Use of the Integral Transforms for Estimation of Instantaneous Frequency

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1. Purpose/Introduction

Estimation of the instantaneous signal frequency is at present an important task in the measurement [1]. Choice of the suitable method depends on the type of the measured signal and on the computing capacity of the equipment used.

Power-line frequency can serve as an important example. Its deviations from nominal value indicate the imbalance between the generation and the consumption of energy in the net. It can even indicate serious malfunction in a part of the power distribution net. Measurement of instantaneous power-line frequency f_i is important also in instrumentation. Changing the length of the first integration tact according to instantaneous power-line period time allows to achieve very high value of the SMRR (series-mode interference rejection ratio) of integrating digital voltmeters. High value of SMRR at the frequency of the power-line voltage and its multiples is one of the conditions for the highest accuracy measurement of the DC voltage with additive AC disturbance. In both the above-mentioned applications, measurement of the instantaneous frequency should be done in short time (preferably as real time measurement). It is therefore very important to implement suitable methods at some of the digital signal processors.

Scope of this contribution is oriented mainly to measurement of the power-line instantaneous frequency. Therefore our experiments use signals with frequencies around 50 Hz.

2. Subject & Methods

Definition of the frequency as the number repetitions of a periodic phenomenon in one second cannot be used directly for non-stationary signals [2]. The instantaneous frequency of a non-stationary real signal $x(t)$ is defined as

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\{\arg z(t)\} \tag{1}$$

where $\mathbf{z(t)} = x(t) + jH[x(t)] = a(t)\, e^{j\omega(t)}$ is the so-called *analytic signal* and H is the symbol used for the Hilbert Transform (see e.g. [2]).

Powerful tools for measurements of frequency changes with time are the so-called time-frequency representations of signal (TFRs) [3]. TFRs provide the observer with the possibility to detect simultaneously changes of the signal both in time domain and in frequency domain. Best known of them is the *Short-Time Fourier transform* (STFT), the basic generalisation of the Discrete Fourier Transform (DFT). It is the basic *linear TFR* and its definition gives Eq.2, where *x(t')* is real signal:

$$STFT_x(t, f) = \int_{t'} x(t')\gamma^*(t'-t)e^{-j2\pi f t'}dt' \tag{2}$$

Its error caused by leakage can be reduced by using interpolation techniques, like the one referred to in in [4]. Interpolated STFT will be labelled ISTFT in the following text.

The *Wigner-Ville distribution* (WVD) and its refinements (e.g. PWVD, the pseudo VWD and the smoothed pseudo-WVD) are examples of *quadratic TFRs* [3], [5], [8]. Definitions of the WVD, PWVD and SPWVD present Eqs.3-5, where *x(t)* is analytic (complex) signal:

$$WV_x(t, f) = \int_{\tau} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\tau \tag{3}$$

$$PWV_x(t,f) = \int_\tau h(\tau)\, x\left(t+\frac{\tau}{2}\right) x^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi f\tau}\, d\tau \qquad (4)$$

$$SPWV_x(t,f) = \int_\tau h(\tau) \int_s g(s-t)\, x\left(s+\frac{\tau}{2}\right) x^*\left(s-\frac{\tau}{2}\right) ds\; e^{-j2\pi f\tau}\, d\tau \qquad (5)$$

Application of window(s) in PWVD and SPWVD can be seen from Eqs.4 and 5.

We have implemented both the STFT and the PWVD at the 16-bit fixed-point digital signal processor ADSP-2181. The two methods were compared also by signals generated by computer and signals get by sampling an analogue signal produced by a modern function generator (HP 33120A) and processed in MATLAB. A comparison of other usable methods was presented by authors in [6].

Using the STFT for f_i estimation means finding frequency bin with maximum value of the STFT of the instantaneous amplitude signal spectrum. The short FFT is applied on the windowed signal and some interpolation technique could be used to diminish the leakage effect caused by non-synchronisation between signal and sampling. Finding the f_i using the VWD includes the Hilbert transform of the signal, computation of the VWD kernel, windowing, FFT computation and finding the frequency bin with WVD maximum. Other transforms (e.g. fast Hartly transform) can be used instead of the FFT. Since the DFT spectrum is periodic with frequency f_S corresponding to the normalized frequency $\theta_s = \omega T = 2\pi$, and the PWVD is periodic concerning the frequency variable with $\theta = \pi$, sampling frequency must be twice the signal Nyquist rate to prevent aliasing if the WVD is computed using FFT of a real sequence. This oversampling is not necessary, if analytic signal is used for the PWVD computation. [7].

2. Results

We have worked with sequences of signal samples gained both by computer simulations and by measurement of physical signals. The signal processing was performed in MATLAB and Signal Processing Toolbox and Time-Frequency Toolbox [5] were used. A part of our experimental results is shown in following figures. PWVD and SPWVD results are not presented in Figs.1 and 2. They are similar to the WVD results. The measured signal was sinusoid with four harmonic components with random phase and amplitudes several percents of the fundamental.

