OPTIMISATION OF NMR COILS BY GENETIC ALGORITHMS

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Abstract

Several variants of genetic algorithms are tested assuming their use for magnetic field homogeneity of an NMR coil optimisation. A new genetic operator of modified elitism has been developed basing on results of the tests. Magnetic field of an RF coil is calculated, results of the calculation are verified by measuring and genetic algorithm with the new genetic operator is applied on magnetic field homogeneity optimisation. The optimisation of the coil with genetic algorithms is compared to optimisation by randomised deterministic optimisation method and their qualities are discussed. The optima calculated by the both optimisation methods are depicted with contour plots in order to prove importance of the optimisation.

Keywords: optimisation, genetic algorithm, randomisation, magnetic field homogeneity.

1. Introduction

Magnetic field homogeneity is a significant parameter of RF coils for NMR tomograph. For achieving higher quality demands it is usually optimised. However, when an object function for magnetic field homogeneity optimisation is created, it can be found out that it is multimodal one (with more optima).

Stochastic optimisation methods are the most convenient tool for multimodal functions optimisation, mainly genetic algorithms. Genetic algorithm is able to find global optimum of a multimodal function. It needs no start point of calculation, no derivatives, no gradient, even the optimised function does not have to be continuous in the whole search space. Genetic algorithm uses various genetic operators for searching and its efficiency depends on them significantly.

Recent papers on magnetic field optimisation with help of genetic algorithms have dealt with gradient [1] and RF [2] coils for NMR tomograph optimisation. Magnetic field of coil calculated according to an appropriate algorithm is optimised by generally described genetic algorithm. Although the structure of genetic algorithm is widely known, it is possible to develop genetic operators, increasing its efficiency for particular tasks, easing their solving.

The purpose of this paper is to examine the optimisation of NMR coils by genetic algorithms, to present a new genetic operator of modified elitism and to compare efficiency of genetic algorithm with such operator versus some known genetic operators. Magnetic field homogeneity of a simplified RF coil (infinitely long saddle-shaped coil) for NMR tomograph is optimised by genetic algorithm containing the new operator. The results are verified by optimisation with randomised deterministic method and qualities of the both optimisations are compared. The calculated magnetic field of the idealised coil is graphically depicted and compared with values measured on its simplified realisation. Results of optimisations are depicted as magnetic field contour plots and significance of the optimisation is proved in this manner.

2. Genetic algorithms

Object function for magnetic field homogeneity optimisation can have more optima and one of them is the global. Deterministic methods with result depending on their start point mostly search quickly but the found optimum many times is only local one. Searching with stochastic optimisation methods takes a longer time but their result does not depend on the start point or initial guess and the found optimum is prevailingly global one. Genetic algorithms [3], [4], [5] emulating evolution process in the live nature are the most efficient stochastic optimisation method. Different variants of genetic algorithms provide with different effectiveness. It depends also on the implemented genetic operators and on parameters setting.

2. 1. Tests of genetic algorithms

The following tests have been performed to judge the power of different genetic operators. However the object function for the magnetic field homogeneity of the investigated coil is not appropriate for testing, because the evaluation of its function values takes a long time. Therefore the function [6] (Fig. 1.)

$$f(x) = e^{0.001x} \cdot (0.5 + \cos^2 x) \tag{1}$$

was used for testing, supposing that it is similar to the coil object function (basing on authors' experiences).

Each test consisted of 1,000 trials, during that the test function was optimised with genetic algorithm of a particular structure. Whereas the global minimum of the test function
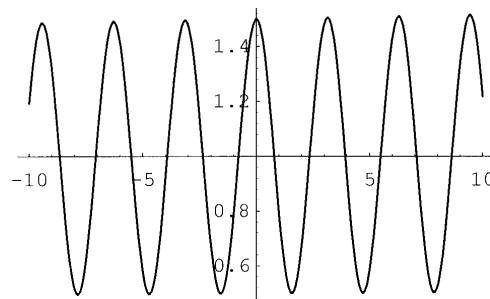

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{0.496088, {x -> -7.85423}}
{0.497649, {x -> -4.71264}}
{0.499215, {x -> -1.57105}}
{0.500786, {x -> 1.57055}}
{0.502362, {x -> 4.71214}}
{0.503942, {x -> 7.85373}}
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Fig. 1. Graph of test function with the calculated minima.

is known results of the trials have been scored as correct and wrong and probability of successful run p(suc) has been recalculated after each trial:
p(suc)=(number of correct calculations) / (total number of calculations).

