Magnetic Field of Spiral-shaped Coil

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Abstract. *Spiral-shaped coil is one of several basic RF coils used in magnetic resonance instruments. It is appropriate mainly for surface measurements. Magnetic field of spiral-shaped coil was calculated using the Biot-Savart law and numerically computed. A sample coil was made and its magnetic field was measured using a magnetic resonance method. The verification showed very good qualitative and quantitative agreements between the calculated and measured values of the magnetic field.*

Keywords: magnetic field, magnetic resonance, spiral-shaped coil, NMR

1. Introduction

Although Nuclear Magnetic Resonance (NMR) tomographs are manufactured and disposed by many commercial firms a researcher can come across a moment when the supplied equipment is insufficient. Such part is for instance an RF coil. It must excite protons in a sample and/or convert RF magnetic field from the sample into electrical signal. Several parameters of the coil must be considered during a design. One of them and very significant is magnetic field of the coil and its homogeneity. Homogeneity or sensitivity of the coil can be increased by optimisation. Knowledge of the magnetic field is inevitable in the both cases. Magnetic field can be calculated in more manners. Calculation using the Biot-Savart law is rather frequent because it can be applied on a coil with an arbitrary shape. The spiral-shaped coil is very frequently used RF coil for construction of surface RF coils or arrays of NMR coils. It can be used for systems working at low as well as at high magnetic field. The purpose of the article is to present equations for computing the magnetic field of spiral-shaped coil based on the Biot-Savart law. Correctness of the equations was verified by NMR method described in [1].

2. Subject and Methods

Magnetic field of a three-turn spiral-shaped coil was calculated using the Biot-Savart law (see a two-turn spiral-shaped coil in Fig. 1). The calculation was made in a vector form, that is why the calculation is very mistake-proof. Calculations in vector form were performed very successfully using the program package Mathematica (Wolfram Research Inc., Champaign, IL). The computed magnetic field was compared with the magnetic field measured on a sample of the spiral-shaped coil using the NMR method [1]. Measurement was performed on 1.5 T NMR system (Gyroscan NT, Philips, Best, the Netherlands).

3. Results

Look at a two-turn spiral-shaped coil in Fig. 1. The application of the Biot-Savart law yielded the following equations:

$\mathbf{B} = \frac{\mu_o I}{4\pi} \oint \frac{d\mathbf{s} \times (\mathbf{P} - \mathbf{s})}{|\mathbf{P} - \mathbf{s}|^3}$ is magnetic field calculated using the Biot-Savart law [2], where

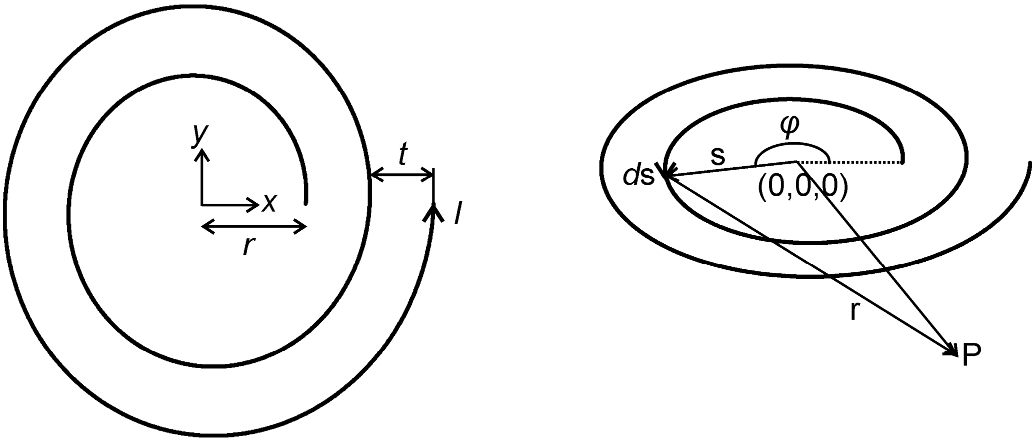

Fig. 1 Two-turn spiral-shaped coil.

I is current through the coil,

P is vector pointing to the observer and

s is vector pointing to centerline element of the coil conductor d**s**.

$\mathbf{P} = \mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z$ is vector pointing to the observer.

