Performance of Likelihood Calibration Method by Gruet from the Posterior Point of View. Simulation Study

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Abstract. *Most calibration methods proposed in the literature are of likelihood type, i.e. they guarantee some statement about conditional probabilities p(Y|X) where X is quantity of interest and Y its surrogate. However, more natural would be to investigate posterior properties of calibration methods, i.e. statements about probabilities p(X|Y). Here, posterior behaviour of likelihood calibration method by Gruet will be examined by simulation from joint distributions of (X,Y) with different strength of dependence between X and Y.*

Keywords: Calibration, Likelihood vs. Posterior Methods

1. Introduction

The situation when direct measurement of some quantity X is difficult and its values are infered from measurements of another quantity Y is frequently encountered in many scientific branches. Such inference may make use of some physical law about relationship between X and Y, or of set of paired measurements of (X,Y), supposedly obtained at the same conditions as unpaired measurements of Y with unknown counterparts of X which we would like to estimate. This second approach is called calibration.

There were many approaches proposed for calibration considered as a solution to a probabilistic problem. Most of them could be named likelihood ones, since they guarantee the validity of some statement about probabilities p(Y|X) – in the calibration task unknown value(s) of X is parameter of interest. But considering the often random nature of both X and Y, it would be more useful to know the posterior properties of calibration method - something about probabilities p(X|Y) – given unpaired measurements of Y (and previously obtained set of paired measurements of (X,Y)), what can be said about unknown corresponding values of X?

2. Subject and Methods

Here we concentrate only on one calibration method, proposed in (Gruet, 1996), giving (under measured unpaired value(s) of Y) simultaneous interval calibration estimates B(Y) for unknown x. This belongs among likelihood methods, since it guarantees (under suitable conditions) that

$$\liminf_{n\to\infty} P(P(x \in B(Y) \mid \{(X_i, Y_i); i=1,...,n\}) \geq 1-\alpha ; \forall Y) \geq 1-\delta.$$

The posterior performance of this method was investigated by simulation in the following way: From the given joint distribution function H(x,y)=C(F(x),G(y)), where C(.,.) is a copula and F(.), G(.) are marginal distribution functions of X and Y, respectively, *n* pairs (X_1,Y_1), ..., (X_n,Y_n) were generated and (parameters of) simultaneous calibration band $\{(Y,B(Y))\}=\{(Y,x);L(Y) \leq x \leq U(Y)\}$ for given α and δ was computed according to (Gruet, 1996). Then approximation $\hat{\psi}$ of "posterior probability content" of the band, i.e. approximation of

$$\psi := P(\{y; |P(X<L(y)|Y=y)-P(X<U(y)|Y=y)| \geq 1-\alpha\})$$

was computed. This was repeated *N* times. The number

$$\Psi := \#\{j=1,...,N;\ \hat{\psi}_j = 1\}/N$$

was finally compared with coverage $1\text{-}\delta$.

Simulations were carried out for the following values of parameters:

$C(u,v)=C_\theta(u,v)=\Phi_\theta(\Phi^{-1}(u),\Phi^{-1}(v))$, i.e. bivariate normal copula described in (Klaassen & Wellner, 1997), where Φ is distribution function of N(0,1), Φ_θ is distribution function of bivariate normal distribution with zero expectations, unit variances and correlation θ; θ = 0.9 or 0.99; F, G are Unif(0,1) or N(0,1); n= 4, 10, 20, 40, 100, 200; α=0.05; δ=0.1; N=100.

Since approach of (Gruet, 1996) makes use of kernel regression, the choice of kernel and particularly of bandwidth can have an influence on calibration. Here we use Epanechnikov kernel and the bandwidth is computed separately for each simulated calibration set by method implemented in npcdensbw function of np package of R software.

3. Results

The results of simulations rounded to two decimal points are in the table.

n	θ	F	G	Ψ	F	G	Ψ	F	G	Ψ
4	0.9	U(0,1)	U(0,1)	0.94	N(0,1)	N(0,1)	0.97	U(0,1)	N(0,1)	0.65
10		U(0,1)	U(0,1)	0.94	N(0,1)	N(0,1)	0.83	U(0,1)	N(0,1)	0.98
20		U(0,1)	U(0,1)	0.94	N(0,1)	N(0,1)	0.81	U(0,1)	N(0,1)	0.91
40		U(0,1)	U(0,1)	0.8	N(0,1)	N(0,1)	0.74	U(0,1)	N(0,1)	0.56
100		U(0,1)	U(0,1)	0.6	N(0,1)	N(0,1)	0.08	U(0,1)	N(0,1)	0.29
200		U(0,1)	U(0,1)	0.5	N(0,1)	N(0,1)	0	U(0,1)	N(0,1)	0.2

n	F	G	Ψ	θ	F	G	Ψ	F	G	Ψ
4	N(0,1)	U(0,1)	0.91	0.99	U(0,1)	U(0,1)	0.98	N(0,1)	N(0,1)	0.95
10	N(0,1)	U(0,1)	0.78		U(0,1)	U(0,1)	0.98	N(0,1)	N(0,1)	0.92
20	N(0,1)	U(0,1)	0.85		U(0,1)	U(0,1)	0.98	N(0,1)	N(0,1)	0.9
40	N(0,1)	U(0,1)	0.68		U(0,1)	U(0,1)	0.98	N(0,1)	N(0,1)	0.71
100	N(0,1)	U(0,1)	0.68		U(0,1)	U(0,1)	0.98	N(0,1)	N(0,1)	0.37
200	N(0,1)	U(0,1)	0.38		U(0,1)	U(0,1)	0.88	N(0,1)	N(0,1)	0.07

n	F	G	Ψ	F	G	Ψ
4	U(0,1)	N(0,1)	0.89	N(0,1)	U(0,1)	0.86
10	U(0,1)	N(0,1)	0.99	N(0,1)	U(0,1)	0.91
20	U(0,1)	N(0,1)	0.98	N(0,1)	U(0,1)	0.98
40	U(0,1)	N(0,1)	0.86	N(0,1)	U(0,1)	0.89
100	U(0,1)	N(0,1)	0.88	N(0,1)	U(0,1)	0.73
200	U(0,1)	N(0,1)	0.8	N(0,1)	U(0,1)	0.63

4. Discussion and Conclusions

As can be seen from the table, simulation results are coarse and inconclusive. This is in part due to small value of N. It seems that the results are better when dependence between X and Y is stronger, as it could be expected, and also that different margins can affect the calibration results differently, but it is not convincing.

It transpires that the bandwidth is very influential parameter of the method. The bandwidth selection method used here leads to inappropriate results – calibration bands are often very wide and their boundaries are rather insensitive to value of Y. Therefore, it can be suspected that corresponding computed values of Ψ given in the table are unrealistically high.

Suitable and sufficiently universal method of bandwidth selection for calibration method by Gruet has to be found yet. It seems, however, that there are cases when the value of Ψ could be considerably changed by change of bandwidth, but large portion of the calibration band would remain constantly unadvantegeously wide regardless of bandwidth.

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