Fig. 2. shows successions p(suc) varying as number of trials for genetic algorithms containing different genetic operators. First only the basic genetic operators of one-point crossover and mutation (course (1)) have been used. After 1,000 trials the genetic algorithm achieved the value p(suc)=0.686. Chromosomes were evaluated by the binary standard code.

Parameters: p_{cros} is probability of crossover, p_{mut} is probability of mutations and k is length of binary string, representing chromosome.

Course (2) shows succession p(suc) for the same experiment, but two-point crossover was used. The value achieved after 1,000 trials was p(suc)=0.744. The difference between one-point and two-point crossover is not very significant for this object function and the set parameters.

The succession, depicted by course (3) shows results for the same genetic algorithm, but the binary Gray code was used for chromosomes evaluation. The value achieved after 1,000 trials p(suc)=0.964. The time of optimisation was a little longer for the Gray binary code had to be recalculated into the standard binary code by the program.
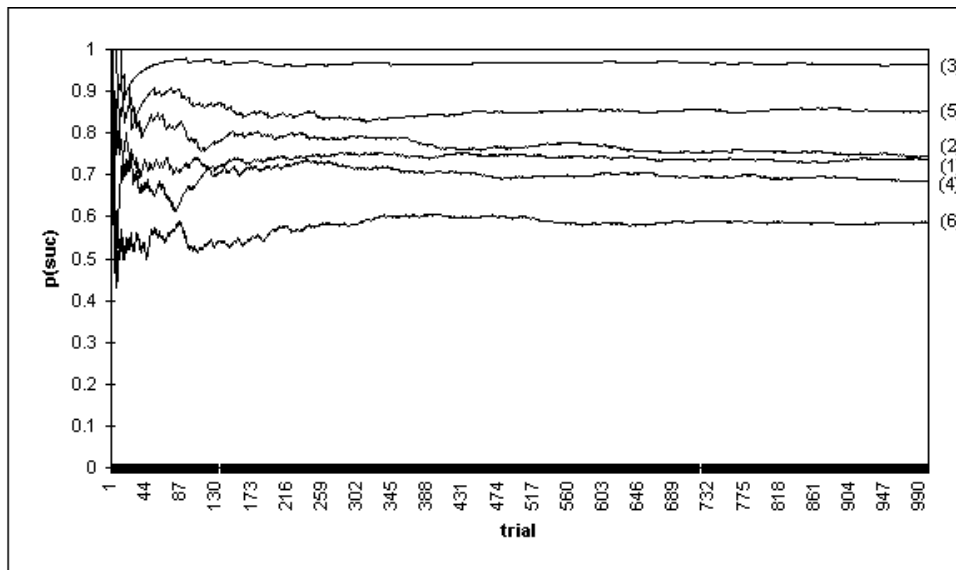

Fig. 2. Graph displaying sequences of successful run probability varying as number of trials for genetic algorithms with different genetic operators. For basic genetic operators (one-point crossover) and standard binary coding of chromosomes (course (1)) achieved p(suc)=0.686.
The same with two-point crossover (course (2)): p(suc)=0.744.
Basic genetic operators (one-point crossover) and binary Gray code coding of chromosomes (course 3)): p(suc)=0.964.
Basic genetic operators (one-point crossover), standard binary coding of chromosomes and operator of inversion (course (4)): p(suc)=0.736.
Basic genetic operators (one-point crossover), standard binary coding of chromosomes and operator of standard elitism (course (5)): p(suc)=0.852.
Basic genetic operators (one-point crossover), standard binary coding of messy chromosomes (course (6)): p(suc)=0.586.
Parameters: $p_{cross} = 0.75$; $p_{mut} = 0.008$; ($p_{inv} = 0.001$; for inversion, $p_{index} = 0.1$; $p_{value} = 0.9$; for messy chromosomes mutations), k=18; 200 chromosomes / 50 generations.

