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MEASUREMENT AND IMAGING OF LONG SAMPLES USING NUCLEAR MAGNETIC RESONANCE

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Abstract

NMR - measurement and imaging of a spin density distribution of long samples needs a special arrangement of the radio-frequency and gradient coils placed on a long cylinder. The paper deals with a method of calculation and optimisation of four-wire radio-frequency coil geometry where the homogeneity in the central region as a transverse cross-section of a measured cylinder is evaluated. The 3D plot and contour plot of the real part of generated field is presented.

1. Introduction

Nuclear Magnetic Resonance (NMR) measurement and imaging of proton density of biological structures needs to create basic physical conditions, -stationary, gradient and radio frequency (RF) magnetic fields. A biological object placed into the centre of a measurement region is absorbing the energy during the RF excitation pulse generated by a coil supplied by an external source - RF transmitter. After absorption is finished, the object is radiating the energy back to the space. This energy is detected by a RF receiving coil, signal is amplified, demodulated and after A/D conversion submits data. Data accumulated in a buffer of a computer are processed by Fast Fourier Transform and organized in a matrix. The matrix is representing a spatial distribution of the measured parameters (proton density, relaxation times etc.), [4, 5].

A special case occurs when one needs to measure and image of long samples, in most cases in a shape of very long cylinder, where the length l = (10 to 100) r, where r is radius of the cylinder. In this case some problems occurs, especially:

a) generating of a homogeneous RF field in all the sample, [1, 2, 3, 7],

b) detection of the radiated RF signal with high sensitivity supposing to minimise diameter of the sample,

c) using the magnetic gradient in all the length of the object with maximal linearity, [6].

The conditions a) and b) are solved in the present paper using a mathematical model of the RF field optimised by computer methods.

2. Magnetic field of four parallel conductors.

In the next analysis we suppose two conditions:

a) In a rectangular coordinate system (x,y) we suppose that conductors are perpendicular to the (x,y) plane.

b) The lengths of the conductors are supposed to be infinite in comparison with the investigated region neglecting the diameter of conductors and the influence of supply - wires.

According to Fig.1 we suppose four conductors placed on the surface of a cylinder drawn as four points on the circle with the left - right symmetry. Conductors are fed by currents +I, -I. The stationary magnetic field $\mathbf{B}_0$ orientation is parallel to the conductors.

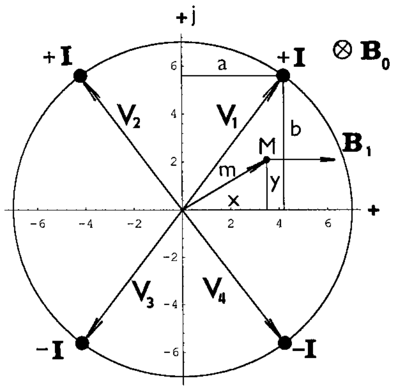

Fig.1 Four conductors placed on a cylinder supplied by currents +I and -I are generating the RF field in the central region. The field $\mathbf{B}_1$ in the point M is calculated.

Let us investigate the magnetic field strength $\mathbf{H}(\mathbf{m})$ in a point M represented by a position vector $\mathbf{m} = x + \mathbf{j}y$. The position of wires is determined by vectors $\mathbf{v}_i = a_i + \mathbf{j}b_i$. Then the general expression for the magnetic field in the point M supposing $\mathbf{m} \neq \mathbf{v}_i$ according to [4] is given by the equation

$$\mathbf{H}(\mathbf{m}) = \mathbf{j}\frac{I}{2\pi}\sum_{i=1}^{n}\frac{1}{\mathbf{m}^* - \mathbf{v}_i^*} \qquad (1)$$

where $\mathbf{m}^* = x - \mathbf{j}y$, $\mathbf{v}_i^* = a_i - \mathbf{j}b_i$.
Supposing the conductor directions of the currents +I and -I according to Fig.1 respecting the symmetry and position vector configuration:
$\mathbf{v}_1^* = \mathbf{v}^*$, $\mathbf{v}_2^* = -\mathbf{v}$, $\mathbf{v}_3^* = -\mathbf{v}^*$, $\mathbf{v}_4^* = \mathbf{v}$,
we can write for the magnetic field strength:

$$\mathbf{H}(\mathbf{m}) = \\ = j\frac{I}{2\pi}\left[\frac{1}{\mathbf{m}^* - \mathbf{v}^*} + \frac{1}{\mathbf{m}^* + \mathbf{v}} - \frac{1}{\mathbf{m}^* + \mathbf{v}^*} - \frac{1}{\mathbf{m}^* - \mathbf{v}}\right] \qquad (2)$$

