Problems concerning the measurement of form profiles of non-closed cylindrical surfaces of machine parts

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Abstract: *The measurement methods applied so far have concerned full (closed) profiles. However, in a number of cases the contact surfaces are not full and their profiles determined in appropriate cross-sections are non-closed. Specialised instruments are used for such measurements. Of particular interest are those constructed mainly for the needs of the bearing industry and used to measure the quality of rolling surfaces of rings. In such cases, a correct measurement requires selecting an appropriate reference circle, in relation to which the deviation of the analysed non-closed profile will be determined. The paper deals with a theoretical analysis of the applied reference circles, on the basis of which it was found that the deviation determined for particular reference circles can differ considerably. The work discusses also the theoretical determination of a reference circle, which will lead to the computerisation of measurements of non-closed profiles.*

Key words: *roundness, non-closed profiles, reference circle*

1. Introduction

An important problem of basic experimental research and, in consequence, industrial practice is the necessity to regularly measure and assess the geometrical surface structure (GSS) of elements of machine parts, because it has a great influence on the dynamic state of mating elements within particular assemblies of ready products [1]. Therefore, the major aim of the engineering industry, particularly precision industry, is to minimise the occurrence of some particular cases of surface irregularities, such as form deviation, waviness, or surface roughness. This is particularly important for elements assuring the rotary motion, such as rolling and slide bearings, or machine tool spindles, for non-rotating elements with linear displacements, such as engine pistons, and also for elements of machine tools moving along guides. In the majority of cases, the mating elements have cylindrical surfaces, which can be treated as closed surfaces (e.g. the journal and pan of a slide bearing, or the piston and the cylinder of a combustion engine).

However, frequently, the mating surfaces are not full and their profiles determined in certain cross-sections are non-closed, like in the case of associated elements of ball-shaped joints, some types of guides, splined connections or rolling bearings.

Measurement of non-closed profiles with curvilinear surfaces is at the early stage of development, which was possible thanks to the dynamically developing co-ordinate measuring technology applying special impulse or scanning measuring heads [2]. The optical technology has an important impact too. The methods of discrete image processing enable the evaluation of profiles of any nominal form [3]. The problem refers also to the measurement of the form profiles of non- closed surfaces, whose obtained profile in cross sections has a nominal profile being a circular sector or some other curvilinear line. For this type of measurement, the profilographometric instruments, the so-called formographs, are applied because of their universal character. However, they may be applied only to small machine parts. This group of instruments produced for a dozen years include, e.g.:

- Form Talyrond (made by Taylor Hobson from Great Britain)
- Conturoskop (made by Perthen from Germany)
- Kształtograf PG-2/200 (made by IOS from Cracow, Poland).

2. The significance of measurements of non-closed profiles used in the bearing industry

The specialist instruments are also used. They are developed mainly for the needs of the bearing industry and are used for measurement of quality of rolling bearing surfaces. This group of instruments includes e.g. Rotary-Talysurf, produced for quite a long period of time by Taylor-Hobson, and Talytrac by the same manufacturer. The practice shows that one of the most important parameters of a mounted bearing is the level of vibrations, which allows a quantitative analysis of its vibrations. Errors of the race profile and the rolling elements (such as balls or rollers) prove to have the most essential, basic influence on the occurrence of vibrations in a rolling bearing. They are deviations of the real profile of active surfaces of rolling bearings from the nominal profiles determined by such reference elements as the circle, cylinder or sphere. The deviations necessitate reciprocal displacements of component parts of a bearing during operation and are a direct cause of a vibration of the rolling bearing. Thus, the problem of accuracy in the manufacturing of particular rolling elements is extremely important. This refers particularly to the mated elements of ball bearings, in which balls move along the races of the inside and outside ring. To assure appropriate mating of the ball with the ring race, the balls must form a closed circular cylindrical surface that is not full and is called a cylindrical sector.

