

# Noise properties of the SQUID-systems for measurement of AC magnetisation characteristics of HT<sub>c</sub> superconductors by the compensation method

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## Abstract

This contribution deals with analysis of the influence of the topical noise sources on the resulting sensitivity of the measuring system in magnetically unshielded environment. The geomagnetic noise in the measuring ambient, the noise component of the magnetisation field and the intrinsic noise of the superconducting quantum gradiometer (SQG) are considered. The results of the theoretical analysis are applied in the system with the magnetisation coils and the distance of the superconductor sample from the  $x_s = 10,7$  cm. In typical conditions of magnetically unshielded Faraday chambers placed at the border of the town area, it is possible to achieve the spectral sensitivity of the order of  $(10^{-9} \div 10^{-10}) \text{ Am}^2\text{Hz}^{-1/2}$ .

## 1. Introduction

In compensation method of the magnetisation characteristics measurement the measured HT<sub>c</sub> sample is inserted to the magnetisation field of two serially in phase opposition connected coils with different diameters of the turns (Fig. 1). The main part of the magnetisation field is generated by the coil with smaller diameter - the magnetisation coil. The second coil (coaxially placed) - compensation

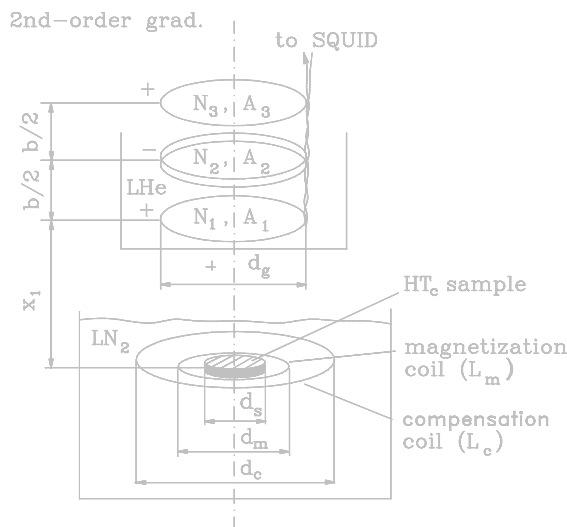


Fig. 1. Magnetisation and sensing part of the measuring system (simplified).

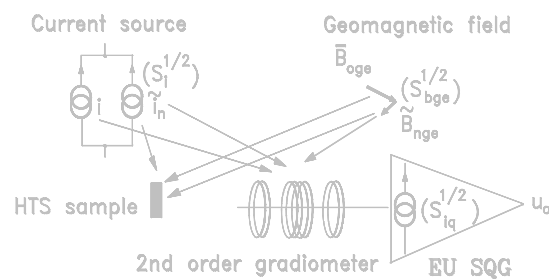


Fig. 2. Sources of the noise signals and their action in the transfer system.

coil (with greater diameter) - compensates the magnetic field of the magnetisation coil in the location of the sensing 2nd order gradiometer in the state without the sample. Therefore AC magnetic field, generated by the coil set immersed into the liquid Nitrogen, does not induce a signal on the output of the electronic unit of the superconducting quantum gradiometer (EU SQG). After inserting the HT<sub>c</sub> sample into the coil set and after its transition to the superconducting state, the compensation is

disturbed. A signal proportional to the magnetisation of the sample will appear on the EU SQG output. An example of the magnetisation curves, measured at gradual increasing of amplitude of the magnetisation field of the triangle shape with the frequency of  $f_m = 0.1$  Hz, is in Fig. 3.

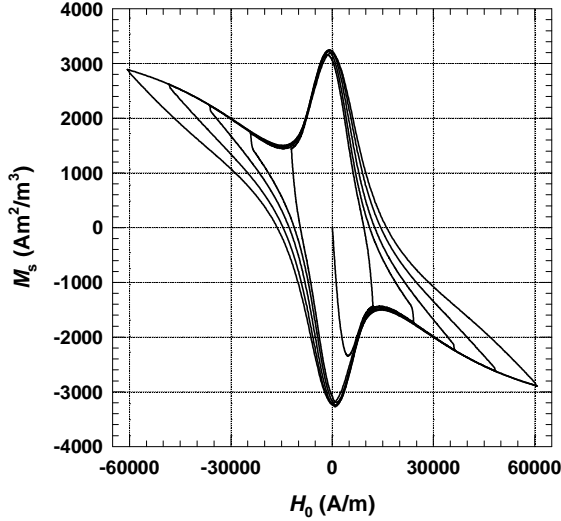


Fig. 3. Magnetisation curves of the HT<sub>c</sub> material YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7.8</sub>/Ag (80 wt%).

