The Magnetoplasmic Measurements of the Carrier Density and Mobility in Semiconductors

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Abstract. Many semiconductor materials manufactured by the help of nanotechnology have the charge carriers of different type and mobility. Already existing carrier density and mobility measurement methods are not accurate enough for the case of several carrier components. The use of the magnetoplasmic waves provides a simple and most precise way to determine the density and mobility of each type of the carriers (electrons and/or holes).

Magnetoplasmic Waves may be excited in semiconductors when the strong magnetic field $H$ is applied. The semiconductor sample becomes partially transparent under these conditions. In the case of magnetoplasmic resonance within each of carrier groups the transparency coefficient has maximum. For fixed values $H$ and excitation frequency $\omega$ the density and mobility of every carrier type can be found.

Dispersion relation for two types of charge carriers is obtained and resonance curves are calculated.

Keywords: Magnetic measurement, imaging, magnetic susceptibility, helicons, calculation.

1. Introduction

Many semiconductor materials manufactured by the help of nanotechnology have the charge carriers of different type and mobility. Already existing carrier density and mobility measurement methods are not accurate enough for the case of several carrier components. The use of the magnetoplasmic waves (helicons) provides a simple and more precise way to determine the density and mobility of each type of the carriers (electrons and/or holes).

Magnetoplasmic waves may be excited in semiconductors when the strong magnetic field $H$ is applied and large Hall currents may exist. In the case of magnetoplasmic resonance within each of carrier groups the transparency coefficient has maximum. For fixed values $H$ and excitation frequency $\omega$ the density and mobility of every carrier type can be found.

2. Subject and Methods

Let us consider a semiconducting plate with an electric coil placed on its surface. We shall for the sake of simplicity assume the semiconductor to be infinitely large. Let the semiconductor surface be parallel to the $xy$ plane and the $z$-axis be directed perpendicular to it. Then the electrical current has only one component along the $y$-axis, and the magnetic has one along the $x$-axis. We assume, that the field $H_x$ varies with time according to a harmonic law

$$H_x = H \cos \omega t, |z| = a,$$  \hspace{1cm} (1)

Where $a$ is the thickness of the plate along $z$-axis [1]. The currents induced in the semiconductor sample are directed in such a way so as to counteract the penetration of the field. As a result the varying magnetic field within the semiconductor will be other than zero.
only to a certain depth (skin depth). If the semiconductor plate is at the same time placed into the strong magnetic field \( H_x = H_0 \), the Hall currents \( j_x \) and \( j_y \) appear and helicon magnetoplasmic waves may be excited. Assuming that the conductivity of the plate is provided by electrons with an isotropic mass we have the following equations of motion for the current components \( j_x \) and \( j_y \):

\[
\begin{align*}
\frac{d}{dt} j_x + \tau^{-1} j_x - \frac{eH_0}{mc} j_y &= \frac{Ne^2}{m} E_x, \\
\frac{d}{dt} j_y + \tau^{-1} j_y + \frac{eH_0}{mc} j_x &= \frac{Ne^2}{m} E_y
\end{align*}
\]

where \( N, e, m \) and \( \tau \) are the density, charge, mass and collision time of the electrons, \( E_x \) and \( E_y \), are the varying electrical field, and \( H_0 \) is the static magnetic field along the z-axis.

The equations of motion (2) must be solved along with the Maxwell equations

\[
\text{rot} \vec{\varepsilon} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \quad \text{rot} \vec{\varepsilon} = -\frac{1}{\varepsilon} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}
\]

The solutions of the system of equations (2), (3) were sought in the form [2]

\[
\vec{\varepsilon} = \text{Re} \vec{E} \exp(-i\omega t), \quad \vec{H} = \text{Re} \vec{H} \exp(-i\omega t), \quad \vec{j} = \text{Re} \vec{I} \exp(-i\omega t)
\]

where the complex amplitudes \( \vec{E}, \vec{H} \) and \( \vec{I} \) depend on only coordinate \( z \) and have the components \( x \) and \( y \). Taking into account that \( \omega \ll \tau^{-1} = 10^{11} - 10^{13} \) Hz and ignoring the displacement currents in comparison with conductive ones we obtain a system of equations for the complex amplitudes of the varying magnetic field \( H_x \) and \( H_y \):

\[
c^2 \frac{\partial^2}{\partial z^2} H_x + i\omega \delta_{12} H_x + i\omega \delta_{12} H_y = 0, \quad c^2 \frac{\partial^2}{\partial z^2} H_y + i\omega \delta_{21} H_y - i\omega \delta_{12} H_x = 0
\]

where the components of the conductivity tensor \( \sigma \) are

\[
\begin{align*}
\sigma_{11} &= \sigma_{22} = \sigma_0 \frac{1}{1 + \left( \frac{eH_0 \tau}{mc} \right)^2}, \\
\sigma_{12} &= \sigma_0 \frac{\frac{eH_0 \tau}{mc}}{1 + \left( \frac{eH_0 \tau}{mc} \right)^2}, \\
\sigma_0 &= \frac{Ne^2 \tau}{m}
\end{align*}
\]

