USING OF REGRESSION METHOD TO CALCULATE TRANSFORMATION EQUATION OF INTEGRATED HUMIDITY SENSORS

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ABSTRACT
Results of experiments concerning humidity measurements have been presented in the paper. Integrated humidity sensors were used for the experiments, which transformed the value measured into a corresponding capacity. The structure of a measuring system was described, and the requirements concerning particular elements of the measuring track were defined. Using the regression method, an analytical equation was modelled, combining the output capacity value of the sensor with the examined humidity changes. With the use of the experimental results as well as the least square method, the values for the above equation’s parameters were determined. The confidence interval for each ordinate with the assumed probability 1- α in the examined range of humidity changes was also determined.

1. INTRODUCTION
Beside temperature and pressure, humidity forms a set of parameters which have a very relevant influence on many physical phenomena and technological processes. Hence the need to determine humidity. Humidity changes have an influence on operational reliability and restrictions of elements and devices. Therefore humidity measurement is an exceptionally important issue. Several main principles of humidity measurement may be distinguished. They have different applications and give information on different quantities expressing humidity. The most important principles are as follows [1]:
- removal of humidity from the air and the measuring of the quantity of water acquired in this way;
- balancing of water vapour contained in the air with the second phase, and the measurement of this state’s parameters;
- observation of temperature decrease, caused by water evaporation from a moistened substance into the outside air;
- changes in mechanical or electrical parameters of solid, influenced by relative humidity of the outside air.

2. HUMIDITY SENSORS
Now many humidity sensors are produced, which vary in working rule, resistance to outer conditions, measurement range, or errors. As exemplary humidity sensors, the following may be enumerated [2]:
- ceramics-based ERH300 humidity sensor, manufactured by Elmwood Sensor Inc. It is characterised by brief response time and good sensitivity concerning relative humidity changes in the range of 10% to 90%. The working of this sensor consists in generating impedance of resistance character at the output, as a property of dopped material located on the surface of aluminium oxide absorbing humidity. This sensor indicates relative humidity changes within seconds.
- HC1000 humidity sensor, manufactured by E&E Elektronik GmbH – works in the range of 0 and 100% of relative humidity, and in the temperature range between -40°C and +115°C. The sensor may be fully moistened and is resistant to pollution substances. It is manufactured in voltage and current configuration.
- relative FE09/2 humidity sensors, manufactured by MELA Sensortechnik GmbH, that went under examination. The structure of measuring circuit was described, the transformation characteristics of these sensors was determined and the results of model description of transformation equation of the sensor were presented.
3. EXPERIMENTAL RESEARCH

FE 09/2 sensors consist of a system of electrodes covered with a semiconductor protecting layer, located on ceramic base. Their working is based on the rule of output capacity $C$ change in the function of relative humidity $RH$. Basic parameters of these sensors are the following: measuring range - between 0 and 100% of relative humidity, working temperature – between -40°C and 140°C, basic capacity - (135±10) pF, response time – 10 s, linearity < ±2% of relative humidity.

The transformation characteristics of this sensor was determined on the measuring system shown in Fig.1.

![Fig.1 Scheme of measuring system to determine the transformation characteristics of FE09/2 humidity sensor](image)

A calibrated sensor was put into a hermetically closed vessel in the environment of vapours of saturated standard solution. The vapours of this solution transpired into the field where the examined sensor was located through semi-permeable membrane. Depending on the kind of solution used for the experiment, appropriate value of relative humidity was acquired. The measurement of sensor capacity $C$ dependent on humidity value $RH$, was performed with RLC - HP4263B digital meter, which error is equal to ±0.1% of the measured value. The temperature inside the vessel was measured with DTM 1010 digital thermometer. As sensor co-operating with the thermometer, a nickel-chromium thermo-element was used. In the temperature range considered, the temperature was measured with the error of $\Delta T = \pm (0.5K \times \text{digit})$, and the error concerns the digital thermometer together with the nickel-chromium thermo-element. Special thread connection between humidity and temperature sensors inside the vessel and the thermometer and the RLC meter prevents from the exchange of temperature and humidity with the outside environment. Statement of measuring results of capacity $C$ in the function of relative humidity changes $RH$, performed in the system shown in Fig.1, was presented in Table 1.

![Table 1. Statement of measuring results concerning the determination of transformation characteristics of FE 09/2 humidity sensors](image)

<table>
<thead>
<tr>
<th>Standard solution</th>
<th>relative humidity</th>
<th>sensor 1 capacity</th>
<th>sensor 2 capacity</th>
<th>sensor 3 capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium chloride LiCl</td>
<td>12%</td>
<td>137 pF</td>
<td>138 pF</td>
<td>138 pF</td>
</tr>
<tr>
<td>Magnesium chloride MgCl$_2$</td>
<td>33%</td>
<td>142 pF</td>
<td>143 pF</td>
<td>142 pF</td>
</tr>
<tr>
<td>Sodium chloride NaCl</td>
<td>75%</td>
<td>155 pF</td>
<td>156 pF</td>
<td>155 pF</td>
</tr>
<tr>
<td>Potassium chloride KCl</td>
<td>84%</td>
<td>159 pF</td>
<td>160 pF</td>
<td>160 pF</td>
</tr>
<tr>
<td>Barium chloride BaCl$_2$</td>
<td>90%</td>
<td>162 pF</td>
<td>163 pF</td>
<td>163 pF</td>
</tr>
</tbody>
</table>

All the measurements were conducted in temperature $T=25°C$. There are many possibilities of presenting the experimental data of Table 1 by means of an equation illustrating relations of the values of measurement-based variables. When selecting the shape of empirical equation that presents the experimental data, one should keep in mind two postulates. First – the equation should in the best possible way present the dependence among variables values that result from the measurement. Second – it should contain the least possible number of constants. Analysing the measurement results in Table 1, one can assume that it is possible to use linear regression to describe the dependence of the influence of relative humidity changes $RH$ on changes in output capacity $C$. 