Also, to assure good quality mating of balls with bearing races, it is necessary to measure the form profile and surface waviness of the whole race. The measurement must be made in the plane perpendicular to the ring symmetry axis (Fig. 1) so as to determine roundness profile or surface waviness for a closed profile. Such measurements are made with the above-mentioned Talyrond-type non-reference instruments. However, the balls may move all over the race, so it is essential that measurements should be made in the planes intersecting the ring symmetry axis (Fig. 2). As a result, one obtains a non-closed profile, whose nominal or the so-called adjacent element will be a circular sector determined by the value of the radius r . The obtained profile becomes the basis for the evaluation of accuracy as for the manufacturing of bearing races by determining the deviation of a race profile, which is an important parameter of correctness of the manufacturing of a bearing ring according to the adopted branch standards. Up to the present, measurement of this deviation for a race profile has been made in laboratory and industrial conditions by means of the above-mentioned Rotary-Talysurf instruments, which are adjusted to measurement of races for bearings with a diameter of up to 70 mm. For bigger rings, the Polish bearing industry applies the MDL-27 instruments produced by FŁT "Kraśnik" in Kraśnik.

In the instrument, the tips of the measuring sensor move in accordance with changes in the value of the bearing race radius. The obtained electrical signal, transformed and amplified, makes it possible to register the measured form profile of the race in the form of a diagram in rectangular co-ordinates. Additionally, after selecting an appropriate measuring section, it is possible to measure the surface roughness of the race applying the parameter R_a determined by means of a traditional indicating instrument. The obtained in rectangular co-ordinates form profile of the race will enable determining the form deviation (see Fig. 3).

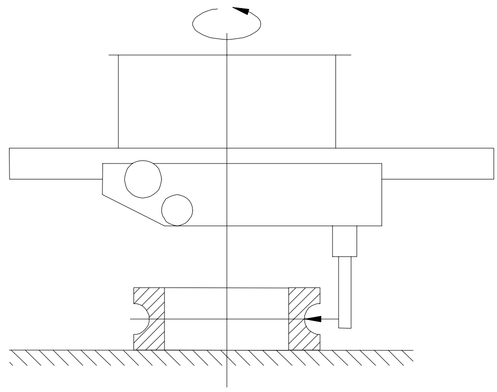

Fig. 1. Principle of roundness measurement made in the plane perpendicular to the ring symmetry axis.

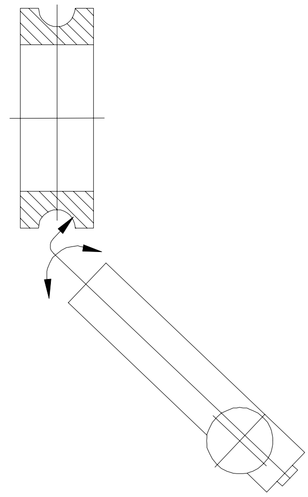

Fig. 1. Principle of bearing raceway roundness measurement made in the plane intersecting the ring symmetry axis.

However, a correct measurement with such instruments requires accurate setting of the object on the measuring table in relation to the rotation axis of the measuring sensor as well as precise selecting of the angle and radius of rotation for the measuring sensor. Additionally, while a measurement is made, it is necessary to manually change the amplification and the determination of a profile deviation on the basis of the diagram of a form profile registered with little accuracy is time-consuming and may contain gross errors. The measurement is therefore a dull activity of little efficiency. The BEFORM computer program has been developed in order to modernise the instrument for non-closed profile measurement.

3. Reference circles for non-closed profiles

A form profile can be evaluated in relation to various reference elements. A deviation of a closed roundness profile can be determined, for example, in relation to the mean square circle, the least circumscribed circle, the greatest inscribed circle or the minimum zone circles. In the case of non-closed profiles with small values of the central angle, the circle circumscribed about a profile or the circle inscribed of a profile cannot be defined correctly. The BEFORM program enables evaluation of a profile only in relation to the mean square circle and the minimum zone circles. The program applies also a reference circle called a three-point circle defined according to the bearing industry manufacturing standards.

