Application of the Earth's magnetic field and accelerometry to the measurement of net knee extensor torque

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Introduction

We have encountered measurement of net knee extensor torque in two situations in medical rehabilitation. One is the assessment of spastic torque in a paralysed leg subjected to the pendulum test [1]. The subject sits or lies on a couch, with the lower leg protruding over the edge, free to swing when released from a horizontal position. A flaccid leg executes a damped pendular motion about the vertical. If a spastic torque is induced, however, a disturbance to this pattern is observed. A "nonlinear observer," of second order, based on feedback control theory, was devised by de Paor [2] to estimate the underlying torque from the record of angle of the shin with respect to the vertical, versus time. In our experiments [3], the angle was measured with an accurate, though cumbersome, electromechanical goniometer. This involved two potential dividers, mounted on straps around the upper and lower leg, connected by a telescopic tube.

The second application is the monitoring progressive strengthening of muscles in the legs of a paraplegic person, prior to restoring standing and limited walking ability under Functional Electrical Stimulation [4].

This paper presents a new angle measurement and torque estimation scheme. The angle of the shin is measured with a light, inexpensive two-axis magnetoresistive bridge (Honeywell, HMC 1022), sensing the swing of the leg through the Earth's magnetic field. The plane of swing is magnetic north-south. The transducer is currently hard-wired to a computer, but it is planned to couple it via a miniature radio transmitter. A single-axis accelerometer (Monitran, MTN/7000-5) is mounted on the leg, below the knee at a specific distance, based on anthropomorphic data due to Winter [5]. This enables the gravity nonlinearity in the equation of leg swing to be cancelled out, linearises the observer and reduces its dynamical order.

Magnetoresistive angle transducer.

The principle is shown schematically on Fig.1. The Earth's magnetic field makes angle "dip" with respect to the horizontal, and the shin makes angle θ to the vertical. Components of the magnetic flux censity are B_1 and B_2 along and perpendicular to the shin, respectively.

Geometry gives

$$B_1 = B\cos(\theta + \pi/2 - \text{dip})$$
$$B_2 = B\sin(\theta + \pi/2 - \text{dip}) \qquad (1).$$

$$\theta = \text{dip} - \pi/2 + \tan^{-1}(B_2/B_1) \qquad (2).$$

The quotient B_2/B_1 does not preserve the signs of B_2 and B_1, so we interpret eqn.(2) as

$$\theta = \text{dip} - \pi/2 + \arctan(B_2/B_1) \text{ for } B_1>0$$
$$= \text{dip} + \arctan(B_2/B_1) \text{ for } B_1<0 \qquad (3).$$

Accelerometry applied to linearisation and order reduction of the observer.

In over three hundred pendulum tests on paralysed legs, we have found that the dynamics of the swinging lower leg are described accurately by the equation of motion

$$d^2\theta/dt^2 + 2\zeta\omega_n d\theta/dt + \omega_n^2\sin(\theta) = T_e/J \quad (4).$$

In eqn.(4), the symbols have the interpretations given below.

$$\omega_n = \sqrt{([mgc]/J)} \quad (5)$$

is the angular frequency of small undamped oscillations about the vertical; m is the mass of the lower leg; g is acceleration due to gravity; c is distance of centre of gravity of the lower leg below the centre of the knee; and J is moment of inertia of the lower leg for rotation about the knee. If L is length of lower leg from centre of knee to heel, Winter [5] gives $c = 0.606L$. The moment of inertia is $J = mr^2$, where r is radius of gyration. Winter [5] gives $r = 0.735L$. Taking $g = 9.81\ ms^{-2}$, eqn. (5) yields $\omega_n = 3.317/\sqrt{(L)}$.

$$\zeta = F/(2J\omega_n) \quad (6)$$

damping ratio for small oscillations, where F is viscous friction coefficient. In our experiments and those reported by Bajd and Vodovnik [6], $\zeta \approx 0.125$. This is a fascinating finding, for, taking the mass of the lower leg to be proportional to L^3, it implies that F is proportional to $L^{4.5}$. Why this should be so is still a mystery to us.
T_e is net knee extensor torque, primarily the resultant of quadriceps and hamstrings. Our problem is to derive an accurate estimate of T_e/J from the record of θ vs. t. We solved this previously by the nonlinear second order observer described by de Paor [2].
Currently, we have mounted a single-axis accelerometer at a distance δ below the centre of the knee. Its output voltage is

$$v = \alpha[\ \delta d^2\theta/dt^2 + g\sin(\theta)] \quad (7).$$

Rearranging eqn. (7), setting

$$g/\delta = \omega_n^2 \quad (8),$$

and subtracting from eqn.(4) gives

$$2\zeta\omega_n d\theta/dt = T_e/J - v/[\alpha\delta] \quad (9).$$

The figures given by Winter [5] yield $\delta = 0.891L$.

