Numerical Evaluation of Signal in Resistive Plate Chamber

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Abstract: *Equations for the numerical evaluation of output signals from the resistive plate chamber are presented in the paper. The field of moving charges is described by means of scalar transient electric potential. The displacement and conductive currents are induced in the slightly conductive layer covering pick-up electrodes of an arbitrary shape. These electrodes are connected to current-controlled operational amplifiers and so their potential can be set to zero. The potential equations are discretized by the Finite Element Method and the potential distribution due to the moving charge is then evaluated. The results are used to find the current passing through the rectangular pick-up electrodes. In conclusion the distribution of the total current in the electrodes as a function of the moving charge position and the layer conductivity and permittivity is given.*

1. INTRODUCTION

A Resistive Plate Chamber (RPC) is a new type of particle detector. It is now under development at CERN, Geneva. RPC consists of one or two weakly conducting panels bounding a gap filled with a gas. The panels are covered from the outer side by electrodes. The cathode is a continuous sheet of metal. The anode consists of strips or generally of arbitrarily shaped metallic slices.

The operation of resistive counter is easy to explain. A high-energy primary particle passing through the detector ionizes some atoms of the gas. A DC voltage difference (of about 1 kV) applied to the electrodes accelerates the ions and electrons and so an avalanche of charges is generated. Friction in the gas slows down these charges. The displacement currents caused by the motion of fast free electrons as well as the conductive currents in the panel close via the electrodes. The slow motion of heavy ions can be neglected. Individual currents flowing through the anode slices are amplified and measured. Evaluating these signals makes it possible to get information about the position of the high-energy particles in the gap.

One is interested to evaluate the time dependence of the current in dependence on the instantaneous charge position. A simple theory for signals induced in one pick-up conductive strip is developed in [1]. The average velocity of free charges, which is around $5 \cdot 10^4$ m/s, is low in comparison to the velocity of light, and so the relativistic effects are neglected in the paper. Finite conductivity of the anode panel is also neglected whereas its relative permittivity other than 1 is respected. The electron avalanche is approximated by a point charge moving with constant velocity. A fully analytical solution is given in the form of infinite sum of integrals, which are evaluated numerically and presented in graphical form. It is proven in [2] and [3] that the quasi-static description of the field is sufficient for the usual charge velocity. Many further methods for evaluating the current in a strip electrode can be found in [3]. All these analytical methods are limited to the strip shape of anode slices, which are infinite in one direction. Some results obtained for the rectangular shape of the pick-up slices while neglecting the panel influence are in [4].

It was found that the limits of analytical evaluation of currents could be removed by applying an appropriate numerical method for the solution of field equations. The Finite Element Method (FEM) is probably the best one. Some theoretical formulae based on FEM are derived in [5]. This paper presents a complete numerical solution of the problem, which enables the introduction of variable electron cloud velocity and even an evaluation of its velocity during the motion. A new program solving the equations for three-dimensional potential field was written and carefully tested. A postprocessor evaluates the total conductive and displacement currents through the anodes of an arbitrary shape. Some results are presented in graphs at the end of the paper.

2. Basic Equations and Boundary Conditions

Let us assume the arrangement of the RPC such as in Fig. 1. An avalanche of electrons moves in a gaseous filled volume V_1 in the direction of the z-axis. The field of the slowly moving charge in V_1 satisfies

$$\mathrm{div}\boldsymbol{D} = \mathrm{div}\varepsilon_0 \boldsymbol{E} = Q\,\delta(\boldsymbol{R}_0 + \boldsymbol{v}t)\,. \tag{1}$$

Here $\boldsymbol{D}$ is the electric flux density, $\boldsymbol{E}$ is the field intensity, δ is the 3D Dirac function, Q is the value of the moving equivalent point charge with initial position $\boldsymbol{R}_0$ and velocity $\boldsymbol{v}$. Volume V_2 represents a lossy dielectric of conductivity γ and of permittivity ε. The field in V_2 is described by the continuity equation

$$\mathrm{div}\boldsymbol{J}_{\mathrm{tot}} = \mathrm{div}\left(\gamma\,\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}\right) = 0 \tag{2}$$