A good way how to overcome so called Hamming cliffs, occurring if binary standard code is used for chromosomes evaluation, is using the operator of inversion. Results of optimisation by genetic algorithm, containing the operator of inversion are depicted by course (4). The value achieved after 1,000 trials was p(suc)=0.736. It is not very high probability and further attempts showed that the value could be improved by increasing the time of calculation (using more generations for each trial) but the shorter time of calculation is the

main advantage versus Gray binary code, because Hamming cliffs do not occur if Gray binary code is used. Parameter p_{inv} is probability of inversion.

Course (5) in Fig. 2. shows p(suc) varying as number of trials for genetic algorithm with the genetic operator of the standard elitism, where the best one chromosome survives into the next generation, where it replaces the weakest chromosome. This genetic algorithm contains in addition to the genetic operators of one-point crossover and mutation also the operator of the standard elitism. The value p(suc)=0.852 achieved with the binary standard code after 1,000 trials can be even improved by using the binary Gray code for chromosomes evaluation.

Genetic algorithm with messy chromosomes [5] is a variant developed for deceptive functions optimisation, ie object functions with more minima with Hamming cliffs among them. Course (6) shows succession of p(suc) for genetic algorithm with messy chromosomes, varying as number of trials. The value achieved after 1,000 trials p(suc)=0.586 is not very high but if the algorithm found the global minima, it found it very quickly. (The results of the tests can be rather different for a different test function and parameters settings.) The reasons were probably high value of probability of value mutation p_{value} (chromosomes consisted of pairs: index+value and each term of the pair was muted with its own probability p_{index} and p_{value}) and using template: the best chromosome from the previous generation. That gave orientation to the next studies. It must be stressed that the above tests results are valid for the selected conditions. They may be rather different for a different test function and for different genetic algorithms parameters settings.

2. 2. Genetic algorithm with modified elitism

The previous attempts revealed that the ability of genetic algorithms to find the global optimum can be increased by setting higher value of mutation probability p_{mut}, using
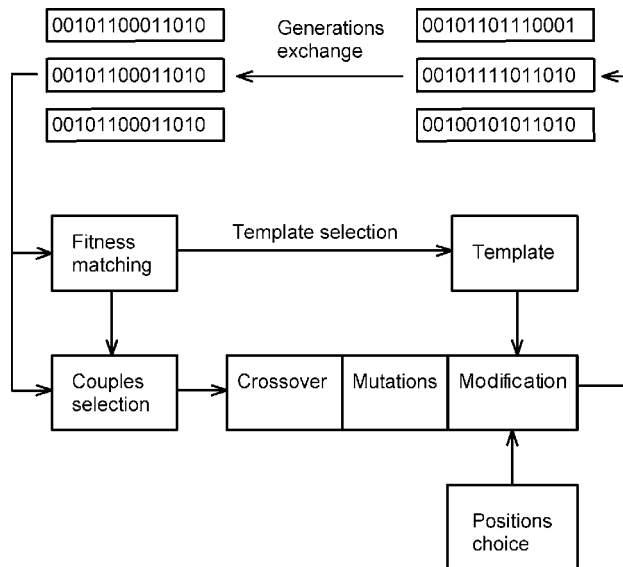

Fig. 3. Genetic algorithm with the operator of the modified elitism

the best chromosome of the generation as a template. This modified elitism (Fig. 3.) can be described as follows: Population consists of chromosomes, created by binary strings of the length k, determined by the applied accuracy of variable approximation. After evaluating the

function values of object function in each generation, the best chromosome is selected as a template and after reproduction consisting of crossover and mutations a set of k_{max} integers with values from 1 to k is randomly generated for each chromosome of the population. Whereas some integers can be generated more times the cardinality of such set is almost always less than k_{max}. After removing multiple elements of the particular set belonging to the particular chromosome the integers determine positions of the chromosome where its information is replaced by information of the template. After repeating the operation for all chromosomes of the population this new generation replaces the old and its evaluation follows

