Magnetic Field of Saddle-shaped Coil

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Abstract. *Saddle-shaped coil is one of several basic RF coils used in magnetic resonance instruments. Magnetic field of saddle-shaped coil was calculated using the Biot-Savart law and numerically computed. Magnetic field of a sample coil was measured using magnetic resonance method. To verify the calculation correctness, the calculated and measured values were compared showing excellent results.*

Keywords: magnetic field, magnetic resonance, saddle-shaped coil, NMR

1. Introduction

RF coil is an important part of Nuclear Magnetic Resonance (NMR) tomograph. It must excite protons in a sample and/or convert RF magnetic field from the sample into electrical signal. Several parameters of the coil must be considered during a design. One of them and very significant is magnetic field of the coil and its homogeneity. Homogeneity or sensitivity of the coil can be increased by optimisation. Knowledge of the magnetic field is inevitable in the both cases. Magnetic field can be calculated in more manners. Calculation using the Biot-Savart law is rather frequent because it can be applied on a coil with an arbitrary shape. The saddle-shaped coil was very frequently used RF coil in the beginning of the NMR. Nowadays it was replaced by some new kinds of coils in the machines with high magnetic field but it is still important for imaging at low magnetic field. The purpose of the article is to present equations for computing the magnetic field of saddle-shaped coil based on the Biot-Savart law. Correctness of the equations was verified by NMR method described in [1].

2. Subject and Methods

Magnetic field of a one-turn saddle-shaped coil was calculated using the Biot-Savart law (see Fig. 1). The calculation was made in a vector form, that’s why the magnetic field for a more-turn coil can be derived very simply adding the magnetic field from the next turns. Calculations in vector form were performed very successfully using the program package Mathematica (Wolfram Research Inc., Champaign, IL). The computed magnetic field was compared with the magnetic field measured on a sample of the saddle-shaped coil using the NMR method [1]. Measurement was performed on 1.5 T NMR system (Gyroscan NT, Philips, Best, the Netherlands).

3. Results

A one-turn saddle-shaped coil is depicted in Fig. 1. The application of the Biot-Savart law [2] yielded the following equations:

$\mathbf{B} = \dfrac{\mu_o I}{4\pi} \oint \dfrac{d\mathbf{s} \times (\mathbf{P} - \mathbf{s})}{|\mathbf{P} - \mathbf{s}|^3}$ is magnetic field calculated using the Biot-Savart law, where

I is current through the coil,
P is vector pointing to the observer and
s is vector pointing to centerline element of the coil conductor d**s**.

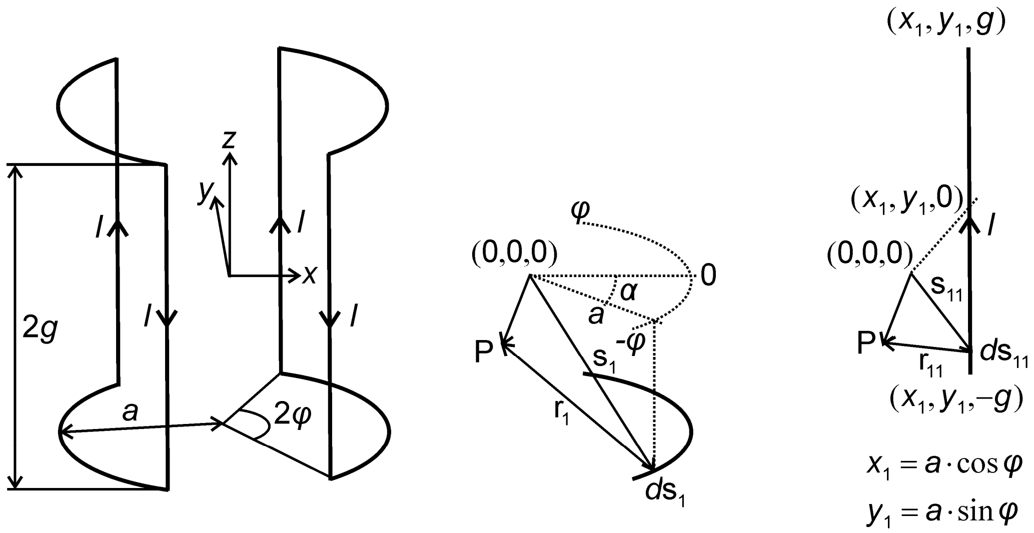

Fig. 1 One-turn saddle-shaped coil with details for the first arc and vertical.

Magnetic field of a saddle-shaped coil arcs:

$\mathbf{P} = \mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z$ is point of the observer in which the magnetic field is calculated.

$\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unity vectors in the directions of the x, y, z axis,

$\mathbf{s}_1 = \mathbf{i}a\cos\alpha + \mathbf{j}a\sin\alpha - \mathbf{k}g$,

$\mathbf{s}_2 = \mathbf{i}a\cos\beta + \mathbf{j}a\sin\beta - \mathbf{k}g$,

$\mathbf{s}_3 = \mathbf{i}a\cos\gamma + \mathbf{j}a\sin\gamma + \mathbf{k}g$ and

$\mathbf{s}_4 = \mathbf{i}a\cos\xi + \mathbf{j}a\sin\xi + \mathbf{k}g$ are points of the arcs.