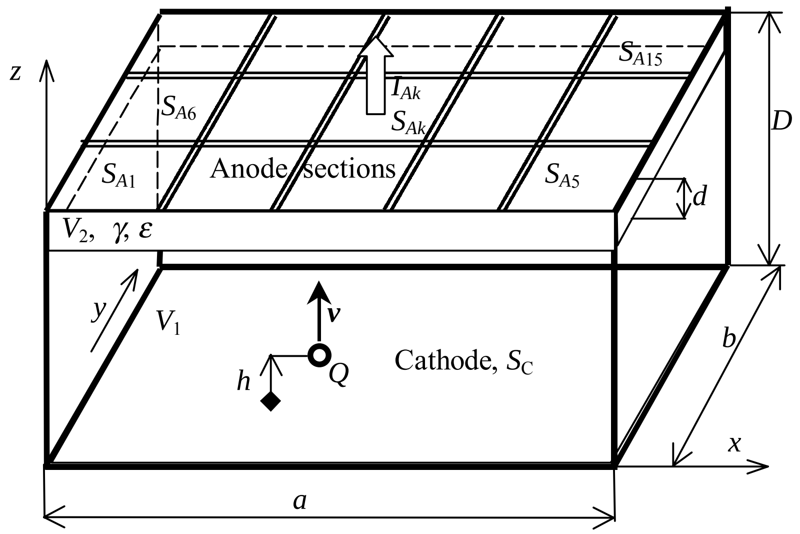


Fig. 1. An example of the resistive plate chamber

The field $\boldsymbol{E}$ is considered to be irrotational and so the scalar potential $U(x, y, z, t)$ is introduced by the relation $\boldsymbol{E} = -\mathrm{grad}U$. The superposition of the accelerating DC field on the electrodes together with the field of the moving charge is used. Valid for the field of the moving charge are the boundary conditions (see Fig. 1): $U_C = 0$ on the cathode surface S_C and $U_{Ak} = 0$ ($k = 1$ to P) on each of P anode slices. For the sake of simplicity, the Neumann condition $\partial U/\partial \mathrm{n} = 0$ is applied to the lateral surfaces S_N. The integral of current density $\boldsymbol{J}_{\mathrm{tot}}$ over k-th anode S_{Ak} is the anode current I_{Ak} , k = 1, … , P

$$\int_{S_{Ak}} \boldsymbol{J}_{\mathrm{tot}} \cdot d\boldsymbol{S} = \int_{S_{Ak}} \left(\gamma\,\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{D}}{\partial t}\right)\cdot d\boldsymbol{S} = I_{Ak} \tag{3}$$

$d\boldsymbol{S}$ is directed out of V_2.

3. Finite Element Solution

Equations (1) and (2) are solved using the FEM. Since the volumes V_1 and V_2 are of regular geometry, they are discretized into N_E regular brick elements with N_N nodes. The potential in both regions is approximated from the nodal values $U_j(t)$, $j = 1, \ldots, N_N$

$$U(t) = \sum_{j=1}^{N_N} U_j(t)\,W_j(x, y, z)\,. \tag{4}$$

Here $W_j(x, y, z)$ are approximating functions set up from the shape functions of the brick elements [6].

Equation (1) is discretized into a system of equations for the nodal potentials of nodes i in V_1. For these equations the following relation can be derived:

$$\sum_{j=1}^{N_N} \frac{dU_j}{dt} \int_{V_1} \varepsilon_0 \, \mathrm{grad} W_i \cdot \mathrm{grad} W_j \, dV = Q \, \mathrm{grad} W_i \cdot \boldsymbol{v}(\boldsymbol{R}). \tag{5}$$

For the nodal potentials of the internal nodes i in V_2 and those on the S_{12} layer between V_1 and V_2 it holds that

$$\sum_{j=1}^{N_N} U_j \int_{V_2} \gamma \ \mathrm{grad} W_i \cdot \mathrm{grad} W_j \, dV + \sum_{j=1}^{N_N} \frac{dU_j}{dt} \int_{V_2} \varepsilon \ \mathrm{grad} W_i \cdot \mathrm{grad} W_j \, dV = 0 \,. \tag{6}$$

The details are given in [5]. Equations (5) and (6) form a system, which has the matrix form

$$\underline{\boldsymbol{K}}\,\underline{\boldsymbol{U}} + \underline{\boldsymbol{C}} \frac{\mathrm{d}\underline{\boldsymbol{U}}}{\mathrm{d}t} = \underline{\boldsymbol{F}} \,. \tag{7}$$

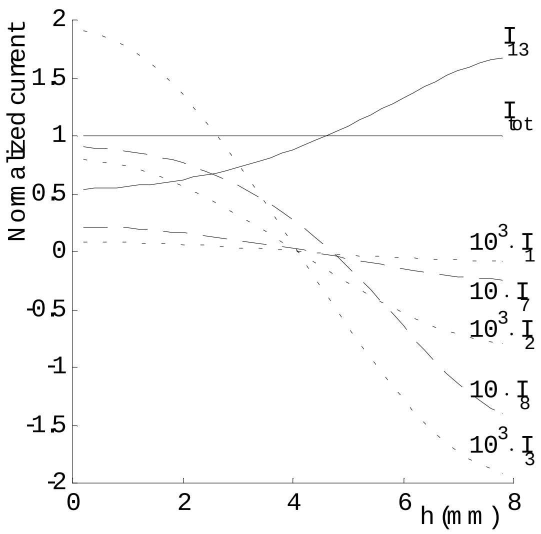
Here $\underline{\boldsymbol{U}}$ is a vector of nodal voltages $\underline{\boldsymbol{U}} = [U_1(t), U_2(t), \ldots, U_{N_N}(t)]^T$. Once $\underline{\boldsymbol{U}}$ is calculated, the total current I_k of each anode ($k = 1, \ldots , P$) is found from integral (3). It is shown in [5] that the accuracy of integration can be significantly improved by transforming the surface integral into the volume one over the layer of elements having at least one node on S_{Ak}. Denoting M_k the set of these nodes, (3) is evaluated numerically from

$$I_k = \sum_{\forall i \in M_k} \sum_{j=1}^{N_N} \left(U_j \int_{V_2} \gamma \ \mathrm{grad} W_i \cdot \mathrm{grad} W_j \, dV + \frac{dU_j}{dt} \int_{V_2} \varepsilon \ \mathrm{grad} W_i \cdot \mathrm{grad} W_j \, dV \right). \tag{8}$$