and a new template is selected. For chromosomes evaluation the binary standard code as well as the binary Gray code can be used. Whereas crossover and mutations ensure global searching of the whole space, the described modified elitism simultaneously with high probability of mutations p_{mut} provides searching in the local surroundings of template, formed by the best chromosome of the population in current generation. The value of the probability of successful run for genetic algorithm, which together with genetic operators of one-point crossover and mutation contains also the operator of the modified elitism converged to the value p(suc)=0.971 after 1,000 trials with binary standard code for chromosomes evaluation and can be further improved by using the binary Gray code. For example genetic algorithm with modified elitism and Gray binary code for chromosomes evaluation achieved after 1,000 trials p(suc)=0.999, it means only one failure during the experiment. Co-operation of the modified elitism with standard elitism yielded p(suc)=0.934 after 1,000 trials. It is the value worse than for genetic algorithm only with the modified elitism but better than for genetic algorithm only with the standard elitism, moreover the genetic algorithm became monotonous.

3. Homogeneity of coil magnetic field

3. 1. Calculation of magnetic field

The magnetic field of a simplified coil was selected: infinitely long, infinitely thin conductors on surface of infinitely long cylinder. Despite of its simplification, the subject is a theoretical platform for many RF coils of NMR tomograph consisting of sets of symmetrical conductors quaternions (eg long saddle-shaped coils). Fig. 4. displays symmetric placing 4 conductors on surface of cylinder and the currents orientation.

In the next analysis we suppose two conditions:

a) In a rectangular co-ordinate system (x,y) the conductors are perpendicular to the (x,y) plane.

b) For simplicity the lengths of the conductors are supposed to be infinite in comparison with the investigated region neglecting the diameter of conductors and the influence of supply-wires.

According to Fig. 4. we suppose four conductors symmetrically placed on the surface of a cylinder drawn as four points on the circle with the left-right symmetry. Conductors are fed by currents +I, -I. The stationary magnetic field $\mathbf{B}_o$ orientation is parallel to the conductors.

Let us investigate the magnetic field strength $\mathbf{H(m)}$ in a point M represented by a position vector $\mathbf{m}$=x+$\mathbf{j}$y. The position of wires is determined by vectors $\mathbf{v}_i = a_i + \mathbf{j}b_i$. Then the general expression for the magnetic field in the point M supposing $\mathbf{m} \neq \mathbf{v}_i$ according to [7] is given by the equation

$$\mathbf{H(m)} = \mathbf{j}\frac{1}{2\pi}\sum_{i=1}^{n}\frac{I_i}{\mathbf{m}^* - \mathbf{v}_i^*}, \tag{2}$$

where $\mathbf{m}^* = x - \mathbf{j}y$, $\mathbf{v}_i^* = a_i - \mathbf{j}b_i$.

Supposing the conductor directions of the currents +I and -I according to Fig. 4. respecting the symmetry and position vector configuration for a quaternion of conductors:

$$\mathbf{v}_1^* = \mathbf{v}^*, \mathbf{v}_2^* = -\mathbf{v}, \mathbf{v}_3^* = -\mathbf{v}^*, \mathbf{v}_4^* = \mathbf{v},$$
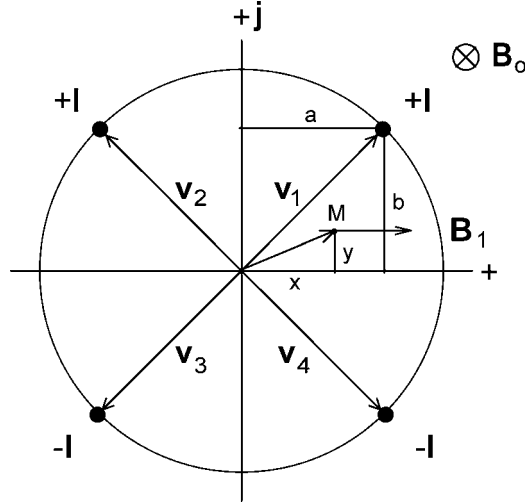

Fig. 4. A symmetric quaternion of conductors placed on a cylinder, supplied by currents +I, -I is generating the RF field in the central region. The field $\mathbf{B}_1$ in the point M is calculated.

