# Modeling and Measuring of the Changes of the Magnetic Field Next to Non-Ferromagnetic Substances for Magnetic Susceptibility Calculation

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Abstract. This article is focused on the principles of the magnetic susceptibility measurement of the non-ferromagnetic substances using NMR tomography. Magnetic susceptibility is calculated from changes of the magnetic field close to the cylinder shaped specimen. COMSOL program was used for modeling experiment results. Consequently, model results were compared to MR tomograph measured data. Changes of the magnetic field in space of xy cut of the specimen were integrated in both cases. Before integrating of the magnetic field changes was necessary to filter, unwrap and detect accurate position of the specimen. Magnetic susceptibility calculated from measured data is in comparison to theoretical value, as well as model value, slightly different. Difference of those values is lower than 5%. Due to the used model and measurement were extended principles of the NMR method for magnetic susceptibility measurement.

Keywords: COMSOL, Non-ferromagnetic, NMR, Magnetic Susceptibility

# 1. Introduction

Magnetic susceptibility is the physical quantity describing material properties in the external magnetic field. Magnetic susceptibility is defined as ratio between magnetization  $\mathbf{M}$  of the material in the magnetic field and the field intensity  $\mathbf{H}$ :

$$\mathbf{M} = \boldsymbol{\chi}_{\mathrm{m}} \, \mathbf{H} \, . \tag{1}$$

All materials can be classified by value of magnetic susceptibility into three groups. This article is focused on the non-ferromagnetic (paramagnetic and diamagnetic) materials. These materials have defined magnetic susceptibility in range  $-1 < \chi_m << 1$ .

Non-ferromagnetic substation placed in homogeneous magnetic field  $B_0$  react with this field and create the local changes of the magnetic field close to the specimen. Changes of the magnetic field depend on the magnetic susceptibility of used material. NMR method of measurement magnetic susceptibility is based right on measurement of these changes [1] and [2]. Next paragraphs show calculating of the magnetic susceptibility from distribution of the magnetic field around the specimen in COMSON environment. Consequently, value of the magnetic susceptibility is calculated from the measured data and compared with theoretical and from the model calculated value.

# 2. Subject and Methods

Model and real specimen used the same space configuration of the sample as is shown in Fig.1. Cubic space (40x40x40 mm) filled by a deionised water has magnetic susceptibility  $\chi_{m1} = -13e \cdot 10^{-6}$ , labeled by number (1) in Fig. 1. In the center of this space is placed the sample of mentioned materials with different magnetic susceptibility  $\chi_m$ . The samples are

cylindrical 10 mm in length and 4 mm in diameter, labeled by number (2) in Fig. 1. Sample materials was used aluminum ( $\chi_{mAl} = 2.2 \cdot 10^{-5}$ ) and copper ( $\chi_{mCu} = -9.2 \cdot 10^{-6}$ ). No. (3) is the cutting plane (*xy* plane) placed in the center of the cylinder. In this article is always used this plane.



Fig. 1. Left - Configuration of the modeled system, cube filled by an air with sample. Right – copper and aluminum cylinders.

Modeling of the changes of the magnetic field close to the specimen in COMSOL environment COMSOL environment was used for calculating of the magnetic field distribution. Created configuration (in Fig. 1) use mesh of the final elements. Equations for the electromagnetic field are used for calculation of the magnetic field distribution. In the same way was calculated reaction magnetic field  $\Delta B$ , as

$$\Delta B = B - B_0. \tag{2}$$

In article [3] is used equation for calculation of the magnetic field form the 1D cut. We extend this equation for 2D space use, xy plane in the center of the cylinder (No. (3) in Fig. 1). Integral of the reaction field  $\Delta B_w$  round the cylinder divided by space of the cylinder cut  $S_{\text{cylinder}}$  give as differential magnetic susceptibility

$$\Delta \chi = -\frac{\iint\limits_{S} \Delta B_{\rm w} \cdot dx dy}{S_{\rm cylinder} \cdot B_{\rm 0}}.$$
(3)

### Calculation of the magnetic susceptibility from data measured by MR tomograph

As a measurement method we chose Gradient echo (GE) method. GE is very sensitive to inhomogenities of the static magnetic field, what can be useful for susceptibility measurement. By this method measured image is phase coded image  $\Delta \varphi$  used for reaction field  $\Delta B$  calculation by the following equation:

$$\Delta B = \frac{\Delta \varphi}{\gamma \cdot T_{\rm E}},\tag{4}$$

where  $T_{\rm E}$  is time to echo. More about GE you can see in [4], [5].

In Fig. 2 is shown how are the MR data processed. First of all are the data convert by the FFT from *k*-space to the 2D spatial function of intensities. Next step is image filtering, correcting of the image and phase unwrapping. As an input for Eq. 4 and for reaction field, calculation is used this unwrapped image. Only one image from whole set is used for magnetic susceptibility calculation as was mentioned above. Cylinder position in the image is marked by detection algorithm and whole space of the cylinder is set to zero. From resulting image we can easily calculate the  $\Delta B_w$  integral and establish this value to Eq. 3 for magnetic susceptibility calculation.



Fig. 2. Block diagram of the process of magnetic susceptibility calculation from MR data.

## 3. Results

Reaction field distribution  $\Delta B_w$  in the measured MR image is shown in Fig. 3. Measured data are processed by algorithm from Fig. 2. Result of this algorithm is matrix 80 x 75 px. Magnetic field changes are shown in Fig. 3, on the left 2D, on the right the 3D image of distribution of the reaction field.



Fig. 3. Reaction field  $\Delta B_{w}$  in xy plane in the center of the aluminum cylinder. On the left 2D, on the right the 3D image of distribution of the measured reaction field.

Summary of the results are presented in Table 1. Middle column are values obtained from COMSOL model, right column are values calculated from MR measurement. Measurement was provided on a MR tomograph at the Institute of Scientific Instruments, Academy of Sciences of the Czech Republic (ISI ASCR). The MR tomograph dispose of static field flux density  $B_0 = 4,7$  T, nuclei resonance frequency is 200 MHz.

Material	$\chi_{\text{model}} [x 10^{-6}]$	χ <sub>NMR</sub> [x10 <sup>-6</sup> ]
Aluminum	21,484	21,713
Copper	-8,940	-9,401

 Table 1.
 Tables Values of the magnetic suseptibility for alumnum and copper cylinders calculated from model and MR data.

## 4. Discussion and Conclusions

The results are compared with sample properties publish in literature. The causal agent is the measurement of magnetic susceptibility in static magnetic field  $B_0 = 4.7$  T and the material saturation. Magnetic susceptibility is published in literature as  $\chi_{AI} = 22 \cdot 10^{-6}$  for aluminum and as  $\chi_{Cu} = -9 \cdot 10^{-6}$  for copper. Due to calculation of the magnetic susceptibility from COMSOL model we verified that the Eq. 3 give us right results. How you can see in Table 1 the difference between the value calculated form model and the value calculated from measured data is very small. This difference is caused by size of the mesh of the model. Deviation of the obtained magnetic susceptibilities did not exceed 4,5%. Reached accuracy of this method makes it very suitable for magnetic susceptibility measurement of the non-ferromagnetic materials. Future research of this method holds possible ways for improvement of these method possibilities.

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