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The regression equation is formulated in order to enable the prediction of the values of dependent variable \( C \), based on the values of independent variable \( RH \). This prediction is more or less accurate, according to lower or higher degree of dependence of both variables. The measure of the dependence between these variables is correlation coefficient. If it reaches values close to the boundary values \( \pm 1 \), one can infer that the dependence between the variables is strong, and that it is linear.

Based on the results of the experiment shown in Table 1, the estimators of the correlation coefficient \( r_{RH,C} \) of the examined indicators were determined. After the calculations, the result obtained was equal to \( r_{RH,C} = 0.996 \). Hence, a linear regression equation was accepted to describe the dependence between the variables \( RH \) and \( C \):

\[
C = \alpha_1 + \beta \cdot RH
\]

(1)

Estimators \( a \) and \( b \) of parameters \( \alpha_1 \) and \( \beta \) were determined from trial. Number values of these parameters were determined using the least squares method. According to the method, the parameter values were selected so in order to minimize the sum square of differences between observed values \( \cdot C \) and the estimate expected value \( \cdot C = a + b \cdot RH \). Parameters values \( a \) and \( b \) were determined using the computer program, hence the empirical regression equation was assumed:

\[
\cdot C = 133.38 + 0.32 \cdot RH
\]

(2)

Fig. 2 presents the experimental characteristic \( C = f(RH) \) for sensor 2, in the form of measuring points, and transformation characteristic of the same sensor in the form of regression line with the use of equation (2). Assuming the normal distribution of variable \( C \) with the given value of variable \( RH \), it is possible to prove [3] that parameters \( a \) and \( b \) are unbiased estimators \( \alpha_1 \) and \( \beta \) respectively, and their distribution from the trial is related to Student’s distribution \( t \) for \( k = n - 2 \) degrees of freedom. This enables to determine confidence intervals for \( \alpha_1 \) and \( \beta \) as well as for the expected value \( E(C) = f(RH) \) for the determined value \( RH = RH_i \). Making the calculations for many values of \( RH \), we obtain the confidence intervals that were determined according to dependences [4]:

\[
P(a - k_a < \alpha_1 < a + k_a) = 1 - \alpha
\]

(3)

\[
P(b - k_b < \beta < b + k_b) = 1 - \alpha
\]

(4)

\[
P\left(\frac{\cdot C - k_\cdot}{\cdot C} < E(C) < \frac{\cdot C + k_\cdot}{\cdot C}\right) = 1 - \alpha
\]

(5)

where:

\[
k_a = t_{\alpha k} \cdot S_{\cdot C} \sqrt{\frac{1}{n} + \frac{\bar{RH}^2}{\sum_{i=1}^{n} (RH_i - \bar{RH})^2}}
\]

(6)
\[ k_b = t_{qk} \cdot S_c \frac{1}{\sqrt{\sum_{i=1}^{n} (RH_i - \overline{RH})^2}} \]  \hspace{1cm} (7)

\[ k_c = t_{qk} \cdot S_c \frac{1}{n} \frac{1}{\sqrt{\sum_{i=1}^{n} (RH_i - \overline{RH})^2}} \]  \hspace{1cm} (8)

\[ S_c = \sqrt{\frac{\sum_{i=1}^{n} [C - (a + b \cdot RH)]^2}{n - 2}} \]  \hspace{1cm} (9)

\( S_c \) - standard deviation of the trial of the observed values \( C \) in relation to \( C \), \( t_{qk} \) - standardized variable of Student’s distribution \( t \), read out from the Student’s distribution tables for the confidence interval \( 1 - \alpha \) and the number of freedom degrees \( k = n - 2 \).

Having computed the values of \( C \), a confidence interval for each ordinate \( C_i \) with the assumed probability \( 1 - \alpha \) may be determined, based on the dependence (5). Using the dependences (5), (8), and (9) number values of the confidence interval for the transformation characteristic of sensor 2 with probability \( 1 - \alpha \) were determined. A statement of computation results is given in Table 2.

Table 2 Confidence interval values for transformation characteristic of sensor 2, determined with probability \( 1 - \alpha = 0.95 \)

<table>
<thead>
<tr>
<th>Relative humidity</th>
<th>Sensor capacity</th>
<th>Determined confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>pF</td>
<td>pF</td>
</tr>
<tr>
<td>12</td>
<td>138</td>
<td>130,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>142,9</td>
</tr>
<tr>
<td>33</td>
<td>143</td>
<td>138,6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>147,6</td>
</tr>
<tr>
<td>75</td>
<td>156</td>
<td>152,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>159,9</td>
</tr>
<tr>
<td>84</td>
<td>160</td>
<td>154,3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>163,2</td>
</tr>
<tr>
<td>90</td>
<td>163</td>
<td>155,7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>165,5</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

Static transformation characteristics of humidity sensors FE09/2 were determined on a designed and performed measuring position. The measurement results obtained, set up in Table 1 and presented for an exemplary sensor in Fig. 2, testify to a good linearity of the sensors examined. It gave reason to accept the linear pattern for the description of static transformation equation of the sensors presented. The confidence intervals determined show a good accuracy of the model in the form of the regression line, expressing the dependence of output sensor capacity \( C \) in the function of changes in relative humidity \( RH \).

5. REFERENCES