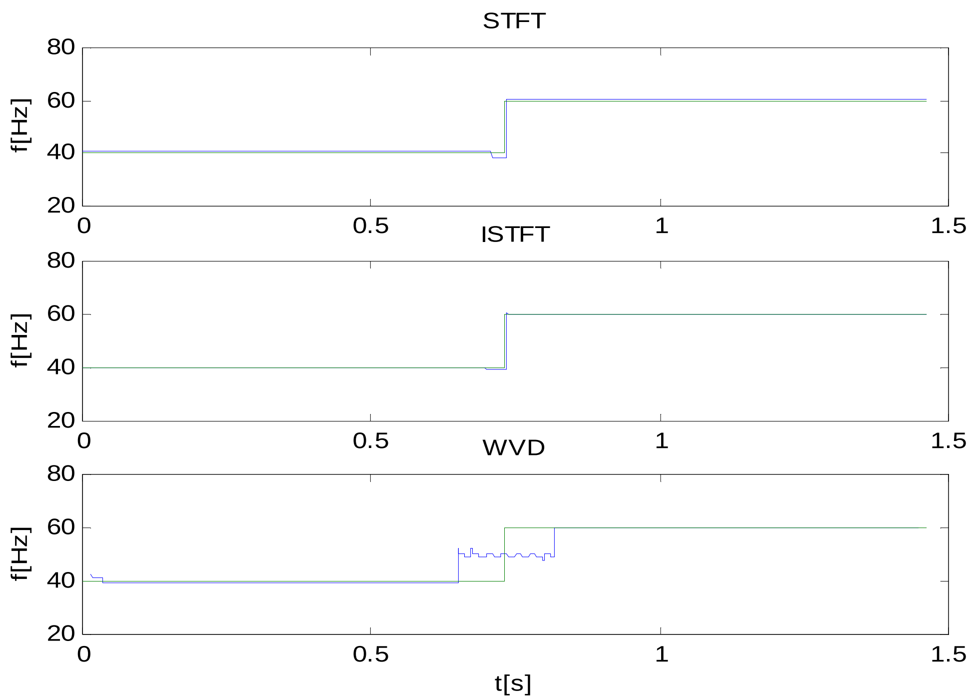

Fig.1 An abrupt change in signal frequency from 40 Hz to 60 Hz and corresponding outputs of instantaneous frequency gained by STFT, interpolated STFT and by WVD

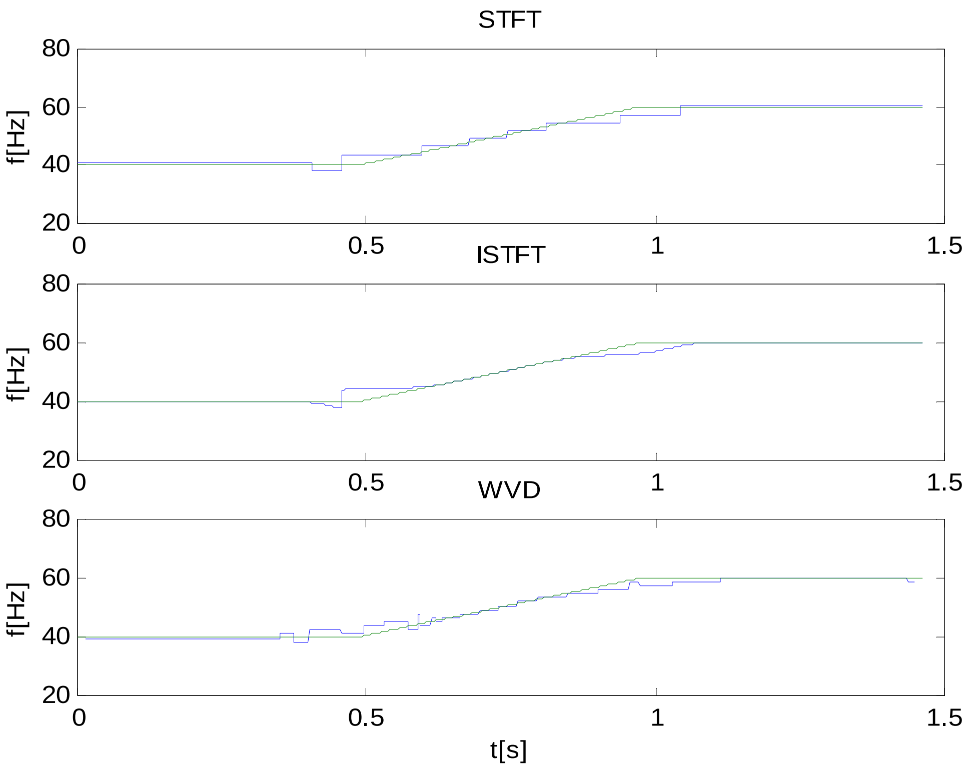

Fig.2 Linear change in signal frequency from 40 Hz to 60 Hz and corresponding outputs of instantaneous frequency gained by STFT, interpolated STFT and by WVD.