on the gradiometric antenna. With the non-ideal compensation, also the useful component of the magnetisation field, corresponding to the current component  $\mathbf{i}$ , has also a character of disturbing signal acting on the gradiometer. Similarly as the magnetisation field, also the noise ( $\tilde{\mathbf{B}}_{nge}$ ) and the unidirectional ( $B_{oge}$ ) components of the geomagnetic field act on the sample and the gradiometer (non-ideally balanced). The unidirectional component causes the unidirectional magnetisation of the sample, which is in some cases undesired. The source of the EU SQG intrinsic noise is represented by the input equivalent noise current with the spectral density  $S_{iq}$ . A detailed theoretical analysis of the noise and transfer properties of the measuring system was given in [1]. This analysis shows that the spectral density of the equivalent noise magnetic moment (spectral sensitivity) is given by the relation:

$$S_{mD}^{1/2} = K_{\Phi m}^{-1} \left[ \chi_{ext}^2 S_i V_s^2 K_{sy}^2 D^{-2} + \left( \chi_{ext} |V_s K_{\Phi m} / \mu + |\beta_r| N_1 A_1 \right)^2 S_{bge} + 4L_{in}^2 S_{iq} \right]^{1/2} [\text{Am}^2 \text{Hz}^{-1/2}] \quad (1)$$

where it holds:

D.....rate of the reduction of the magnetisation current by the "noiseless" current divider (reduction of the useful and the noise components, usually in decade steps  $D=1, 10, 100, \dots$ )

$\chi_{ext}$  ....external magnetic susceptibility of the sample,  $\chi = \chi_{ext} / (1 - N\chi_{ext})$  ...volume susceptibility of the sample

N.....demagnetisation factor, depending on the sample's shape ( $0 \leq N \leq 1$ )

$K_{\Phi m} = 0.5\pi^{1/2} \mu N_1 A_1 \left[ (A_1 + \pi x_1^2)^{-1/2} - 2(A_1 + \pi x_2^2)^{-1/2} + (A_1 + \pi x_3^2)^{-1/2} \right]$  - transfer constant of the symmetric 2nd order gradiometer

$S_{iq}$ .....spectral density of the equivalent input noise current of the EU SQM

$S_i$ .....spectral density of the noise current of the current source (at  $D = 1$ )

$S_{bge}$ ....spectral density of the geomagnetic field noise component

$A_1$ .....area of one gradiometer's turn,  $N_1$ .... number of turns of the gradiometer's first coil

$\beta_r$ .....gradiometer relative imbalance ( $\beta_r \sim 10^{-4}$ ),  $\Delta f$ .... noise width of the transmission band

## 2. Analysis

The noise component of the measuring system output signal is induced by the noise sources shown in Fig. 2. In the following text, all time-dependent quantities are considered as the instantaneous values (unless given otherwise). The noise component of the magnetisation field is excited by the noise component ( $\tilde{\mathbf{i}}_n$ ) of the current source feeding the magnetisation coil set. It acts on the HT<sub>c</sub> sample (of the shape of pellet) and, by a non-ideal compensation, also directly

$V_s$ .....volume of the sample,  $\mu$ ....permeability of vacuum  
 $x_1, x_2, x_3$  ....distances of the gradiometer sections from the sample ( $x_1 = x_s, x_2 = x_1+b/2, x_3 = x_1+b$ )  
 $K_{sy} = K_{\Phi m} (K_{Hi}^{(mo)} - K_{Hi}^{(co)})$  ....constant, where  $K_{Hi}^{(mo)}, K_{Hi}^{(co)}$  are the transfer constants of the magnetisation and the compensation coils.