After algebraic transformation we get the general equation for magnetic field strength in the position M in the form:

$$\mathbf{H}(\mathbf{m}) = j\frac{I}{\pi}\left[\frac{v^*}{\mathbf{m}^{*2} - \mathbf{v}^{*2}} - \frac{v}{\mathbf{m}^{*2} - \mathbf{v}^2}\right] \qquad (3)$$

By substitution: $\mathbf{m}^* = x - \mathbf{j}y$, $\mathbf{v}^* = a - \mathbf{j}b$ and $\mathbf{v} = a + \mathbf{j}b$,
to the equation (3) we get eqn. (3) in the form of a function of the complex variable in the form:

$$\mathbf{H}(\mathbf{m}) = H_{\mathrm{Re}}(x,y) + \mathbf{j}H_{\mathrm{Im}}(x.y) \qquad (4)$$

Supposing the direction of the stationary magnetic field $\mathbf{B}_0$ according to Fig .1 and physical principles of NMR, it is evident that only the real part of the expression (4) is useful for measurement. After simple algebraic transcription we can write the resultant expression as

$$H_{\mathrm{Re}}(x,y) = \\ \frac{I}{\pi}\left[-\frac{R}{Q^2+R^2} + \frac{S}{Q^2+S^2} - \frac{R}{T^2+R^2} + \frac{S}{T^2+S^2}\right] \qquad (5)$$

where $Q = x - a$, $R = y - b$, $S = y + b$, $T = x + a$

Considering that $b = \sqrt{r^2 - a^2}$ for testing the equation (5) after substitution: $a = r$, $b = 0$, we naturally get $H_{\mathrm{Re}}(x,y) = 0$.

3. Optimising and plot of the radio frequency magnetic field

A character of the magnetic field distribution generated by four parallel conductors according to Fig. 1 represented by equation (5) is authorizing to state that this function has the continuous derivate of the second order and it is matching to the Laplace equation $\Delta\mathbf{H} = 0$ in the selected central region and our function is considered to be harmonic. According to the principle of a harmonic function, the investigated function $H_{\mathrm{Re}}(x,y) = \mathbf{H}(x,y)$ attains maximal and minimal values on the central region, in our case on a circular line.

The contour plot of the function (5) for $r = 1$, $x \in \{-0.8; 0.8\}$, $y \in \{-0.8; 0.8\}$ is illustrated in Fig.2.

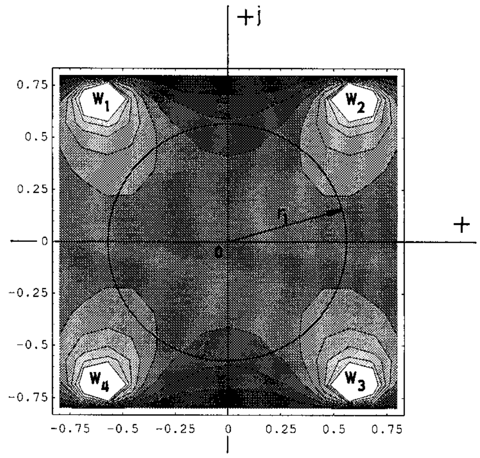

Fig. 2 The relative contour plot of the optimised RF field generated by four conductors. The evaluated circular line has the relative radius $r_1 = 0.6$.