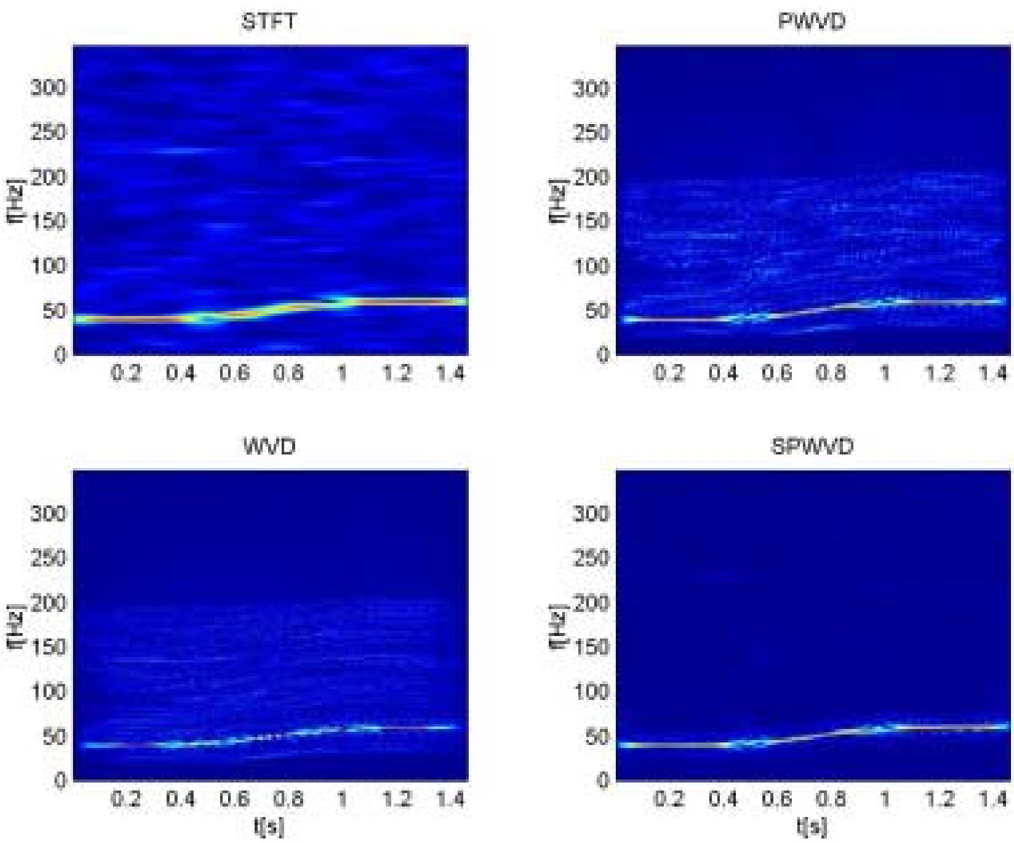

Fig.3 Time-frequency signal representations gained for the sinusoidal signal with frequency change according to Fig.2. Signal is mixed with additive white Gaussian noise, SNR=0 dB.

4. Discussion/Conclusion

Whereas using STFT requires short window for high resolution in time domain but long window for high resolution in frequency domain, the SPVWD allows independent resolution control in time and frequency, since windows with different length can be used in either domain. In both cases the instantaneous frequency is found as frequency corresponding to the frequency bin with the highest value (of spectrum or energy). Higher resolution can be achieved using interpolation in the frequency domain, e.g. that proposed in [4].

As can be seen from the figures above, the best of the investigated methods seems to be the interpolated STFT. It is interesting that the interpolation designed for linear frequency sweep improves measurements also for certain non-linear sweeps [4]. Using ISTFT leads to very low errors also for signals with constant frequency which can be considered as swept frequency signals with the sweep rate 0 Hz/s. This can be seen from Figs.1 and 2, ISTFT. The ISTFT proved itself to be the best method also for signals with abrupt changes in frequency (see Fig. 1). Possible improvement of the WVD and its refinements could be reached also by suitable interpolation. We would like to try it in future. Errors of all methods depend also on the window lengths used.

Our experiments have shown as well, that the influence of higher-order harmonic components and of noise is negligible, even for SNR=0 dB. The additive noise influences the TFR of signal (see Fig.3), but does not influence significantly position of frequency bins determining the fundamental harmonic component of the signal. Resolution in frequency domain in influenced both by the FFT length and the sampling frequency. If no antialising filter is used in the system, the sampling frequency must be chosen according to the highest non-negligeable harmonic components of the signal.

4. Acknowledgement

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