If a deviation of a roundness profile is much smaller than the real dimensions of the measured object, then the equation of a reference circle in the polar system will be

$$R_n(\varphi, a, b, r) = r + a\cos\varphi + b\sin\varphi,\ \beta_1 \le \varphi \le \beta_2,$$

where r is the radius of the circle, a and b are the parameters of the centre, and, finally, β_1 and β_2 are the angular co-ordinates of the starting and end points of the profile. Without losing generality, we assume that $\beta_1 = -\alpha,\ \beta_2 = \alpha.$. The minimum zone circle will be the circle adjacent to the profile so that the condition $R_n(\varphi, a, b, r) \ge R(\varphi), \forall\varphi \in [-\alpha, \alpha]$ is satisfied and the roundness deviation of the profile.

$$\Delta R(r,a,b) = \max_{\varphi\in[-\alpha,\alpha]} (R(\varphi) - R_n(\varphi, r, a, b)) - \min_{\varphi\in[-\alpha,\alpha]} (R(\varphi) - R_n(\varphi, r, a, b))$$

achieves minimum. The minimum zone circle parameters are defined by the relation

$$(a,b) = \arg\min_{(a,b)} \Delta R(0,a,b).$$

The task to determine the parameters of the reference circle centre is a task of non-linear programming with a non-differentiable objective function. In the program, the problem is solved using the Nedler-Mead algorithm. The determination of the optimal values of the parameters a and b with an accuracy of 0.001 μm requires that about 50 iterations of the algorithm should be carried out.

The mean circle is one for which the sum (or integral) of the squares of the profile distance from the circle reaches minimum. Thus

$$(r,a,b)_{LS} = \arg\min_{(r,a,b)} \int_{-\alpha}^{\alpha} (R(\varphi) - R_n(\varphi, r, a, b))^2 d\varphi \cdot$$

The determination of the mean square circle is considerably less troublesome because the problem can be solved analytically from the necessary conditions of optimality. By comparing to zero the partial derivatives of the integral after r, a and b, we obtain

$$\begin{bmatrix} \alpha + \sin 2\alpha & 0 & 2\sin\alpha \\ 0 & \alpha - \sin 2\alpha & 0 \\ 2\sin\alpha & 0 & 2\alpha \end{bmatrix} \cdot \begin{bmatrix} a_{LS} \\ b_{LS} \\ r_{LS} \end{bmatrix} = \begin{bmatrix} \int_{-\alpha}^{\alpha} R(\varphi)\cos\varphi d\varphi \\ \int_{-\alpha}^{\alpha} R(\varphi)\sin\varphi d\varphi \\ \int_{-\alpha}^{\alpha} R(\varphi) d\varphi \end{bmatrix}.$$

It should be noted that the determination of the reference circles defined above is possible only by using computer-aided measuring devices. In the bearing industry, a deviation was determined from the reference circle going through the profile centre and both ends of a profile. This method of determination of a deviation is used in the BEFORM program and the reference circle defined in this way is called a three-point circle. By definition, the parameters of the three-point circle satisfy the equation

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 1 \\ \cos\alpha & \sin\alpha & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_{3P} \\ b_{3P} \\ r_{3P} \end{bmatrix} = \begin{bmatrix} R(-\alpha) \\ R(\alpha) \\ R(0) \end{bmatrix}$$

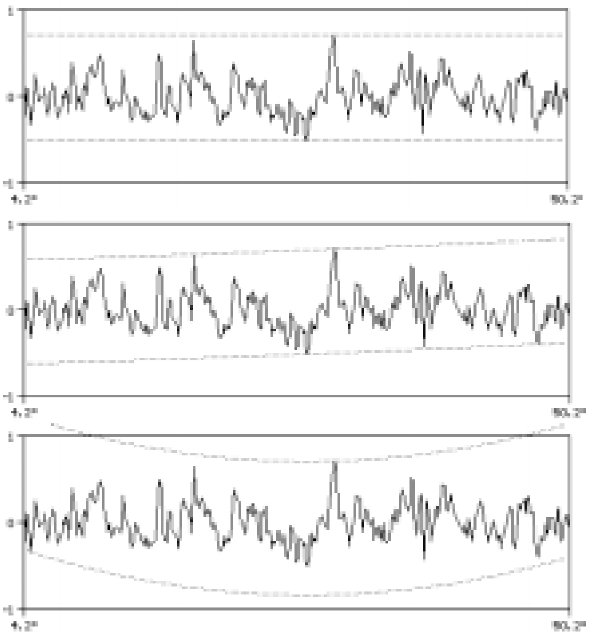

Fig. 3. Roundness deviation of a sample profile analysed by means of the mean square circle, the minimum zone circle and the three-point circle.

