Roundness: Determining the Reference Circle for MCCI and MICI System

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Abstract. Roundness measurements are used to evaluate and control the quality of cylindrical objects. There are several methods how to find the reference circle. The roundness error is then defined as the maximum peak-to-valley distance from the reference circle. In this article we present methods with their basic principles. The aim is focused on finding the minimum circumscribed circle and maximal inscribed circle.

Keywords: circumscribed, inscribed, circle, Voronoi, roundness.

1. Introduction

Roundness measurements are used to evaluate and control the quality of cylindrical objects. Methods used for determining the roundness are the least-squares circle (LSC), maximum inscribed circle (MIC), minimal circumscribed circle (MCC) and minimum zone circle (MZC) method. These methods are defined as follows [2][4].

 - Least square circle (LSC): It is a circle which separates the roundness profile of an object by separating the sum of total areas of the inside and outside it in equal amounts. The roundness error then can be estimated as the difference between the maximum and minimum distance from this reference circle

- Minimum circumcised circle (MCC): It is defined as the smallest circle which encloses whole of the roundness profile. Here the error is the largest deviation from this circle

- Maximum inscribed circle (MIC): It is defined as the largest circle that can be inscribed inside the roundness profile. The roundness error here again is the maximum deviation of the profile from this inscribed circle.
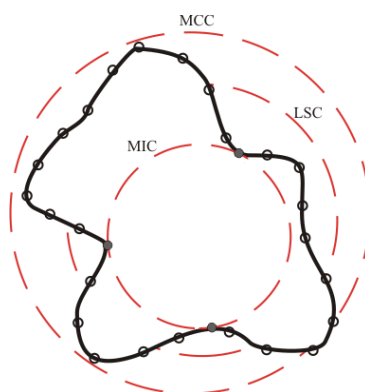

Fig. 1.   Overview of roundness methods

- Minimum Zone circle (MZC): Here two circles are used as reference for measuring the roundness error. One circle is drawn outside the roundness profile just as to enclose the whole of it and the other circle is drawn inside the roundness profile so that it just inscribes the profile. The roundness error here is the difference between the radius of the two circles.

2. Subject and Methods

For evaluating the roundness error from the actual measurement, a reference circle must be established from the measurement data to minimize the maximum deviation between the reference circle and the actual one. The roundness error is then defined as the maximum peak-to-valley distance from the reference circle.

In this chapter we describe the basic principle of finding the reference circle for deviation calculation.

Taylor's maximum material principle
This principle is invented by Wiliam Taylor in 1905 as GO gage. This principle is still used to inspect the maximal material condition (MMC) and least material condition (LMC). Principle states: "The maximum material limits of as many related elements or dimensions as practicable should be incorporated in the go gage, whereas the minimum material limit of each related element or dimension may be gaged only by individual minimum material limit gages or gaging methods." On cylindrical parts, size of gage limits roundness. This is accomplished through checking procedures of two opposing points [2].

Voronoi – Voronoi diagram
Voronoi diagrams are fundamental data structures that have been extensively studied in Computational Geometry. A Voronoi diagram can be defined as the minimization diagram of a finite set of continuous functions. Usually, each of those functions is interpreted as the distance function to an object. The associated Voronoi diagram subdivides the embedding space into regions, each region consisting of the points that are closer to a given object than to the others. We may define many variants of Voronoi diagrams depending on the class of objects, the distance functions and the embedding space. Voronoi diagrams are named after Russian mathematician Georgy Fedoseevich Voronoi who defined and studied the general n-dimensional case in 1908. The application of the diagram can be used in many fields, for example accuracy of parameters, biology, chemistry, network systems, modeling, planning [3].
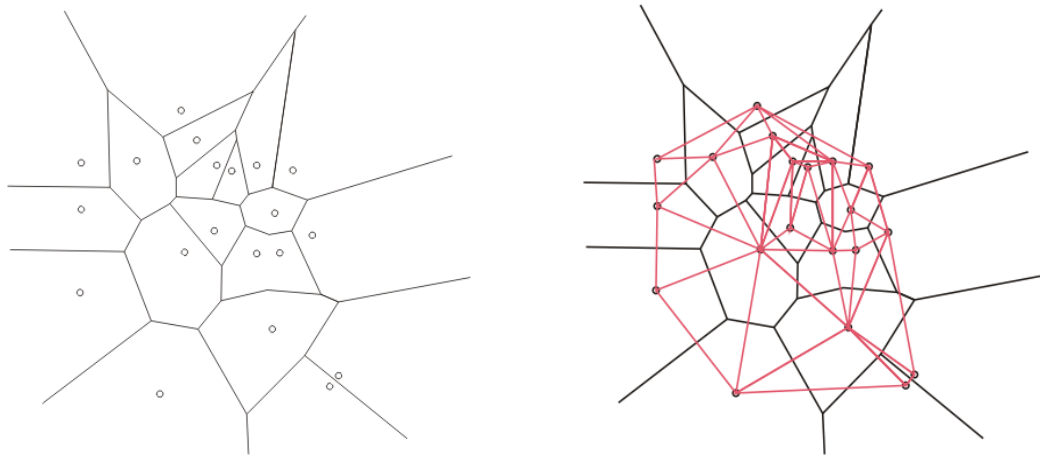

Fig. 2.   Voronoi diagram (left), relationship between Voronoi and the Delaunay triangulation (right).

