Coverage Interval as a Measure of Uncertainty of Measurement

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Abstract. *The paper presents two methods for the evaluation of the expanded uncertainty of measurement result. The first one, in accordance with the classical method, consists in determining the uncertainty of type A and type B together with the coverage factor. The other method consists in calculating the coverage interval, which is a measure of uncertainty, and can be defined in two ways. The whole paper is completed with examples of the determination of measurement uncertainty by applying the latter method, for symmetric as well as asymmetric distribution.*

Keywords: Measurement Uncertainty, Coverage Interval, Monte Carlo Method

1. Introduction

As everybody knows, no measurement or test is perfect and the imperfections give rise to errors in the measurement result. Consequently, every measurement result is only an approximation of the value of measurand and is only complete when accompanied by a statement of the uncertainty of that approximation. It seems appropriate for the experimenter, who takes decisions about the choice of method for evaluating the expanded uncertainty, to be aware of the effects of choosing a given method with respect to its accuracy.

According to document [1], the uncertainty of measurement is the non-negative parameter characterizing the dispersion of the quantity values, being attributed to a measurand based on the information used. The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

It is obvious that expanded uncertainty U is a parameter allowing the determination of the limits of confidence interval comprising an unknown true value with a defined probability $1 - \alpha$. To determine the confidence interval for an arbitrary parameter of population, it is necessary to know the probability distribution of the estimator of that parameter. A detailed description of determining the value of uncertainty of type A and type B as well as the value of combined uncertainty u_c can be found, among others, in documents [2] and [3].

2. Evaluation of Uncertainty

Document [4] sets a new standard for calculating the measurement uncertainty. According to its interpretation, a measure of measurement uncertainty is the coverage interval, which can be defined in two ways. The first definition reads: "coverage interval for a quantity such that the probability that the quantity is less than the smallest value in the interval is equal to the probability that the quantity is greater than the largest value in the interval". The other definition states: "coverage interval for a quantity with the shortest length among all coverage intervals for that quantity having the same coverage probability". Determination of coverage interval, based on the first definition assumes that the probability of values on both sides of the interval is equal. If we assume that the coverage probability equals 0,95, then the interval limits are determined by the quantiles of distribution $G^{-1}(0{,}025)$ and $G^{-1}(0{,}975)$. Symbolically this interval can be denoted as:

$$I_{\mathrm{sym}} = [G^{-1}(0{,}025), G^{-1}(0{,}975)] \tag{1}$$

Determination of coverage interval based on the other definition demands the shortest interval from the set of all intervals for the same probability to be assumed. For the coverage probability 0.95 it will be an interval with the limit values for which the difference of upper value $G^{-1}(\alpha + 0.95)$ and lower value $G^{-1}(\alpha)$ will be the smallest. Symbolically this interval can be written as:

$$I_{\min} = [G^{-1}(\alpha), G^{-1}(\alpha + 0{,}95)], \;\; G^{-1}(\alpha + 0.95) - G^{-1}(\alpha) = \min \tag{2}$$

The numeric procedure recommended by document [4] for calculating the coverage interval for probability 0.95, based on the Monte Carlo method, expects the number of draws M = 10000. Each of 10000 determined values is an element of the set of possible values of the distribution of output value. After the elements of this set are ordered in the form of an increasing sequence of values and after a successive probability is assigned to each of them, starting from p = 0.0001 and ending with na p = 1, the boundaries of coverage interval, according to the first definition, are determined by the values represented by elements with number 250 and number 9750. If the other definition is applied, we will get 500 coverage intervals for probability 0.95. These will be successive intervals between pairs of elements of the set, numbered from 1 and 9501 up to 500 and 10000, respectively. The smallest element should be chosen among them.

3. Experiment Results

As far as the above considerations are concerned, a significant problem appears whether the lengths of both intervals: probabilistically symmetric and the shortest, are close to one another. This problem will be presented with an example of symmetric and asymmetric distribution. The representative of symmetric distribution will be the normal distribution, widely applied in metrological practice. As asymmetric distribution, on the other hand, the chi square - χ^2 distribution was applied. The normal distribution was analysed firstly. Using a generator of random numbers with standardised normal distribution N(0,1), a set of 10000 values was set up. Sorting the set of these values in increasing order, and assigning a successive probability to each of the values, starting from p = 0.0001 and ending with p = 1, we get the numerical distribution function for normal distribution, presented in Figure 1.

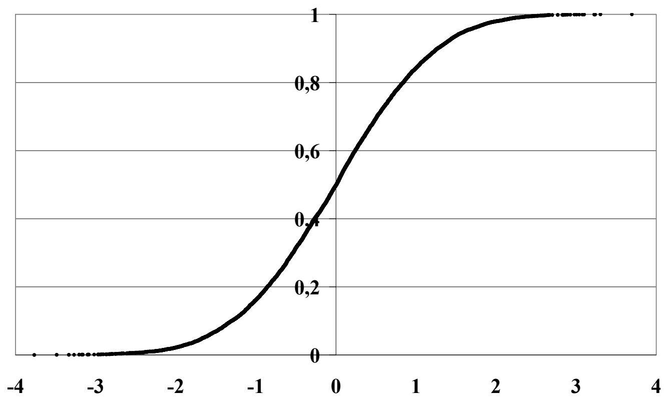

Fig. 1. Numerical distribution function for normal distribution

Based on the calculations carried out, exemplary values of the lengths of coverage intervals were obtained: 3.91 for the shortest interval and 3.93 for the symmetric interval, respectively, which are very close to each other. The obtained values are in accordance with the length of the confidence interval for normal distribution, which for confidence interval 95% is determined by the difference between two quantiles of distribution 1.96 and -1.96. A set of exemplary values of all 500 coverage intervals for this distribution is shown in Figure 2.

