The Iterative Method of Parameters and Centre of Radial Distortion Estimation

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Abstract. *Correction of radial distortion of the image is a very important process during the calibration of non-metric camera intrinsic parameters. The paper treats the topic of estimation of coefficients and the centre of radial distortion of the image. A new method of coefficients and coordinates of the centre of radial distortion estimation in the image is presented. It is based on the iterative correction of coordinates of selected points on the distorted straight lines. It does not require a parametric expression of distorted lines. It is presented verification of method properties on simulated and real images.*

Keywords: Radial Distortion, Image Correction, Straight Line

1. Introduction

During measurement of spatial coordinates of scene interest points by the use of non-metric cameras it is necessary to establish camera model parameters. Model of the camera describes relation of spatial coordinates of scanned point to the coordinates of its image in the camera image plane. Model is established by intrinsic and extrinsic camera parameters. Extrinsic camera parameters express the relationship between coordinates of the point in the space in world coordinate system and coordinates of the same point in camera coordinate system. Relationship between camera and world coordinates of the spatial point is defined by the translation vector and rotation matrix expressing the position and rotation of camera to the world coordinate system. Intrinsic camera parameters are parameters of the model describing relationship between position of the point in the space with coordinates expressed in the coordinate system of the camera and position of the image of the same point expressed in the image coordinate system [1]. Model describes the mechanism of distortion of image point position caused by the imperfection of camera lens as well. The now widely recognized significant source of distortion caused by the camera optics is radial distortion of the image [2] described by model [3]

$$r_d = r(1 + \alpha_2 r^2 + \alpha_4 r^4 + \cdots) \tag{1}$$

where

r is distance of the point in ideal undistorted image to the centre of distortion

r_d is the distance of the point in the radial distorted image to the centre of distortion

α_i are coefficients of radial distortion of the image

There are mainly used models with the second, maximal the fourth order of polynomial in published papers [3]. Existing methods of radial distortion correction can be divided into two groups of approaches to the solution. The first approach (multiple view method) uses the correspondence of images of the scene point in multiple images using constrains among images to correct distortion of their position in images. The second approach characterizes methods using only one image [4]. Methods using one image are usually based on the fact that at the perspective projection (assumption in many models) the ideal image of the straight line in space is straight line. The strategy of methods based on this assumption is based on the detection of potential images of the straight line whose models are quadratic curves well approximating images of lines deformed according model (1). The individual methods differentiate by the ways of approximation of the line images, number of parameters and modification of the model (1) and whether they would calculate centre of radial distortion or whether they would identify centre of radial distortion to the principal point of the image [5]. Method presented in this paper considers the possibility that the centre of distortion and the principal point are not the same. This fact is emphasized in many papers [4], [6]. The submitted method does not require to parameterize distorted image of the straight line.

2. Subject and Methods

Let's suppose to have K sets of images p_{ki} of selected points with coordinates (x_{ki},y_{ki}) that are in the distorted image. Images of p_{ki} should lie on the straight line L_k in ideal undistorted image. Model (1) is replaced by models considering unequal distortion along the coordinate axes of the image that can be for each point p_{ki} expressed by

$$r_{xki} = \frac{r_{dki}}{(1+\beta_{x2} r_{dki}^2 + \beta_{x4} r_{dki}^4)} \quad r_{yki} = \frac{r_{dki}}{(1+\beta_{y2} r_{dki}^2 + \beta_{y4} r_{dki}^4)} \tag{2}$$

where

r_{xki}, r_{yki} are relative coordinates of point p_{ki} to the centre $C=(c_{dx}, c_{dy})$ of radial distortion in undistorted image

r_{dki} is the distance of the image point to the centre of radial distortion in distorted image

β_{x2}, β_{x4}, β_{y2}, β_{y4} are coefficients of the inverse radial distortion along axes x, y

Coordinates (x_{cki},y_{cki}) of points p_{cki} in undistorted image corresponding to the points p_{ki} can be expressed by

$$x_{cki} = r_{xki} + c_{dx} \quad y_{cki} = r_{yki} + c_{dy} \tag{3}$$

As the equation of the straight line L_k isn't known we use straight line $l_k(x,y)$ that approximates all points p_{cki}

$$l_k(x,y) = a_k x + b_k y + c_k = 0 \quad \sqrt{a_k^2 + b_k^2} = 1 \tag{4}$$

Square of perpendicular distance of point p_{cki} to the straight line (4) will be

$$d_{cki}^2 = l_k^2(x_{cki}, y_{cki}) = (a_k x_{cki} + b_k y_{cki} + c_k)^2 \tag{5}$$

In case of ideal undistorted image sum of distances (5) of all points p_{cki} equals to zero. All corrected points lie on the straight lines (3). We don't know neither corrected coordinates (3) at the start of solution or equations of approximating lines (4). Therefore correction of radial distortion of the image is based on the iterative improvement of values of unknown parameters such as the centre of radial distortion $C=(c_{dx}, c_{dy})$ and parameters β_{x2}, β_{x4}, β_{y2}, β_{y4} are calculated. As the objective is to minimize cost function of 6 parameters $\omega=[c_{dx}, c_{dy}, \beta_{x2}, \beta_{x4}, \beta_{y2}, \beta_{y4}]^T$ defined by

$$J(\omega) = \min_\omega \sum_{k=1}^{K} \sum_{i=1}^{nk} \left(l_k(x_{kic}, y_{kic})\right)^2 \tag{6}$$

where n_k is number of points corresponding to the k^{-th} line. In j^{-th} step of iteration are calculated partially corrected values of points p_{ckij} coordinates for parameter ω_j. From their parameters of $l_{kj}(x,y)$ are calculated by the use of farthest points of subset of points corresponding to L_k. Then the value of cost function (6) can be calculated. Values of $\omega_{j+1}$for the next step of iteration will be established from the values of parameters near the previous value. Such a value of parameter will be chosen for that value of cost function (6) is less than the value of this function for ω_j.