The Voronoi diagram we can use for determining the minimal circumscribed circle or maximal inscribed circle. The circumscribed circle we can construct from the boundary points of diagram, which lies farthest from center of diagram. The inscribed circle has the center on Voronoi vertex or on Voronoi edge, which intersect convex hull. The radius is a distance of

circle center and convex vertex. For a calculating the convex and Voronoi diagram we use the Delaunay triangulation. Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation. The Delaunay condition states that a triangle net is a Delaunay triangulation if all the circumcircles of all the triangles in the net are empty. The triangulation was invented by Boris Delaunay in 1934. The relationship between Voronoi and the Delaunay is on the fig 2. The convex vertex represents the measured value.
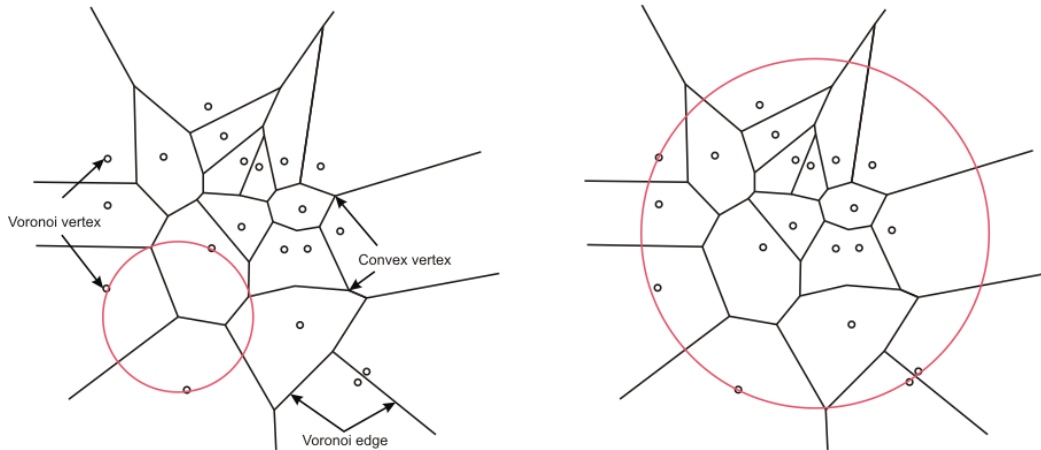

Fig. 3.   Determining the inscribed (left) and circumscribed (right) circle using Voronoi.

Permutation method
Simplest way to find the MIC or MCC is to make the permutation from measured points. The each three point set are calculated. The condition for these points is that the centre of circumscribed triangle by circle must be within these tree points and all point must lie in (MIC) or out (MCC) from the circle (Fig.4). The second pass calculates with the pairs of points, where the circle is in the middle and the radius is the half distance of these points. This is not the best for many points and not fastest method to find the circles.
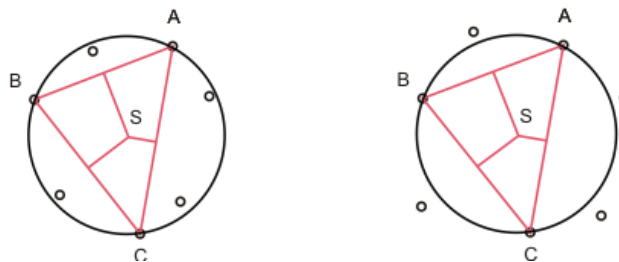

Fig. 4.   Demonstration of condition for circumscribing(left) and inscribing(right) circle

The diameter and the centre position of this circumscribed circle around the triangle are following [1][5]:

$$diameter = \frac{|AB|.|BC|.|CA|}{2.S_{abc}}$$ (1)

where

      A, B, C   vertex of triangle

      S_{abc} area of triangle

$$Sx = \dfrac{1}{2\begin{vmatrix} B_x - A_x & B_y - A_y \\ C_x - A_x & C_y - A_y \end{vmatrix}} \begin{vmatrix} |A|^2 & A_y & 1 \\ |B|^2 & B_y & 1 \\ |C|^2 & C_y & 1 \end{vmatrix} \qquad Sy = \dfrac{1}{2\begin{vmatrix} B_x - A_x & B_y - A_y \\ C_x - A_x & C_y - A_y \end{vmatrix}} \begin{vmatrix} A_x & |A|^2 & 1 \\ B_x & |B|^2 & 1 \\ C_x & |C|^2 & 1 \end{vmatrix} \qquad (2)$$

where

S_x, S_y centre of circle

Linear time method

In computational complexity theory, an algorithm is said to take linear time, or O(n) time. We can state that, there exist O(n3) such circles, and each takes O(n) time to check, for a total running time of O(n4). Elzinga and Hearn gave an O(n2) algorithm in 1972, and the first O(n logn) algorithms were discovered by Shamos and Hoey (1975), Preparata (1977), and Shamos (1978). Nimrod Megiddo showed in 1983 that his ingenious prune-and-search techniques for linear programming that adapted to find the minimal enclosed circle. This method reduced the number of circles (points) from calculation in every step by the special formula. The latter algorithm finds the median (and in general, the k-th smallest element) [6][7].

3. Conclusions

Each of these methods described above has some advantages and disadvantages. The best choice depends on quantity of points to be checked for reference circle. The question is "Which method is more precise, less time consuming, and reliable?"

Acknowledgements

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