Since $v/[\alpha\delta]$ is known, eqn.(9) shows that T_e/J could be derived by differentiating the graph of θ vs.t. We avoid this, and consequent noise enhancement, by the feedback system

$$y = k\{\theta - x/[2\zeta\omega_n]\}$$

$$dx/dt = y - v/[\alpha\delta] \quad (10).$$

Differentiating the first line of eqn.(10), subject to the second and to eqn.(9), gives

$$dy/dt = \{k/[2\zeta\omega_n]\}\{T_e/J - y\} \quad (11).$$

Eqn.(11) constitutes a linear first order observer: y is tracking T_e/J through a first order lowpass filter of time constant

$$\tau = 2\zeta\omega_n/k \quad (12).$$

Provided that k is chosen so that the filter's passband accommodates all significant frequency components of T_e/J, y is a good estimate of T_e/J.

Results

A simulation experiment shows that the observer described by eqn.(10), properly tuned, yields a very good approximation to the normalised net knee extensor torque, T_e/J.
Fig.2 shows the simulated record of θ vs. t for a pendulum test performed on eqn.(4) subjected to the inset artificial torque spasm T_e/J vs. t. Setting k = 100 gives y vs. t shown on Fig.3. To the same scales, this is practically indistinguishable from T_e/J vs. t. Maintaining k = 100, the real experiment shown by the goniometer record on Fig. 4 was performed. The subject, a 21 year old student, raised her lower leg from the vertical position, $\theta = 0$, held it hovering around $\theta = 1$ radian, then let it drop while simulating a spasm, and finally let it swing, slightly offset backwards from the position $\theta = 0$. We confirmed that the magnetic transducer equations reproduce θ vs. t essentially perfectly. Then we passed the captured records of θ vs. t and v vs. t (accelerometer voltage) through the linear first-order observer, within the simulation program SIMNON, to generate an estimate T_e/J vs. t, shown on Fig.5. Guided by the simulation experiment, we are confident that Fig.5 gives as accurate a record of T_e/J vs. t as could be desired in clinical applications.

Conclusion

Cheap, compact, magnetoresistive bridges can be used to harness the Earth's magnetic field to produce an accurate record of lower leg rotation for use in clinical studies. An accelerometer can be used to linearise the observer originally proposed by de Paor [2] and reduce it to first order, while giving just as close an approximation to the graph of normalised muscle torque versus time. These findings can simplify the instrumentation used in our studies, which are directed to the quantification of spastic torque in paralysed legs and its alleviation by medication or by therapeutic electrical stimulation.

Acknowledgement

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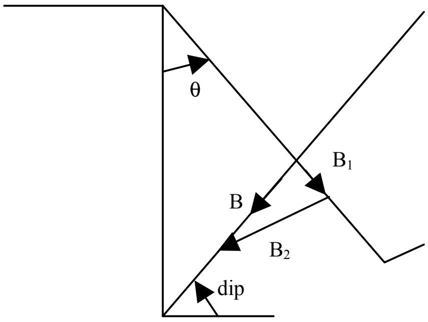

Fig.1 Measurement of θ via Earth's magnetic field

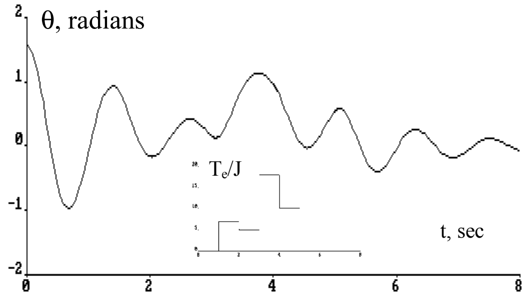

Fig.2 Simulated θ vs. t and underlying spasm

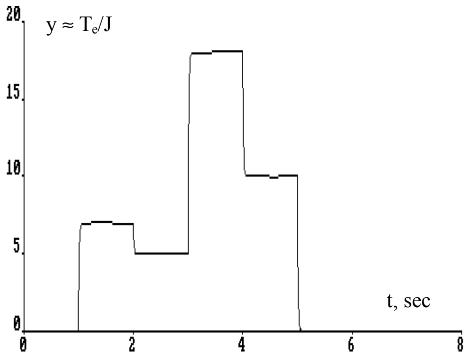

Fig.3 Simulated torque estimated by observer

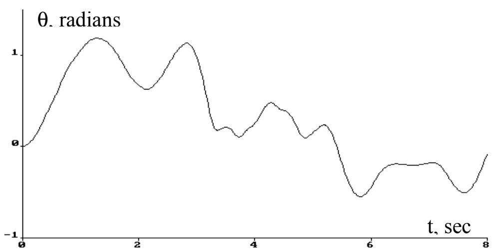

Fig.4 Record of θ vs. t in a real experiment

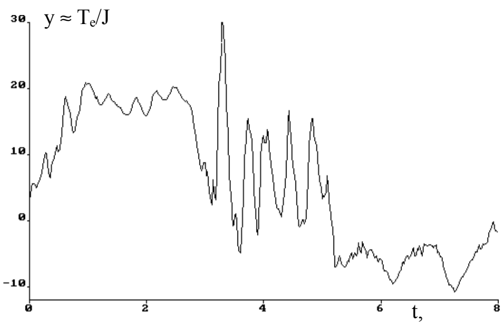

Fig.5 Estimated normalised torque for Fig.4