Detection of Variations in Biomedical Signals Based on Continuous Wavelet Transform Modulus Maxima

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Abstract. *This paper presents an approach to identification of small variations in sampled signals based on continuous wavelet transform and nonlinear solution of the least-mean squares problem. Proposed technique is able to distinguish changes which are not visible by direct observation. The study was performed on signals from an ECG simulator recorded by the ProCardio 8 system. The method could be used in a beat-to-beat morphological analysis of ECG signals.*

Keywords: ECG, continuous wavelet transform, modulus maxima point, Gaussian wavelet

1. Introduction

The continuous wavelet transform (CWT) represents a tool for analysis of signals which can be the best described as aperiodic, noisy, intermittent, transient and so on [1]. CWT transform coefficients (TC) contain detailed information concerning the signal dynamics across the time-scale-amplitude (T-S-A) space. However this representation is redundant and not easy to interpret. Analysis of such TC can be focused on variations of frequency content with respect of time or on discontinuities, edges and sharp transitions. The latter are important in ECG signals that are quasi-periodic type of sequences with repeating fiducial points in P, QRS, T and U waves [2]. These points can be regarded in the time domain as singular points [3]. We analysed these singularities by measurement of the decay of TC computed by the CWT.

2. Methods

It was shown [3] that each sample in a discrete sampled signal can be considered as a point ν with some neighbourhood $[\nu - h, \nu + h]$ and the signal $f(t)$ can be approximated by the function:

$$p_\nu = \sum_{k=0}^{m-1} \frac{f^{(k)}(\nu)}{k!}(t-\nu)^k \tag{1}$$

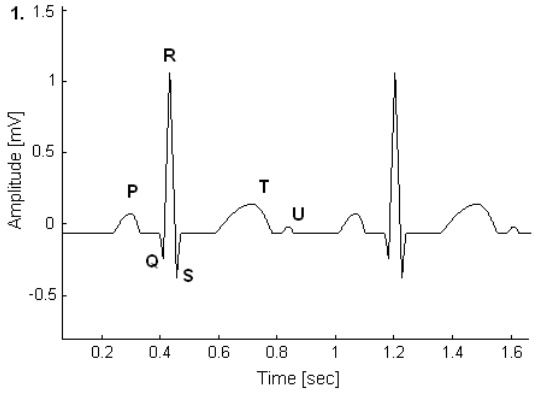
with an approximation error $\varepsilon_\nu(t) = f(t) - p_\nu(t)$ (2). Signal $f(t)$ can be expressed as:

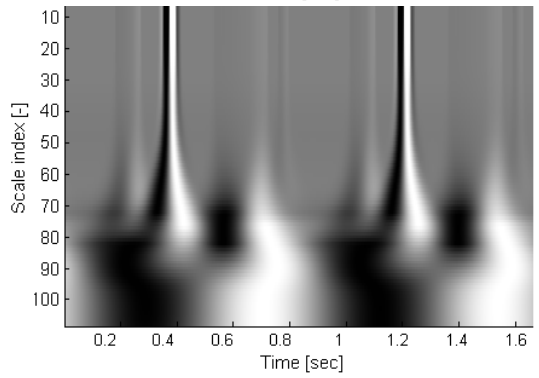
$$f(t) = p_\nu(t) + \varepsilon_\nu(t) \tag{3}$$

where the error of approximation is $|\varepsilon_\nu(t)| \le K|t-\nu|^\alpha$ (4), α is Lipschitz exponent and K is constant that is independent of ν. If exists the wavelet $\psi(t)$ with $n > \alpha$ vanishing moments $\int_{-\infty}^{\infty} t^k \psi(t)dt = 0$ (5) for $0 \le k < n$, for the wavelet transform (WT) is valid:

$$Wp_\nu(u,s) = \int_{-\infty}^{+\infty} p_\nu(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)dt = 0 \tag{6}$$

With respect to Eq.(3), WT can be written as $Wf(u,s) = Wp_{\nu}(u,s) + W\varepsilon_{\nu}(u,s)$ (7) and thus: $Wf(u,s) = W\varepsilon_{\nu}(u,s)$ (8), where $Wf(u,s)$ are amplitudes of TC in u time and s scale. From Eq.(8) it is clear that the TC of WT express the error of approximation of function $f(t)$ by Eq.(1). It is also obvious that the amplitudes of TC are related with Lipschitz exponent. We used this dependence to measure the decay of amplitude of TC to characterize singularities that correspond to fiducial points in analyzed ECG signal. For this purpose we computed the modulus maxima points (MMPs) across the T-S-A space by using the equation:





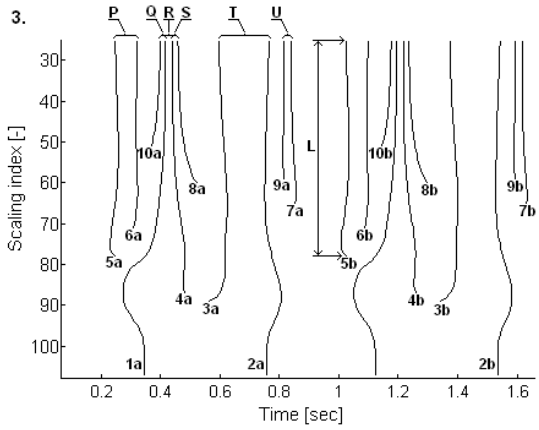


Fig. 1.1. Two similar cycles of recorded ECG signal;

Fig. 1.2. TC computed from signal depicted in Fig. 1.1;

Fig.1.3. MMLs (marked as 1a - 10a according to the length L in T-S-A space) that correspond to the fiducial points in ECG signal depicted in Fig. 1.1.

$$\frac{\partial W f(u_k, s_l)}{\partial u} = 0 \qquad (9)$$

where u_k is k-th value of time and s_l is l-th value of scale. By connecting all MMPs across the scale s in the T-S-A space we created modulus maxima lines (MMLs). CWT of an ECG signal (Fig. 1.1) acquired by the ProCardio-8 system [4] was computed using the first derivation of Gaussian wavelet. We searched TC (Fig. 1.2) in T-S-A space and localized all points where Eq.(9) is valid. From these points we constructed the MMLs. Not only MMLs belonging to the fiducial points but also those belonging to the singularities that represent the noise, were created. MMLs that correspond to the fiducial points were separated by tresholding (Fig. 1.3). From Fig. 1.1 and Fig. 1.3 it is obvious that each MML corresponds to one specific fiducial point (1a to 10a represent the first and 1b to 10b the second activation sequence in the ECG signal). Each MML can be then analyzed in 3D space. This option gives the complete characterization of all fiducial points in the T-S-A space (Fig. 2.1). We analyzed each MML in a 2D projection, namely in the S-A (scale-amplitude) projection (Fig.2.2) and limited the analysis only to the exponential segment of the MML. The analysis was performed with an exponential model:

$$\mathbf{y} = A.e^{(\mathbf{x}-D).B} + C \qquad (10)$$

where $\mathbf{x}$ and $\mathbf{y}$ are values for input segment of MML in S-A projection (Fig. 2.2) and A, B, C, D are approximation coefficients. We solved Eq.(10) for all MMLs associated to the fiducial points in selected segment of the ECG signal, and obtained a set of parameters that are related to behavior of these points in the T-S-A space. Parameters for MMLs in the first and in the second activation sequence of the ECG signal were computed.

3. Results and discussion

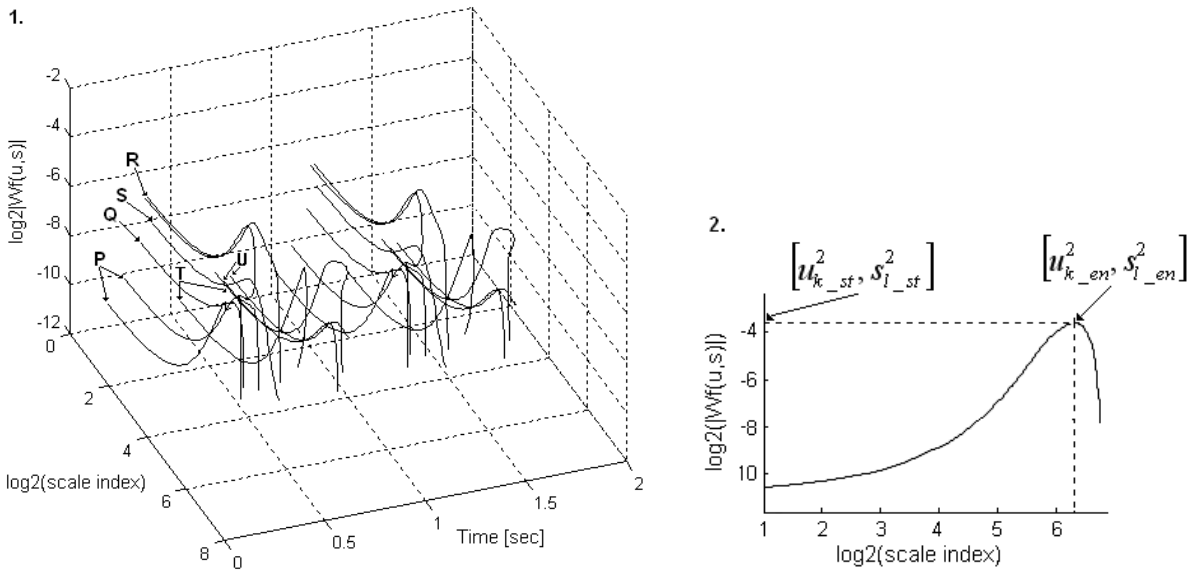


Fig. 2.1. MMLs of ECG signal in T-S-A space. Fig.2.2. MML of ECG signal pertaining to T wave in S-A projection (coordinates $u^2_{k\_st}$, $s^2_{l\_st}$ denote the start and coordinates $[u^2_{k\_en}, s^2_{l\_en}]$ denote the end point of analyzed exponential segment of the maxima line in S-A projection).