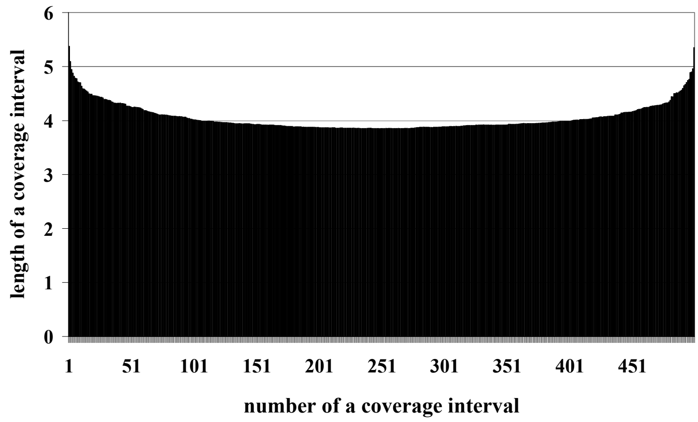

Fig. 2. Numerical coverage intervals for normal distribution

Asymmetric distribution χ^2, with one degree of freedom, can be obtained from normal distribution with the operation of squaring, which is presented in the equation below:

$$\chi_1^2 = [N(0,1)]^2 \tag{3}$$

Carrying out the calculations for the above equation 10000 times, we get a set of values with chi square distribution and one degree of freedom. Similarly as in the previous case, sorting the set of these values in increasing order and assigning to them successive probabilities, we get the numerical distribution function for chi square distribution, presented in Figure 3. Carrying out the calculations for the shortest and symmetric coverage interval, we get for example values: 3.839 and 5.062. Both determined intervals differ substantially, by over 30%, and the value of the shortest interval is close to the value of the quantile of chi square distribution for confidence interval 95 %, which is equal to 3.841. A set of values of all 500 coverage intervals was shown in Figure 4.

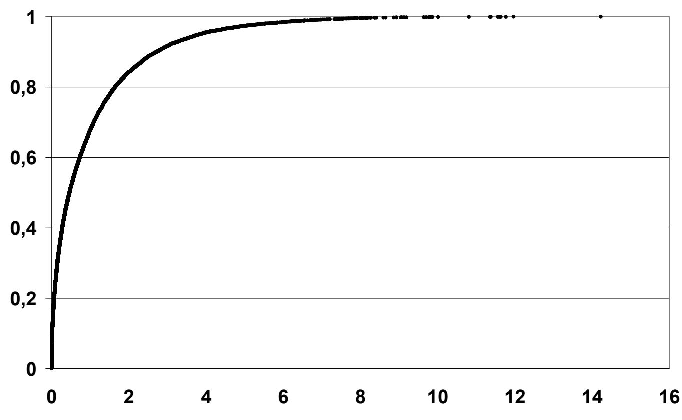

Fig. 3. Numerical distribution function for chi-square distribution with one degree of freedom

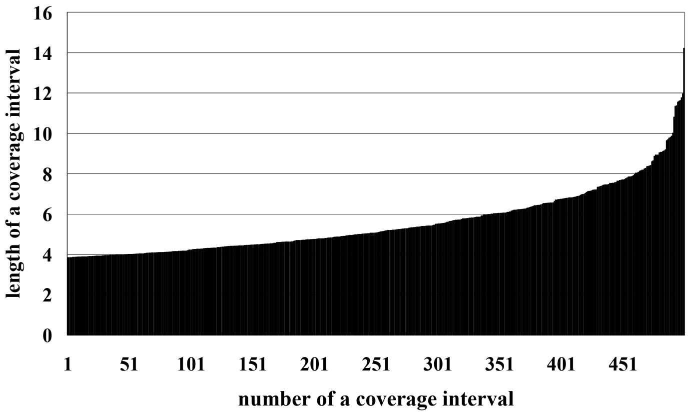

Fig. 4. Numerical coverage intervals for chi-square distribution with one degree of freedom

4. Conclusions

The estimate of a measurand is in practice usually an average value being an expected value of probability distribution. In case of standardised normal distribution, its value equals zero. Around the estimate a coverage interval is formed. For this type of distribution the expected value divides the interval in halves with numerically equal values of expanded uncertainty. The symmetric and shortest coverage intervals are comparable, although due to the computational accuracy of the Monte Carlo method, the shortest interval is usually slightly more narrow than the symmetric interval.

The expected value for χ^2 distribution with one degree of freedom is equal to one. However, this value does not determine the centre of coverage interval, both the shortest and the symmetric. Therefore, the expanded uncertainty cannot in this case be determined, only the boundaries of coverage interval can be determined. We should state, then, whether the values of the limits of this interval were determined for the shortest or for the symmetric one. Such situation can occur if the measurand is defined using a non-linear model.

Expanded uncertainty U, applied generally in metrology as a measure of inaccuracy of measurement result, determines the symmetric dispersion of the measurand value around its estimate. Problems appear when the asymmetric distribution of type chi square or gamma is used. The research results presented in the paper point to the significance of the raised issues, which, however, cannot be widely discussed within the limits of this publication.

References

[1] International vocabulatory of metrology – Basic and general concepts and associated terms (VIM), BIPM, JCGM 200:2008.

[2] Evaluation of measurement data – Guide to the expression of uncertainty in measurement, BIPM, JCGM 100:2008.

[3] Otomański P., Szlachta A. The evaluation of expanded uncertainty of measurement result in direct measurement using the LabVIEW environment, Measurement Science Review, Vol. 8, No 6, 2008, 147-150.

[4] Evaluation of measurement data – Supplement 1 to the “Guide to the expression of uncertainty in measurement” – Propagation of distributions using a Monte Carlo method, BIPM, JCGM 101:2008.