The evaluated circular line has the radius $r_1 = 0.6$. When we define the level of $\mathbf{H}(x,y)$ in the origin as $\mathbf{H}(0,0)$ we can determinate an absolute deviation D_a as:

$$D_a(a,b) = \mathbf{H}(x,y) - \mathbf{H}(0,0) \tag{6}$$

The field in the origin calculated from equation (5) has the form:

$$\mathbf{H}(0,0) = \frac{4I}{\pi} \frac{b}{a^2 + b^2} \tag{7}$$

For plotting the equation (6) it is necessary to express the equation (5) in polar coordinates substituting:

$$Q = r_1 \cos\varphi - a, \quad R = r_1 \sin\varphi - b,$$
$$S = r_1 \sin\varphi + b, \quad T = r_1 \cos\varphi \tag{8}$$

A plot of the equation (6) in rectangular coordinates considering expression (8) for $r_1 = 0.6$, $\varphi \in \{0, \pi/2\}$ can be seen in Fig.3. A criterion of optimisation can be expressed in the simple form as

$$D_a(a,b) = 0 \tag{9}$$

or expressing a difference of two functions defined in the same interval as a mean square deviation or norm of the function:

$$\left\| \mathbf{H}(r_1,\varphi) - \mathbf{H}(0,0) \right\| = \sqrt{\int_0^{\pi/2} \left| \mathbf{H}(r_1,\varphi) - \mathbf{H}(0,0) \right|^2 d\varphi} \tag{10}$$

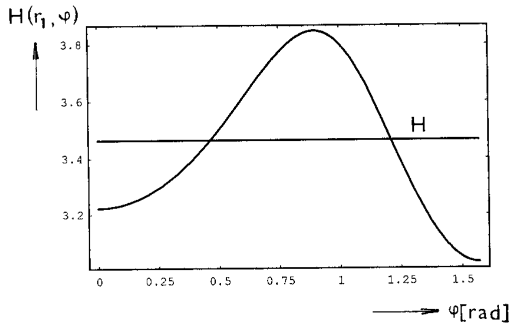

Fig. 3 A plot of the field on the circular line and the field in the origin in rectangular coordinates

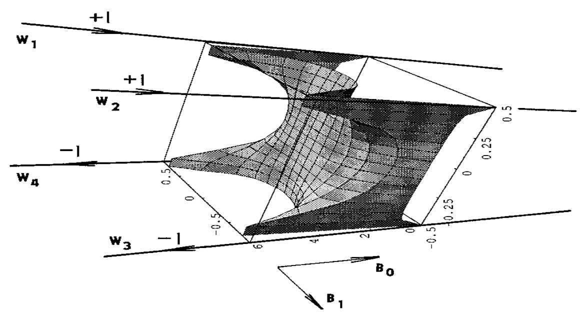

Fig. 4 3D-plot of RF field generated by two long parallel coils placed into the horizontally oriented stationary magnetic field $\mathbf{B}_0$.

Naturally that both criteria (9) and (10) are equivalent from the point of view of the expected result. We have used the equation (10) where all real combinations of a/b parameters were evaluated and a computer procedure has selected the best results, $a_{opt} = 0.483$. Parameter b is determined by its position on the circular line.

4. Conclusion

Measurement and imaging of long samples using Nuclear Magnetic Resonance is a special task where the tendency is oriented to increase the relation of length/diameter of the measuring sensor. That is why the sensitivity of this sensor depends on the sensitivity of a RF transmitting/receiving coil.

As an example we tried to model the RF field of four-conductors arrangement and found an optimal geometrical configuration. A 3D-plot in Fig. 4 shows the RF field generated by four wires in optimal configuration placed into the horizontally oriented stationary magnetic field $\mathbf{B}_0$. In an experimental realization the conductors are creating two long coils connected in series fed by one RF current I. The inner part of these coils can be filled by a glass tube containing water or simply an opening for placing a sample to be investigated (e.g. long botanical samples as stems, shoots or branches of plants).

A special application of the transducer containing water filled tube can be used for simple stationary magnetic field measurement. Using a basic electronic equipment (transmitter, receiver, gradient field generator, A/D converter and a personal computer) one can measure the magnetic field distribution in real time with direct interpretation of results in the form of a 1-D graph or 2-D image on the screen of a monitor.

Acknowledgements

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