we can write for the magnetic field strength of system with p conductors quaternions:

$$\mathbf{H(m)} = \mathbf{j}\frac{1}{2\pi}\sum_{k=1}^{p}\left[\frac{I}{\mathbf{m}^* - \mathbf{v}_k^*} + \frac{I}{\mathbf{m}^* + \mathbf{v}_k} + \frac{-I}{\mathbf{m}^* + \mathbf{v}_k^*} + \frac{-I}{\mathbf{m}^* - \mathbf{v}_k}\right]. \tag{3}$$

After algebraic transformation we get the general equation for magnetic field strength in the position M in the form:

$$\mathbf{H(m)} = \mathbf{j}\frac{I}{\pi}\sum_{k=1}^{p}\left[\frac{\mathbf{v}_k^*}{\mathbf{m}^{*2} - \mathbf{v}_k^{*2}} + \frac{-\mathbf{v}_k}{\mathbf{m}^{*2} - \mathbf{v}_k^{2}}\right]. \tag{4}$$

By substitution: $\mathbf{m}^* = x - \mathbf{j}y, \mathbf{v}_k^* = a_k - \mathbf{j}b_k, \mathbf{v}_k = a_k + \mathbf{j}b_k$,

to the equation (4) we get Eq. (4) in the form of a function of the complex variable as:

$$\mathbf{H(m)} = H_{\text{Re}}(x, y) + \mathbf{j}H_{\text{Im}}(x, y). \tag{5}$$

Supposing the direction of the stationary magnetic field $\mathbf{B}_o$ according to Fig. 4 and physical principles of NMR, it is evident that only the real part of the Eq. (5) should be considered. In this manner the magnetic field for 16 conductor coil ($p=4$) was calculated (Fig. 5.).

The magnetic field was calculated for idealised case (infinitely thin, infinitely long conductors on surface of infinitely long cylinder), measured values were acquired from simplified realisation of the coil (length of the coil $l \cong 2r$, diameter of the wires $\phi \cong 2\,\text{mm}$, ends of coil connecting particular wires into series influenced the resulting magnetic field as well). Therefore the measured magnetic field is rather different from the calculated but the main trend is apparent and the difference has not exceeded 7%.
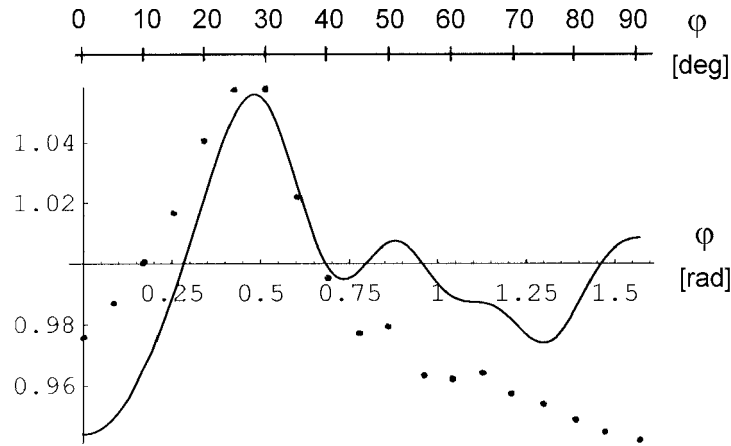
Fig. 5. Magnetic field of the coil with 16 conductors (p=4) placed on cylinder surface calculated and measured on the circular line with $r_1 = 0.75829r$ (r is the radius of the cylinder). Parameters of the coil: $a_1 = 0.10453r$; $a_2 = 0.37461r$; $a_3 = 0.60876r$; $a_4 = 0.84151r$; length of the coil $l \cong 2r = 211\,\text{mm}$. Full line: calculated and unified magnetic field; dots: in the middle of the coil measured and unified values.

3. 2. Optimisation of magnetic field homogeneity by genetic algorithms

The object function for magnetic field homogeneity optimisation was selected as a mean square deviation (or norm) of the magnetic field at circular line with radius r_1 versus the magnetic field in the point (0,0):

$$\left\| \mathbf{H}(r_1,\varphi) - \mathbf{H}(0,0) \right\| = \sqrt{\int_0^{\pi/2} \left| \mathbf{H}(r_1,\varphi) - \mathbf{H}(0,0) \right|^2 d\varphi} , \qquad (6)$$

where φ is the variable of the polar co-ordinates.