4. Verification of the Program and Some Results

A program was written which solves equations (7) and (8) and it was verified on an RPC such as in Fig. 1. Some parameters and dimension of the RPC are taken from [3]. The cathode dimensions are $a = b = 49$ mm, $D = 9$ mm, $d = 1$ mm. The relative permittivity of the panel is $\varepsilon = 3$ and the conductivity varies from 10^{-2} to 10^{-9} S/m. Altogether, 5 x 5 = 25 anode slices of the size 9 x 9 mm are separated by 1-mm gaps. The volume is discretized into 108 000 elements with 115 000 nodes. Charge $Q = 1$ moves with velocity $v = 5 .\ 10^4$ m/s. The time step is 4 ns corresponding to $\Delta z = 0.2$ mm.

It is observed that panel conductivity of less than 10^{-6} S/m has no influence on current distribution. The total current through all electrodes $I_{tot} = \sum_1^{25} I_k$ differs by less than 0.023 % from the theoretical value derived in [3] for the charge moving between two infinite parallel electrodes with two-layer dielectrics. The current growth is apparent for the conductivity $\gamma > 10^{-3}$ S/m.



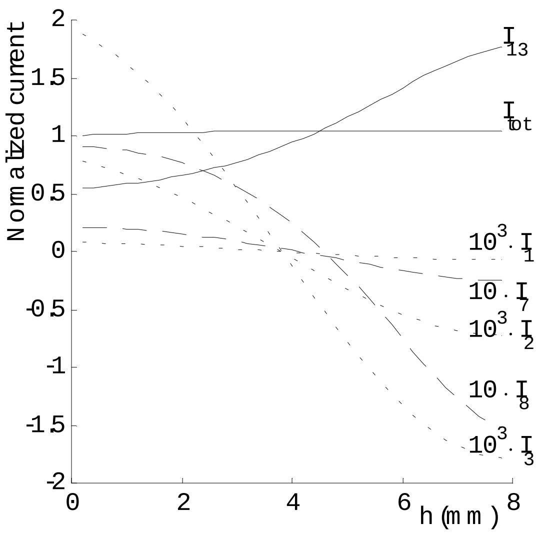


Fig. 2. Normalized currents of the electrodes when charge Q moves towards the center of the middle anode in the z direction. Here $\gamma = 10^{-6}$ S/m (the left figure) and $\gamma = 10^{-3}$ S/m (the right figure)

The currents of the pick-up anodes are normalized by being divided by the total theoretical current ([3])

$$I_{tot} = \frac{\varepsilon Q v}{\varepsilon(D-d)+d}$$

where ε is the relative permittivity of the panel.

The normalized currents in the symmetrical case ($x_Q = y_Q = 24.5$ mm, z between 0.2 and 7.8 mm) are in Fig. 2 as a function of z. Because of the symmetry, only currents of anode 1, 2, 3, 7, 8, 13 is depicted.

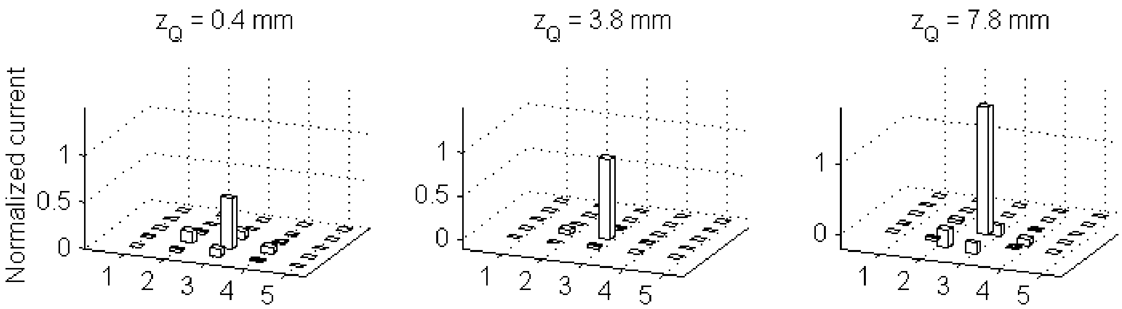


Fig. 3. The normalized currents on the anodes when charge Q moves towards the center of anode No. 8. (Unsymmetrical case)

The current distribution in an unsymmetrical case ($x_Q = 24.5$ mm, $y_Q = 14.5$ mm, z_Q as a parameter) is shown in Fig. 3. Here the charge moves against the center of anode 8.

5. CONCLUSION

Equations (5) to (8) formulate general problem of signals (currents) that are picked up on RPC's anodes with resistive coating. A program evaluating these currents has been written and verified.

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