$\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unity vectors in directions of the x,y,z coordinates.

$\mathbf{s} = \mathbf{i}\left(r + \frac{t}{2\pi}\varphi\right)\cos\varphi + \mathbf{j}\left(r + \frac{t}{2\pi}\varphi\right)\sin\varphi$ is vector pointing to the coil.

$$d\mathbf{s} = \left(\mathbf{i}\left(\frac{t}{2\pi}\cos\varphi - \left(r + \frac{t}{2\pi}\varphi\right)\sin\varphi\right) + \mathbf{j}\left(\frac{t}{2\pi}\sin\varphi + \left(r + \frac{t}{2\pi}\varphi\right)\cos\varphi\right)\right)d\varphi$$

is vector element of the coil in the point determined by the vector **s**.

$\mathbf{R} = \mathbf{P} - \mathbf{s}$ is vector between the observer and the coil.

$$\mathbf{B} = \left(I \cdot 10^{-7}\right)\int_0^{2\pi n} \frac{d\mathbf{s} \times \mathbf{R}}{|\mathbf{R}|^3}$$

is resulting magnetic field in the point **P** due to the current I.

The NMR method used for verification is described in [1]. The magnetic field was measured on the three-turn spiral-shaped coil with the parameters:

I = 4.5 mA, r = 0.01 m, t = 0.007 m, n = 3

The magnetic field was calculated and measured [1,3] in the three basic planes, Fig. 2 depicts the both calculated and measured values on centrelines in the plane xy and yz. It is evident very good agreement between the calculated and the measured values.