The object function was optimised by genetic algorithms with standard and with modified items of elitism. Chromosomes were coded with the binary Gray code. The following 12 runs have been executed: 3 runs of the GA with the standard elitism: 5,000 chromosomes / 16 generations; 3 runs of the GA with the modified elitism: 5,000 chromosomes / 16 generations; 3 runs of the GA with the standard elitism: 5,000 chromosomes / 45 generations; 3 runs of the GA with the modified elitism: 5,000 chromosomes / 45 generations. The object function Eq. (6) has at least two minima according to this analysis. Although the calculated values of them were accurate enough for technical practice they were recalculated by a gradient deterministic method, using the gradient method of Mathematica [8] with the results of the genetic algorithms runs as start points. The found minima are as follows (I=1, r=1, $r_1 = 0.7$):

a) variable no. 1: $\{a_1, a_2, a_3, a_4\}$={0.865971, 0.602245, 0.371815, 0.117363};
minimum1=0.0258517;
b) variable no. 2: $\{a_1, a_2, a_3, a_4\}$={0.998803, 0.779676, 0.457018, 0.156947};
minimum2=0.0241815;
The minimum (b) is the global and it has been found out only in one run of all optimisation runs. During the other 11 runs the minimum (a) have been calculated. It was very difficult to judge whether the genetic algorithm with the modified elitism was better than that with the

standard elitism for the coil optimisation, it would need much more trials but there was the lack of time.

3. 3. Optimisation of magnetic field homogeneity by randomised deterministic method

Optimisation of a multimodal object function by randomised deterministic method is another possibility how to discover the global optimum. The essence of randomisation is repeating optimisation with a deterministic method with random start points. It can be said generally that for a simple object function with few optima, a randomised deterministic optimisation method is the most appropriate. Genetic algorithm would take useless long time investigating an object function with few optima but it is the best method for a function with many optima (eg minima). Number of minima can be simply discovered for function of one or maximal two variables depicting the course of it. Situation is much more difficult for a function of more variables. Supposing a simple relation among its variables it can be analysed for each variable separately. Having a function of n variables, (n-1) variables are replaced by numbers from the competent interval and course of the section of the function is plotted. This should be repeated for all variables and if at least one of the plots is a multimodal complex function, using genetic algorithms seems adequate. The magnetic field function $\mathbf{H(m)}$ is sum of four equal functions (four similar quaternions of conductors) therefore creating only one section of the object function Eq. (6) is enough (Fig. 6.), eg

$$f(a_1, a_2, a_3, a_4) = f(a_1, 0.5, 0.5, 0.5).$$

Varying numbers replacing other three variables would change the course of plots but not number of minima. The object function Eq. (6) seems to be a simple one although with more minima. For its investigation with randomised deterministic gradient method of Mathematica was 1,000 times applied with random start points.
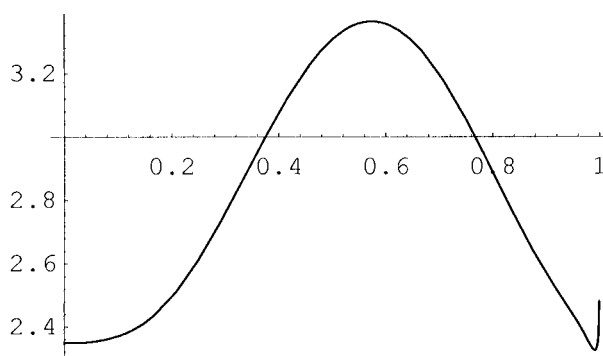

Fig. 6. Section of the object function for magnetic field homogeneity optimisation of NMR coil.

Tab. 1. shows results of the optimisation. As supposed the object function is not too complex, it has three minima. The global minimum (3) and the first local minimum (2) have already been discovered by genetic algorithms optimisation. The second local minimum (1) provides the worst value of the object function Eq. (6) minima.

Calculation of 1,000 minima values at PC Pentium 200 MHz have taken several days but less than optimisation by genetic algorithms and more information have been provided. Gradient of the object function had to be calculated numerically therefore many results were only approximate and had to be recalculated using the original result as a start point. 1,000 runs seem to be sufficient for solid examination of our object function. There is a very little probability here of existing further minima of the object function Eq. (6).