At the same time it holds [1]:

$$i_t = i + \tilde{i}_n \text{ .....total magnetisation current (useful and noise components)}$$

$$H_{mo}^{(t)} = (K_{Hi}^{(mo)} - K_{Hi}^{(co)})i_t \text{ .....total magnetisation field (acting in the state without the sample)}$$

$$m_{su} = \chi_{ext} K_{\Phi m}^{-1} V_s K_{sy} i \text{ .....magnetic moment of measured sample,}$$

$$p_{s/n}^{(D)} = m_{su} / (S_{mD}^{1/2} \Delta f^{1/2}) \text{ .....signal-to-noise ratio on the measuring system output}$$

$$M_s = \frac{m_{su}}{V_s}, \quad M_s = \chi H_o \text{ [Am}^{-1}\text{]....volume magnetisation [} H_o = H_{mo}^{(t)} / (1 + N\chi) \text{].}$$

The equation (1) can be rewritten in the form which expresses the individual influence of considered noise sources on resulting sensitivity

$$S_{mD}^{1/2} = (S_{miD} + S_{miq} + S_{mge})^{1/2} \text{ [Am}^2\text{Hz}^{-1/2}\text{]} \quad (2)$$

**Tab.I Parameters of main parts of the measuring system and computed theoretical values**

$x_s = x_1 = 10.7$ (cm)	$S_{bge}^{1/2} = 1 \times 10^{-9}$ (THz <sup>-1/2</sup> )	$D = 1, \Delta f = 10$ (Hz)
$b = 45$ (mm)	$* S_{bge}^{1/2} = 1 \times 10^{-10}$ (THz <sup>-1/2</sup> )	$S_{mD}^{1/2} = 7.9 \times 10^{-9}$ (Am <sup>2</sup> Hz <sup>-1/2</sup> )
$d_g = 27.2$ (mm)	$S_{il}^{1/2} = 3.10^{-7}$ (AHZ <sup>-1/2</sup> )	$m_{su}^{(D)} _{min} \approx -2,5 \times 10^{-8}$ (Am <sup>2</sup> )
$\beta_r = 10^{-4}$	$S_{iD}^{1/2} = S_{il}^{1/2} / D$	$D = 10, \Delta f = 10$ (Hz)
$d_s = 12$ (mm)	( $D = 1, 10, \dots, 10^4$ )	$S_{mD}^{1/2} = 2.4 \times 10^{-9}$ (Am <sup>2</sup> Hz <sup>-1/2</sup> )
$h_s = 2.5$ (mm)	$S_{iq}^{1/2} = 1 \times 10^{-11}$ (AHZ <sup>-1/2</sup> )	$m_{su}^{(D)} _{min} = -7.6 \times 10^{-9}$ (Am <sup>2</sup> )
$\chi_{ext} = -3.9$	$K_{Hi}^{(mo)} = 2.44 \times 10^4$ (m <sup>-1</sup> )	$D = 100, \Delta f = 10$ (Hz)
$N = 0.74$ ( $\chi \cong -1$ )	$K_{Hi}^{(co)} = 1.55 \times 10^3$ (m <sup>-1</sup> )	$* S_{mD}^{1/2} = 3.9 \times 10^{-10}$ (Am <sup>2</sup> Hz <sup>-1/2</sup> )
$f_m = 0.1$ (Hz)	$K_{\Phi m} = 1.27 \times 10^{-7}$ (Hm <sup>-2</sup> )	$* m_{su}^{(D)} _{min} = -1.2 \times 10^{-9}$ (Am <sup>2</sup> )
$i_{max} = 2$ (A <sub>pp</sub> )	$K_{sy} = 2.9 \times 10^{-3}$ (Hm <sup>-3</sup> )	
$B_{mr max} = 0.029$ (T) (0.058 T <sub>pp</sub> )		

\* in conditions of geomagnetic quiet during the night

These components correspond to the current source noise, EU SQG intrinsic noise and the geomagnetic noise respectively, and it holds

$$S_{miD}^{1/2} = \frac{K_{sy} V_s S_i^{1/2} |\chi_{ext}|}{K_{\Phi m} D}, \quad S_{miq}^{1/2} = \frac{2L_{in} S_{iq}^{1/2}}{K_{\Phi m}}, \quad S_{mge}^{1/2} = \frac{(\chi_{ext} |V_s K_{\Phi m} / \mu + |\beta_r| N_1 A_1)^{1/2}}{K_{\Phi m}} S_{bge}^{1/2} \quad (3)$$

