conclusions

For the bidomain model, linear deformation of the excitable media leads to linear transformation of the dipole moment under the condition of the topological invariability of the repolarization and depolarization processes.

The parameters of linear VCG transformations (eigenvectors, eigenvalues, rotation angles, determinants) may be used as indices of the heart position and haemodinamic changes.

The real interrelation of VCG transformations and myocardium deformations is much more complex. Further theoretical and experimental investigations are needed to assess the validity of the proposed biophysical model and the results obtained.

Acknowledgements

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