$d\mathbf{s}_1 = (-\mathbf{i}a\sin\alpha + \mathbf{j}a\cos\alpha)d\alpha$,

$d\mathbf{s}_2 = (-\mathbf{i}a\sin\beta + \mathbf{j}a\cos\beta)d\beta$,

$d\mathbf{s}_3 = (-\mathbf{i}a\sin\gamma + \mathbf{j}a\cos\gamma)d\gamma$ and

$d\mathbf{s}_4 = (-\mathbf{i}a\sin\xi + \mathbf{j}a\cos\xi)d\xi$ are vector elements of the arcs.

$\mathbf{r}_1 = \mathbf{P} - \mathbf{s}_1$,

$\mathbf{r}_2 = \mathbf{P} - \mathbf{s}_2$,

$\mathbf{r}_3 = \mathbf{P} - \mathbf{s}_3$ and

$\mathbf{r}_4 = \mathbf{P} - \mathbf{s}_4$ are vectors between the observer and the arcs.

$\mathbf{B}_{arc} = \left(I \cdot 10^{-7}\right) \cdot \left(\int\limits_{-\varphi}^{\varphi} \frac{d\mathbf{s}_1 \times \mathbf{r}_1}{|\mathbf{r}_1|^3} + \int\limits_{\pi+\varphi}^{\pi-\varphi} \frac{d\mathbf{s}_2 \times \mathbf{r}_2}{|\mathbf{r}_2|^3} + \int\limits_{\varphi}^{-\varphi} \frac{d\mathbf{s}_3 \times \mathbf{r}_3}{|\mathbf{r}_3|^3} + \int\limits_{\pi-\varphi}^{\pi+\varphi} \frac{d\mathbf{s}_4 \times \mathbf{r}_4}{|\mathbf{r}_4|^3} \right)$ is the magnetic field from the arcs of the one-turn saddle-shaped coil.

Magnetic field of a saddle-shaped coil verticals:

$\mathbf{s}_{11} = \mathbf{i}a \cos \varphi + \mathbf{j}a \sin \varphi + \mathbf{k}z$,

$\mathbf{s}_{22} = \mathbf{i}a \cos(-\varphi) + \mathbf{j}a \sin(-\varphi) + \mathbf{k}z$,

$\mathbf{s}_{33} = \mathbf{i}a \cos(\pi - \varphi) + \mathbf{j}a \sin(\pi - \varphi) + \mathbf{k}z$ and

$\mathbf{s}_{44} = \mathbf{i}a \cos(\pi + \varphi) + \mathbf{j}a \sin(\pi + \varphi) + \mathbf{k}z$ are points of the verticals.

$d\mathbf{s}_{11} = d\mathbf{s}_{22} = d\mathbf{s}_{33} = d\mathbf{s}_{44} = \mathbf{k}dz$ are the vector elements of the verticals.

$\mathbf{r}_{11} = \mathbf{P} - \mathbf{s}_{11}$,

$\mathbf{r}_{22} = \mathbf{P} - \mathbf{s}_{22}$,

$\mathbf{r}_{33} = \mathbf{P} - \mathbf{s}_{33}$ and

$\mathbf{r}_{44} = \mathbf{P} - \mathbf{s}_{44}$ are vectors between the observer and the verticals.

$\mathbf{B}_{vert} = \left(I \cdot 10^{-7}\right) \cdot \left(\int\limits_{-g}^{g} \frac{d\mathbf{s}_{11} \times \mathbf{r}_{11}}{|\mathbf{r}_{11}|^3} + \int\limits_{g}^{-g} \frac{d\mathbf{s}_{22} \times \mathbf{r}_{22}}{|\mathbf{r}_{22}|^3} + \int\limits_{-g}^{g} \frac{d\mathbf{s}_{33} \times \mathbf{r}_{33}}{|\mathbf{r}_{33}|^3} + \int\limits_{g}^{-g} \frac{d\mathbf{s}_{44} \times \mathbf{r}_{44}}{|\mathbf{r}_{44}|^3} \right)$ is the magnetic field from the verticals of the saddle-shaped coil.

$\mathbf{B} = \mathbf{B}_{arc} + \mathbf{B}_{vert}$ is the resulting magnetic field of the saddle-shaped coil in the point **P**.

The NMR method used for verification is described in [1]. The magnetic field was measured on the four-turn saddle-shaped coil with the following parameters:

$\varphi_1 = 1.46\,\text{rad}$, $\varphi_2 = 1.25\,\text{rad}$, $\varphi_3 = 1.0\,\text{rad}$, $\varphi_4 = 0.66\,\text{rad}$, $a = 0.038\,\text{m}$, $g_1 = 0.05\,\text{m}$, $g_2 = 0.045\,\text{m}$, $g_3 = 0.04\,\text{m}$, $g_4 = 0.035\,\text{m}$, I=50 mA.