The boundary conditions have the form

\[
H_x = H, H_y = 0 \text{ if } z = \pm a, \quad \frac{\partial H_x}{\partial z} = \frac{\partial H_y}{\partial z} = 0 \text{ if } z = 0
\]

and the solutions of (5) satisfying (7) are

\[
H_x = \frac{1}{2} H \left( \frac{\cos k_x z + \cos k_y z}{\cos k_x a + \cos k_y a} \right), \quad H_y = \frac{1}{2} iH \left( \frac{\cos k_x z + \cos k_y z}{i\cos k_x a + \cos k_y a} \right)
\]

where \( k \pm \) can be detected from the characteristic equation

\[
c^2 k^2 = \frac{\omega_p^2 \omega}{\omega_p + i\tau^{-1}}, \quad \omega_p = \frac{4\pi Ne^2}{m}, \quad \omega_p = \frac{eH_0}{mc}
\]

The items with the argument \( k \) are caused by the helicon magnetoplasmic waves.
3. Two types of charge carriers

If the semiconductor sample contains two types of electrons with different densities and mobilities the characteristic equation for magnetoplasmic wave vector $k$ can be written as follows

$$k_{\pm}^2 = \omega \mu_0 e N_i u_i \left( \frac{-i \pm u_i B}{1 + (u_i B)^2} \right) + \omega \mu_0 e N_2 u_2 \left( \frac{-i \pm u_2 B}{1 + (u_2 B)^2} \right), \quad (10)$$

where $N_1$, $N_2$ and $u_1$, $u_2$ are the densities and mobilities of the different types of the charge carriers; $\mu_0$ - magnetic permeability of vacuum. The resonant curves in dependence of magnetic field $B$ were calculated from (17) for various values of $N_1$, $N_2$, $u_1$, $u_2$, and angular frequencies $\omega$, and are shown on Fig.1, 2. The resonant curves of amplitude and phase for forward wave $K$ and reflected wave $P$ from InSb plate in dependence of magnetic field $B$ are calculated using program [3] for frequency $f=300$ MHz. Parameters of semiconductor plate: thickness $a=5$ mm, dielectric constant $\varepsilon=15$, charge carriers density $N=0.15*10^{23} \text{ m}^{-3}$, charge carriers mobility $u=5 \text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$.

The first main peak for $B_z = 29T$ corresponds to the resonance of electrons with higher mobility $u_1$ (density $N_1$). The second peak for $B_z = 7T$ is responsible for the resonance of both carriers type with summary density $(N_1 + N_2)$. The heights of both peaks are proportional to the products $(u_1 B)$ and $(u_2 B)$ respectively. The comparison of experimental and theoretical resonant curves provides the possibility to calculate the parameters $N_1$, $N_2$, $u_1$, and $u_2$ for both types of charge carriers.

There are observed, that different sign of mobilities $u_1$ and $u_2$ increase second and higher resonant maximums in resonant curves.

Fig. 1  The resonant curves in dependence of magnetic field $B$ for:

a - $u_1 = -5 \text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$, $u_2 = 5 \text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$, $N_1 = N_2 = 0.15*10^{23} \text{ m}^{-3}$;
b - $u_1 = -2.5 \text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$, $u_2 = 5 \text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$, $N_1 = 0.3*10^{23} \text{ m}^{-3}$, $N_2 = 0.15*10^{23} \text{ m}^{-3}$;
1 - $f=250$ MHz, $K_{\text{max}}=4.01$, 2 - $f=300$ MHz, $K_{\text{max}}=7.19$, 3 - $f=350$ MHz, $K_{\text{max}}=2.74$
4. Experimental verification

The theoretical results were confirmed experimentally for the n-Ge semiconductor material. In Fig. 2 the experimental resonance curve is shown for the case \( f = 25 \text{ MHz} \) and specimen thickness \( d = 3 \text{ mm} \). The relatively weak signal in detection coil for \( B = 1.5 \text{ T} \) pertains to the electrons with a higher mobility \( 0.65 \text{ m}^2\text{V}^{-1}\text{s}^{-1} \), and the stronger maximum for \( B = 8 \text{ T} \) – to the electron with a smaller mobility \( 0.125 \text{ m}^2\text{V}^{-1}\text{s}^{-1} \).

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![Graph showing experimental resonant curve](image)

**Fig. 2** The experimental resonant curve in dependence of magnetic field \( B \)

5. Conclusions

The measurement of the charge densities and mobilities for the charge carriers of various types in semiconductor materials by the help of magnetoplasmic waves can be provided in contactless mode. Many semiconductors thus can be investigated if the high magnetic fields (~30 Tesla) are available. The measurement results are in compliance with the data obtained by the use of already existing methods.

References

