NOISE REDUCTION BASED ON DYNAMICS RECONSTRUCTION

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A new technique of noise reduction in time series is presented. First, the data are embedded into higher dimensional state space.Then, the dynamics of the system is approximated and data are adjusted to satisfy better the approximations to the dynamics. It leads to noise reduction which is more effective than traditonal filtering, especially for complex time series.

• Introduction

Experimental time series are a mixture of deterministic component and random noise. If the noise is predominantly at different frequencies than the frequencies of the signal, then we can reduce the noise by removing the frequency components of the noise. This is the basic idea of linear low-, high- or bandpass filters.

Linear filters eliminate additive noise without distorting the signal in periodic or quasiperiodic cases. They can also be useful for highly oversampled non-linear signals. However, for nonlinear systems, both the power spectrum of the investigated signal and the noise may be broadband. Then, classical filtering methods, that are based on frequency separations are unefficient. They cause distortion because some of the suppressed frequency components are part of the dynamics.

Recently new methods of noise-reduction have been developed. They are based on the theory of nonlinear dynamical systems [1].

• Subject and Methods

In nonlinear noise reduction schemes, the next basic strategy is used. First, from the data an approximation to the evolution law is obtained either globally or locally. In a second step, the trajectory is constructed such that it both satisfies the law and is close to the measured trajectory. The same strategy is behind our method. We briefly introduce the algorithm.

1. The first step in any data analysis based on dynamical systems theory is to reconstruct the state portrait from the time series x_n. Since we measure only one coordinate, it might not be obvious how to reconstruct the state portrait. According to the embedding theory, under some conditions, the state portrait, topologically equivalent to the original one, can be reconstructed from the signal [2], [3]. A very convenient choice for the reconstruction is a set of delay coordinates. Let the data x_n be embedded in a 2k+1-dimensional phase space with delay and "advance" coordinates,

   $$x_n=(x_{n-k}, x_{n-k+1}, ..., x_n, ..., x_{n+k-1}, x_{n+k}) .$$

   The integer 2k+1 is called the embedding dimension. Embedding saves a lot of important properties of the original attractor [2]. Notice that it is essential that both past and future values are used in the embedding. Then the filtering algorithm performs better in points where stable and unstable manifolds are tangent.

2. The reconstructed state portrait can be treated as an attractor of a map *f* whose exact form is unknown. Let us assume that *f* is approximately linear in a small neighborhood of each attractor point **x**:
$$f(\mathbf{x})\approx A\mathbf{x} + \mathbf{b}.$$
The matrix A is the Jacobian of *f* at **x.**
For each embedding vector $\mathbf{x}_n$ we find closest neighbors. They are used to find a local linear approximation of the dynamics in a neighborhood of $\mathbf{x}_n$. To obtain the optimal fit in the least square sense the singular value decomposition is applied. A different A and **b** are computed for each neighborhood.

3. The middle coordinate of $\mathbf{x}_n$ is corrected such that the result both satisfies better the estimated dynamics and remains close to the measured value. The adjustment is done for each point on the attractor.

4. The original time series is replaced by the corrected one and the procedure is repeated until the noise level does not decrease further.

There are some parameters which can be varied in the procedure as the size of the neighborhoods, or the parameter k.. Larger neighborhoods give more stable fits but if they are chosen too large the locality required for the linear approximation is violated. The value of k is limited from above mainly by the fact that neighbors have to be searched in 2k+1 dimensions. For large k, points can be very sparse for limited data sets. The easiest way to optimize this choice is to find the set of parameters for which the average quality of the least squares fits is the best.

To compare our method with a traditonal noise-reduction we applied the general first order model of linear filter. To find the optimal values of coefficients we used the singular value decomposition technique. Offline filtering is assumed to enable the involvement of as much information as possible. Therefore the search for optimal correction of each point in the time series can use both the values from the past and from the future.