As shown in Fig. 3, a roundness deviation can be determined by applying one of the three methods discussed above. In all the three graphs, the line $\varphi = 0$ corresponds to the mean square circle. In the figures, we can see the circles circumscribed about and inscribed of a profile taking into account the centre of an appropriate reference circle. In the example, the roundness deviations for the mean square circle, the minimum zone circle and the three-point circle are $\Delta R_{LS} = 1.22$, $\Delta R_{MZ} = 1.21$, $\Delta R_{3P} = 1.54$ respectively.

However, the application of the three-point circle may have some disadvantages, e.g. a big scatter of the value of the deviation for irregular profiles, such as those shown in Fig. 3. If a component of profile waviness is great, the parameters of the three-point circle, and therefore, the roundness deviation are very much dependent on the co-ordinates of the starting and end points of the profile defined by an operator. In the example shown above, slight changes of these values cause changes of the deviation ΔR_{3P} reaching even 50% of the value of the deviation ΔR_{MZ}.

There is a general opinion that for typical profiles, the mean square deviation can be a dozen or several dozen per cent greater than the minimum zone deviation. The example provides evidence that it is alike with the three-point deviation. The interesting point here is the question how many differences one can expect at worst case in the evaluation of a roundness deviation by means of various reference circles. The problem was analysed in detail in [4] for closed roundness profiles and results for (2 and 3D) straightness, flatness, cylindricity and roundness profiles were given. After studying the results, one can see that the mean square deviation may be even 2 to 3 times greater than the minimum zone deviation dependent on the type of a form profile. An analogous result for a roundness deviation of non-closed profiles is provided in the theorem below.

Theorem: *Let ΔR_{MZ}, ΔR_{LS} and ΔR_{3P} denote the minimum zone deviation, the mean square deviation and the three-point deviation respectively for a non-closed roundness profile with a central angle equal to $2\alpha \le \pi$ Then the following inequalities occur*

$$\Delta P_{LS} \le c_1 \Delta P_{MZ}, \quad c_1 = 1 + \frac{4(1-\cos\alpha)\sin\alpha}{2\alpha - \sin 2\alpha}$$

$$\Delta P_{3P} \le c_2 \Delta P_{MZ}, \quad c_2 = 2 + \frac{1}{4(1+\cos\frac{\alpha}{2})\cos\frac{\alpha}{2}}$$

The obtained inequalities are the best possible in the sense that the coefficients c_1, c_2 on the right side of the inequalities cannot be replaced by smaller numbers.

It should be emphasised that the Theorem 1 shows only the biggest difference that may occur for various definitions of deviations without considering the question whether the analysed profiles have any practical application. It might be essential also to find out how big differences can be expected on average in practice e.g. what is the distribution of the quotient $\Delta R_{LS} / \Delta R_{MZ}$ for certain objects made in a defined technological process. The problem, though very interesting, requires a number of statistical investigations and has not been discussed here.

5. Conclusion

In the majority of measuring instruments, the reference circles for non-closed profiles representing circle segments have been determined using the three-point method. The measurement has been carried out graphically by applying a registered profile of a bearing track. The results, however, have always contained considerable accidental errors and sometimes even excessive gross errors. Besides, while determining the basic parameter, i.e. the so-called profile deviation, a considerable pooled error has been observed. Therefore, investigations into the above mentioned drawbacks of the existing measuring instruments have been commenced.

The aim was to provide a theoretical analysis of the reference circles used for non-closed profiles. It was found that the selection of appropriate reference circles influences the value of the determined deviation of the non-closed profile.

6. References

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