The magnetic field was calculated and measured in the three basic planes, Fig. 2 depicts the both calculated and measured values on centrelines in the planes *xy* and *yz*. It is evident very good agreement between the calculated and the measured values.

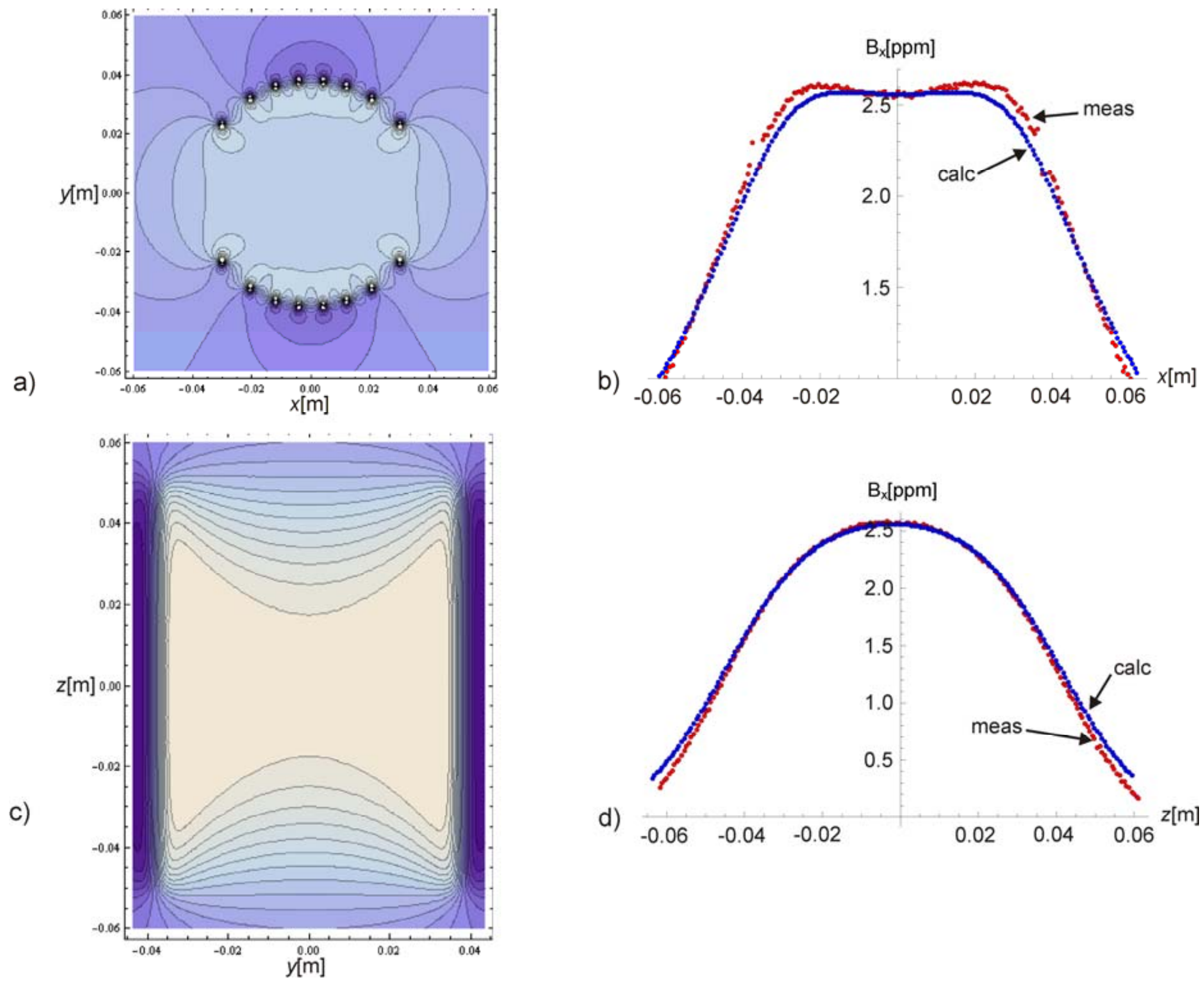

Fig. 2 Comparison between the calculated and the measured magnetic field of the four-turn saddle-shaped coil. The calculated magnetic field in the central plane *xy* a); the measured and calculated values at the centreline in the *xy* plane b); the calculated magnetic field in the central plane *yz* c); the measured and calculated values at the centreline in the *yz* plane d).

4. Discussion and Conclusions

Experiments confirmed correctness of the calculated values. A problem can be the numerical integration; it can spend much time and complicate a subsequent optimisation in such way. The presented equations can be simple used for the magnetic field of more-turn saddle-shaped coils calculation – the magnetic field of the next turns is simply added to the magnetic field of the first turn.

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References

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