### 3. Results of the analysis

The results have been applied in the analysis and following optimisation of the system designed for the distance  $x_s = 10.7$  cm of the sample from the gradiometer. The main parameters are given in Table I. For typical operation modes also the values of computed resulting spectral sensitivities of the system are given. The noise conditions are shown in Figure 4. There are plotted the dependencies of separate components of the spectral sensitivity on the spectral densities of considered noise sources. For the measurements utilising the maximum magnetisation fields ( $D = 1, H_{ot max} =$

$22.8 \times 10^3$  A/m) during the day-time the resulting sensitivity is  $7.9 \times 10^{-9} \text{ Am}^2\text{Hz}^{-1/2}$  and is determined mainly by the noise component  $S_{\text{iD}}^{1/2}$  of the current source. When decreasing the magnetisation field by one order ( $D = 10$ ) the spectral sensitivity of  $2.4 \times 10^{-9} \text{ Am}^2\text{Hz}^{-1/2}$  can be achieved. During the night-time (decrease in geomagnetic noise) and with further decreasing of the magnetisation field again by one order ( $D = 100$ ) it is possible to reach the limit spectral sensitivity up to  $3.9 \times 10^{-10} \text{ Am}^2\text{Hz}^{-1/2}$  with given parameters of the magnetisation set and EU SQG.

#### 4. Conclusions

The results of the theoretical analysis have been verified experimentally with using several modifications of the magnetisation sets with different distances  $x_s$  of the sample from the gradiometer. In measurement with this system ( $x_s = 10.7$  cm) early morning, at the bandwidth  $\Delta f = 10$  Hz and the range  $D = 10$ , the minimum measurable signal was about  $10^{-8} \text{ Am}^2$ . It corresponds to the spectral sensitivity  $\sim 3 \times 10^{-9} \text{ Am}^2\text{Hz}^{-1/2}$  and well corresponds with the computed values.

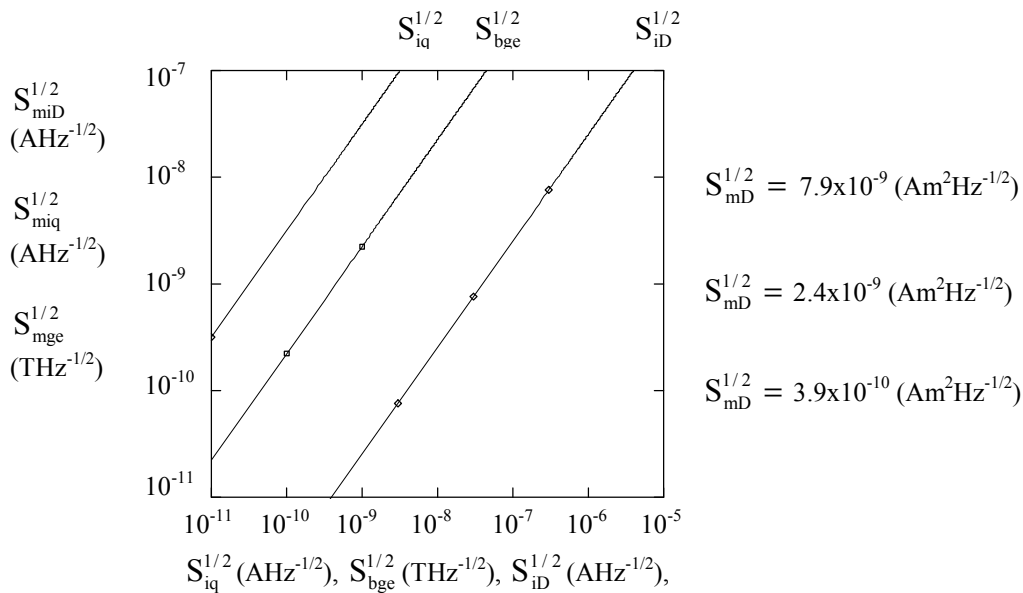


Fig. 4. Dependence of the spectral sensitivity components on the spectral densities of the noise sources.

#### Acknowledgements

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#### References

- [1] V. Zrubec: Theoretical sensitivity limits of the compensation method for magnetization characteristics measurement of  $\text{HT}_c$  superconducting materials using  $\text{LT}_c$  SQUID 2nd order gradiometer, *Cryogenics* 39 (1999) 241–251.