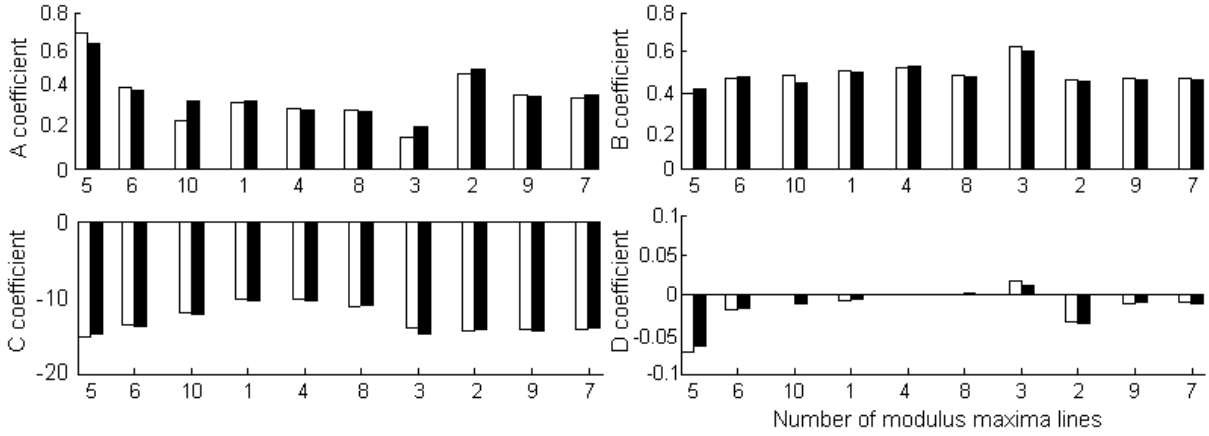


Fig. 3. Coefficients computed from MMLs (Fig. 1.3) for ECG signals depicted in Fig. 1.1 for each line number. Two sets of values of particular coefficient are depicted in each figure that corresponds to the specific MMLs (1a-10a for the first and 1b-10b for the second set depicted in Fig. 1.3). The first set is for coefficients that correspond to the first (white color) and the second set is for coefficients that correspond to the second activation sequence (black color) in the ECG signal.

While no obvious differences between signals were visible in the time domain, noticeable differences were detected in proposed coefficients. The maximal differences in coefficient values (Fig. 3) were localized at positions that correspond to the 5th, 10th and 3th MML. Computed values shown that the maximal difference occurred at 10th MML in all studied coefficients except of C. It corresponds to the onset of the Q wave in ECG signal (compare Fig. 1.1 with Fig. 1.3). In C the maximal difference occurred at 3th MML. The absolute maximal difference among all coefficients, which corresponds to 10th MML, was in coefficient A that refers to a scaling of exponent in the model. Absolute minimal difference that corresponds to that MML was detected in coefficient D that is related to the linear

translation along x axis (this axis expresses log2 (scale index) and it is inversely proportional to the frequency [1]) as it is apparent from Eq.(10) and Fig. 2.2. It is also worth to note that absolute maximal difference from all coefficients (except C that represents the linear translation along y axis – this axis expresses log2 (|Wf (u, s)|)) was detected at position that correspond to the 10th MML and represents the onset of the Q wave. Each coefficient represents different property of particular MML that corresponds to the specific fiducial point. Coefficient D determines frequency offset of each particular MML in the T-S-A space. Coefficient C expresses translation along the axis of amplitude of transform coefficients and determines the amplitude offset of each MML. Another important parameter is length of MML in the T-S-A space. This parameter determines a range of each MML along frequency axis. From Fig. 1.3 it is apparent that the sharpest features of ECG signal (such as R wave and T wave) have the longest corresponding MMLs. This means that frequency content of these features tends from the highest to the lowest frequencies and even very low frequencies are included unlike in the other features of the analysed ECG signal. We can speculate that specific change in activation sequence of ECG signal is related to corresponding change in some property of particular MML in the T-S-A space and thus with the change of coefficient values of the exponential model (with more or less sensitivity).

4. Conclusions

From the presented results we can conclude that even small changes in characteristic features of ECG signals are projected into coefficients of the proposed exponential model. We expect that the measure of these changes is different for each coefficient what means that different type of change in signal is coupled with change of different coefficient and thus it can be detected and evaluated. Each coefficient thus carries different information about the fiducial point in the analyzed signal and is connected with the corresponding singularity. A complete characterization of each activation sequence can be computed what offers new possibilities of the beat-to-beat morphological characterization of ECG signals. Future work will be focused on examination of applicability of the presented approach on real ECG signals [5].

Acknowledgements

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