To prove correctness of the optimisation contour plots (Fig. 7.) of some magnetic fields configurations have been calculated. At first magnetic field of a coil without any

Tab. 1. Results of the optimisation by randomised deterministic optimisation method.

Number	Optimal variable: $\{a_1, a_2, a_3, a_4\}$	Minimum	Occurrence
1	{0.220444; 0.682249; 0.998483; 1.0}	0.0463940	20
2	{0.117363; 0.371815; 0.602245; 0.865971}	0.0258517	725
3	{0.156947; 0.457018; 0.779676; 0.998803}	0.0241815	255

optimisation with all 16 conductors distributed evenly along the cylinder perimeter. Subsequently magnetic fields corresponding to all calculated optima follow. It is evident that the global optimum is not very convenient for practice because two conductors have almost the same position. Homogeneity of the magnetic field inside the coil is very good for all optimised coils. The largest central homogeneous part provides the coil corresponding to the global optimum.
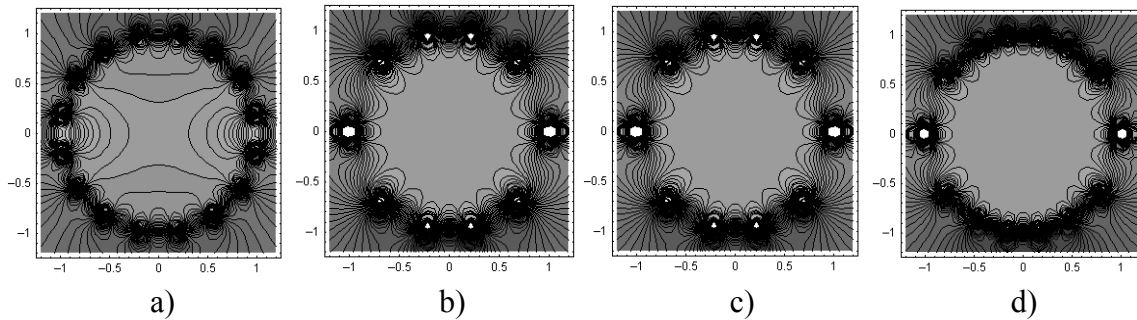

a)   b)   c)   d)

Fig. 7. Contour plots of magnetic fields of some configurations of coils. (a) conductors distributed evenly along the cylinder perimeter. (b) local optimum (1). (c) local optimum (2). (d) global optimum (3).

4. Discussion and conclusion

The acquired results indicate that genetic algorithms could be used as an optimisation method for RF coils of NMR tomograph design, although the task is rather simple for them and does not fully utilise their advantages. The experiments proved that even for genetic algorithm finding the global optimum is not a simple matter. Therefore also for RF coils of NMR tomograph it is necessary looking for new genetic operators increasing efficiency of search in the variables space with the target to find the global optimum of a certain object function. Using genetic algorithm for a simple object function is only a waste of time. Some pros and contras on optimisation of an object function of an NMR coil by genetic algorithms or by randomised deterministic function follow:

- standpoint of time: Randomised deterministic method is more advantageous. The global optimum was known much earlier than by genetic algorithm.
- standpoint of information: Again randomisation is more advantageous because provided information about all optima in the whole space. The only optimum usually found by genetic algorithm need not be appropriate for realisation even though if it is global.
- standpoint of accuracy: The methods are approximately equivalent because recalculation was needed on the both sides. Randomisation would be more accurate using a different method. Genetic algorithm would need a very long time to achieve high accuracy of result

for claims on chased space increase with length of chromosome k (thus with number of object function variables as well) according to the exponential law. Genetic algorithm with chromosomes coded as real numbers could be good compromise.

According to our results it could seem that randomisation is the only convenient method for qualities of NMR coils optimisation. Nevertheless these results are valid only for the examined coil and for the used object function. Generally are the both approaches good and it depends only on solved problem that should be preferred. Genetic algorithms are a young branch of optimisation methods and their use in NMR technique is only starting. The contribution wanted to help in this effort.

Acknowledgements

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