To test the effectiveness of our technique in comparison to traditional linear filtering we have to estimate how much noise both methods remove from the data. In artificial examples the original (noiseless) time series y_n is known. In these cases we can compute the deviation
$$E_0(x)=\sqrt{1/N\sum(x_n-y_n)^2}$$
of the noisy signal x_n from the clean one and the deviation $E_0(x^f)$of the filtered signal x^f_n from the clean one. The corresponding value of $r_0=E_0(x)/E_0(x^f)$ estimates the amount of noise reduction. r_0 is very useful in the process of comparison of different noise-reduction methods.

However, in a typical experimental situation the noise-free data is unknown. Then it is difficult to investigate, how much noise is removed from the data. The so called correlation dimension D_2 is one of characteristics which can be helpful in some of these cases. Its value provides an information about the complexity of the investigated dynamics.

$$D_2 = \lim_{\varepsilon->0} \frac{\ln \sum_{i=1}^{N(\varepsilon)} p_i^2}{\ln \varepsilon} = \lim_{\varepsilon->0} \frac{\ln C_2(\varepsilon)}{\ln \varepsilon},$$

where N(ε) is the total number of hypercubes of side length ε which cover the attractor, and p_i is the probability of finding a point in the hypercube *i.* In order to esimate the correlation dimension, we have to plot *ln* $C_2(\varepsilon)$ as a function of *ln* ε and follow the slope of the obtained curve. This slope

$\nu(\varepsilon)$ is called correlation exponent, and the limit of it for vanishing ε represents the correlation dimension. But the determination of correlation dimension is often a difficult problem [4]. If a flat part plateau is clearly distinguishable at the graph for correlation dimension estimation, then the value of the plateau is taken as the searched dimension. But noise in data destroys the scaling region and makes dimension estimation impossible. On the other hand succesful noise-reduction makes plateau visible. Therefore, dimension estimation from data before and after noise reduction provides a very good tool for filtering quality determination.

• Results and Conclusion

To test the effectiveness of the algorithm, we apply it to a periodic system (noisy sinus function), to an often studied chaotic system (Lorenz attractor) contaminated by noise and to experimental signals (ECG, EEG).

In the case of noisy periodic systems both method reduce the noise succesfully. We obtain approximately the same amount of noise reduction r_0 applying either the linear or the nonlinear filtering.

Lorenz chaotic system,

$$x = 10 (y - x)$$
$$y = -y -xz + 28x$$
$$z = xy - (8/3)z ,$$

has an attractor with correlation dimension of about 2.07. The figure shows that correlation dimension cannot be estimated for data with 10% of noise added. After noise reduction a scaling region is clearly visible and the value of dimension slightly above 2 can be observed. On the other hand, linear filtering was not able to suppress the noise without destroying the dynamics of the system.

The same problems were found filtering real data as ECG or EEG. They resemble chaotic behaviour. Apparent periodicities and also many irregularities are seen in various parts of time series. The continuous part of the spectrum is due to the noise and also due to the irregular but possibly deterministic dynamics. The separation of both parts is impossible with spectral methods. Therefore, we tried to filter by the nonlinear method which was originally developed for chaotic signals despite the fact that the investigated systems (heart and brain) were not proved to be following chaotic behavior. The results are very promising.

Our work demonstrates that linear methods are comparable with our filtering procedure only in case of periodic systems. For nonperiodic signals traditional filtering techniques are ineffective. We have found that new nonlinear noise-reduction leads to much better results especially in the case of complex signals including chaotic and irregular experimental time series.

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References

[1] Kostelich E. J., Schreiber T.: Noise reduction in chaotic time-series data: A survey of common methods. *Physical Review E* 48, no. 3, pp. 1752-1763, 1993;

[2] Takens F.: Detecting Strange Attractors in Turbulence, *Lecture Notes in Mathematics* 898, Springer, Berlin, pp. 366, 1981;

[3] Sauer T., Yorke J. A., Casdagli M.: Embedology, *Journal of Stat. Phys*. 65, pp. 579, 1991;

[4] Krakovská A.: Correlation dimension underestimation. *Acta physica slovaca* 45, no. 5, pp 567-574, 1995;