3. Results

In order to verify the accuracy of the proposed methods simulation experiments and real images of chessboard captured by CCD camera have been used. In both experiments ideal images of points arranged in a grid were examined. Images of grid points in ideal case lie on related vertical and horizontal lines. The ability to correct distorted coordinates of points according to model (2) was in simulations investigated in such a way that corrected coordinates were close to the coordinates of points of ideal undistorted image. Criteria published in [3] have been used to quantify successfulness of correction. Criteria such ARE (Average Residual Error) and MRE (Maximum Residual Error) for deviation of corrected distorted (x_{ci}, y_{ci}) to ideal image points (x_{Ii}, y_{Ii}) and quantities corresponding to the criterion (6) of optimisation such as the mean AREL and maximum MREL deviation from the approximating line were.

$$ARE = \frac{1}{N}\sum_{i=1}^{N}\sqrt{(x_{ci}-x_{Ii})^2+(y_{ci}-y_{Ii})^2} \tag{7}$$

$$MRE = \max_i \sqrt{(x_{ci}-x_{Ii})^2+(y_{ci}-y_{Ii})^2} \tag{8}$$

Criteria AREL and MREL are defined by

$$AREL = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{nk}\sum_{i=1}^{nk}\sqrt{\left(l_k(x_{kic},y_{kic})\right)^2} \tag{9}$$

$$MREL = \max_{i,k}\sqrt{\left(l_k(x_{kic},y_{kic})\right)^2} \tag{10}$$

Table 1. Values of criterion of optimalisation and errors for three simulation experiments and correction of the image of planar calibration object in the form of chessboard captured by CCD camera.

Sample	J(ω)	MRE	ARE	MREL	AREL
Sim 1.	0.0044	0.0918	0.0083	0.0204	0.0013
Sim 2.	186.35	10.0469	0.6216	3.1859	0.2786
Sim 3.	0.0873	1.0167	0.0717	0.1060	0.0060
Real	11.7522	-	-	0.7461	0.0429

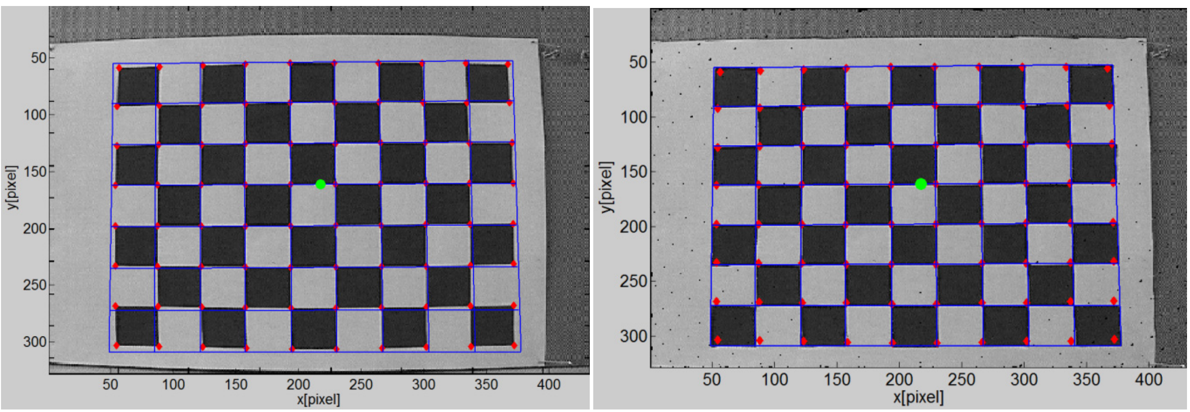

Fig. 1. Distorted and corrected image of the chessboard planar calibration object.

In the table 1 values of mentioned quantities for grid 7x7 points for 3 chosen simulation experiments and correction of real image are presented. In the first experiment image was distorted symmetrically in both directions of the image, in the second and the third the distortion was different in horizontal and vertical direction with parameters in the table. The results of the second experiment are quantified by the described errors in case when symmetrical model of distortion for the correction was used and in the third case when the model thinking different distortion in horizontal and vertical direction was used. Images captured with CCD camera Mitsubishi CCD-400E (fig.1) have been used in experiments with real images. In ideal case there are corner points of chessboard as elements of vertical and horizontal straight lines on the chessboard. Corner points were extracted by Harris corner detector that can calculate coordinates of corner points with sub-pixel resolution. Module from the program [5] has been used to extract corner points. Positions of extracted points are in both images distinguished by the symbol “diamond”. Position of the radial distortion can be seen by the symbol “circle”. Numerical values of errors are in table 1.

4. Discussion

Experiments, some of which were presented in this paper, showed good properties of proposed method of correction of errors due to the image radial distortion. Optimization of non-linear criterion functions brings the risk of instability, or non-correct convergence of solution. In this case, stability and convergence of the solution is provided by the use of points standardized distances to the centre of distortion by the constant greater than the maximum distance of the corrected point to the principal point and choosing the initial value of the centre of radial distortion near the principal point. Variability of choice of initial value of the centre of distortion and of standardization constant didn’t influence the convergence of solution. Initial values of radial distortion coefficients were always set to zero. Criteria AREL and MREL appear to be suitable for assessing the quality of correction in case for that coordinates of points in ideal undistorted image are unknown.

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