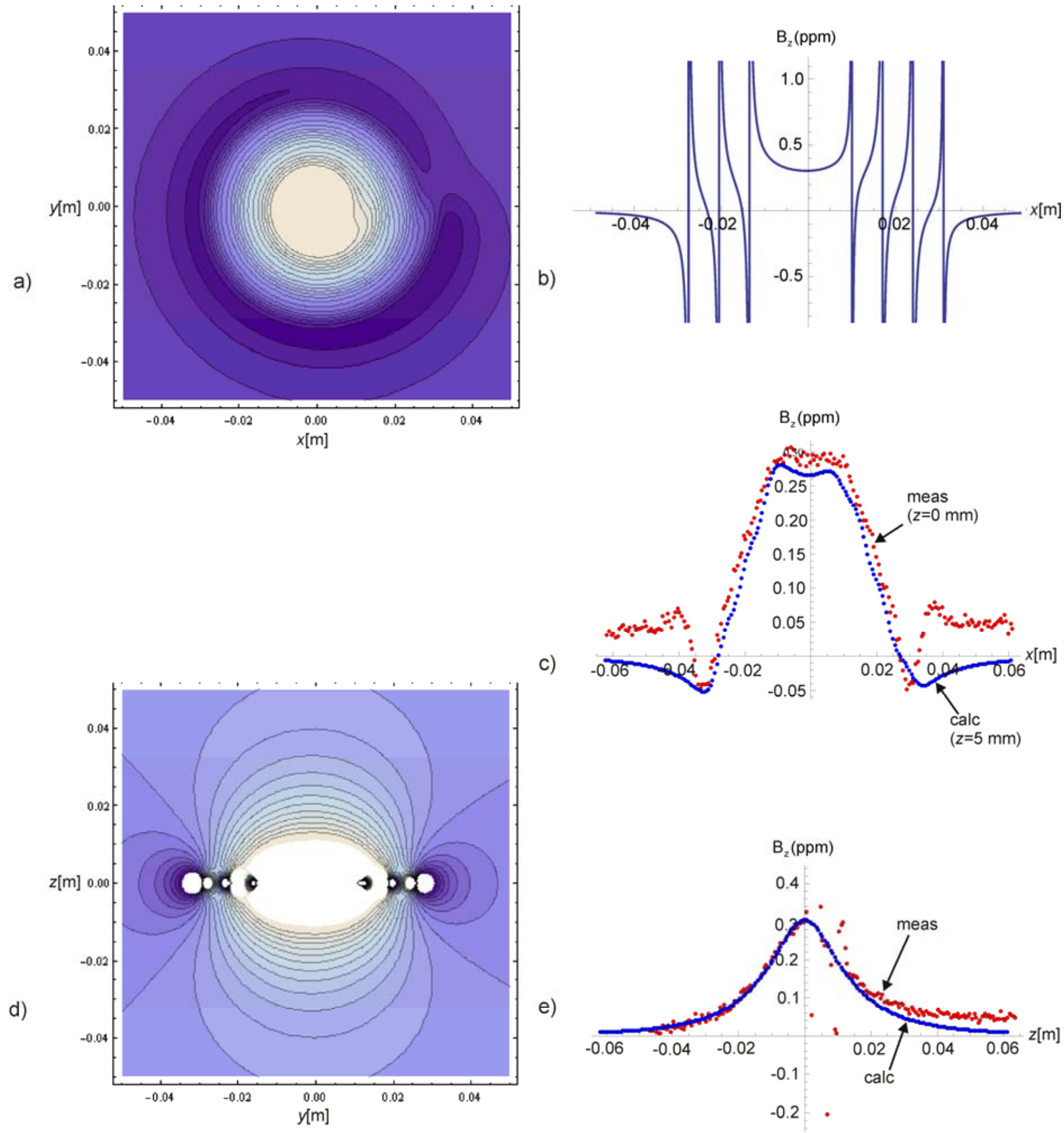

Fig. 2 Comparison between the calculated and the measured magnetic field of the three-turn spiral-shaped coil. The contour plot with the calculated values of the magnetic field in the plane *xy*, *z*=5 mm a); Calculated values at the centreline in the *xy* plane for *z*=0. The discontinuities due to zero distance between the conductor and the observer are obvious. b); A comparison between the magnetic fields at the centrelines in the plane *xy*: the measured for *z*=0, the calculated for *z*=5 mm. Despite of the different *z* coordinates the difference in the magnetic field values has not exceeded 10% in the region of interest c); The contour plot of the calculated values in the plane *yz* for *x*=0 d); A comparison between the calculated and the measured magnetic field at the centrelines in the *yz* plane. The difference in the right part of the graph was caused by a mechanical instability during the measurement but it is beyond the region of interest. e). The noise in the figure e) was caused by a plastic holder at the sample coil.

4. Discussion and Conclusions

Experiments confirmed correctness of the calculated values. Also the numerical integration was completed in relatively short time. The equation so can be used for a subsequent optimisation. Some inaccuracies occurring in the presented results could be caused by some of the following reasons: imperfections in the sample coil (it was rather small), imperfections in the measurement performing, drifting the DC current through the sample coil during the measurement. Whereas the measurement was performed during a stay abroad it can not be repeated simply. The presented equations can be simple used for the magnetic field of a spiral-shaped coil with arbitrary number of turns calculation. Spiral-shaped coil can be used as a surface RF coil for NMR [4,5] or as a device of phase arrays using advantageous signal-to-noise parameters of surface coils to volume measurements. From the presented figures of the magnetic field calculated at z=0 the discontinuity due to zero distance between the observer and the coil conductor is obvious. In the measured figures such discontinuity was not observed. The measured data were processed in different ways to explain the phenomenon e.g. the space close to z=0 was divided into appropriate grid to simulate voxels occurring in NMR measurement and to average the magnetic field within them, but the acquired results were still distant from the measured results. The probable reason is the fact that the measured values are limited by number of excited protons in the sample, but the calculated values can change without limitation. Examination of the phenomenon will continue.

Acknowledgements

Support of the Grant agency of the Slovak Academy of Sciences, grant no 2/0142/08 is gratefully acknowledged.

References

[1] Weis J., Andris P., Frollo I., Ahlström H.: A simple method for mapping the B_1 distribution of linear RF coil. *MAGMA*, 2005, 18: 283-287.

[2] Ilkovic D.: Physics. ALFA, Bratislava, 1972. (in Slovak)

[3] Andris P., Weis J., Frollo I., Ericsson A.: Magnetic field distribution of RF coil measurement using nuclear magnetic resonance. *Journal of Electrical Engineering*, 2002, 53: 32-34.

[4] Andris P., Frollo I.: Varicaps for NMR receiving coil matching and sensitivity changes. *Journal of Electrical Engineering*, 2007, 58: 291-293.

[5] Andris P.: Matching and tuning RF coils for NMR tomograph. *Measurement Science Review*, 2001, 1: 115-118.