# THE SIGMA-DELTA MODULATOR FOR MEASUREMENT OF THE THERMAL CHARACTERISTICS OF THE CAPACITORS

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**Abstract.** *The paper presents a simple and successful design of the sigma-delta modulator with flip-flop sensor for measurement of the thermal characteristics of the capacitors. The theoretical considerations are verified by a laboratory experiment.*

## 1 Introduction

A capacitance of the real capacitors is a function of the temperature and other influences. In Fig.1 can be seen a thermal characteristic of a capacitor.

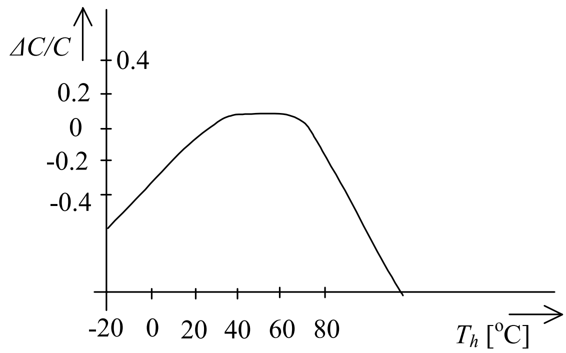


Figure 1: A thermal characteristic of a capacitor

The change of a capacitance, if we assume an influence of the temperature only, can be calculated by formula [6]:

$$\Delta C = \alpha\left(T_{h0}\right) C_0 \Delta T_h \tag{1}$$

where $\alpha(T_{h0})$ is a thermal coefficient and $C_0$ is a capacitance in the point $T_{h0}$, $\Delta T_h = T_h - T_{h0}$ is a thermal change and $\Delta C$ is a capacitive change. The thermal coefficient $\alpha(T_{h0})$ in equation (1) can be calculated by using formula [6]:

$$\alpha\left(T_{h0}\right) = \frac{dC}{C_0 dT_h} \quad \text{in the point } T_{h0} \tag{2}$$

where $(dC/dT_h)$ is a capacitive derivative to temperature and $C_0$ is capacitance in the point $T_{h0}$. The thermal coefficient $\alpha$ is non-linear function of the temperature $T_h$ in practice [6].

## 2 Sigma-delta modulator with flip-flop sensor

The key element of the sigma-delta modulator is flip-flop sensor [1]. The circuit in Fig.2 was introduced in reference [1] as a flip-flop sensor.

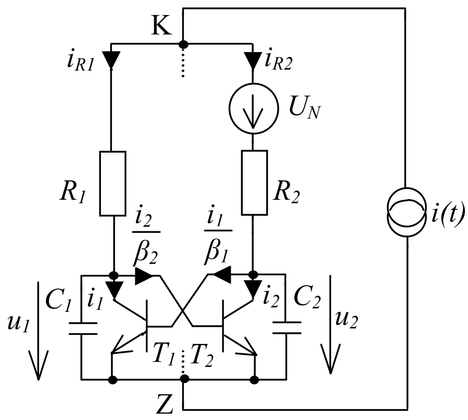


Figure 2: Flip-flop sensor

The flip-flop sensor is part of a class of silicon sensors with a digital output. A standard flip-flop consisting of two transistors and two resistors (see Fig.2) is characterized with two stable states. One of the authors of the patent flip-flop sensor was Lian [1] who showed that flip-flop sensor can be used for measurement of non-electrical quantity and derived formula for calculation of equivalent voltage of the flip-flop sensor controlled by slow-rise control pulse. The principle of measurement is based on this that measured non-electrical quantity will break the value symmetry of the inverters relative to the morphological symmetry axis passing through points K and Z (see Fig.2). However it can be compensated by a voltage $U_N=U_{NE}$ in such way that by repeated connection to a source *i(t)* the 50% state [1] is restored, so that the magnitude of the measured non-electrical quantity will be reflected into the voltage $U_{NE}$, which we will call the equivalent voltage. If needed, however, it is not necessary to stick to the custom of using sensor elements in Fig.2.

It should be noted that in current control we also distinguish between the pulses with a vertical or slow-rise segment of the control pulse (see Fig.3).

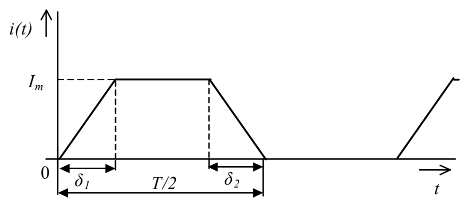


Figure 3: Current control pulse

The control with a vertical-rise segment of the control pulse is characterized by the ratio $I_m/\delta_1$ being such that the currents passing through the capacitors are not negligible compared to the transistors currents of the flip-flop sensors. The notion negligible should be understood in its relative sense. In practice the condition is satisfied if $\delta_1,\delta_2<<R_1C_1$ and $\delta_1,\delta_2<<R_2C_2$ at the same time.

In the case of control with the vertical rise segments of the control pulse the unequal values of capacitances $C_1$, $C_2$ will break the value symmetry of the inverters of the flip-flop sensor but it can be compensated by voltage $U_N=U_{NE}$ [2]. Final formula for the equivalent voltage has the form [2]:

$$U_{NE} = \frac{R\Delta C}{2C} I_m \tag{3}$$

where $I_m$ is an amplitude of the current control pulse (see Fig.3), $C_1=C+\Delta C$, $R=R_1=R_2$ and $C_2=C$.
The flip-flop sensor with feedback is shown in Fig.4a.

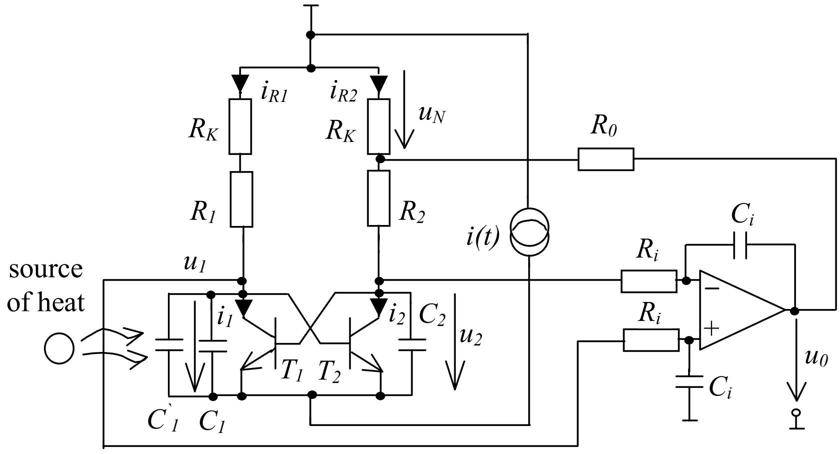


Figure 4a: Flip-flop sensor with feedback

$R_1$ and $R_2$ are the load resistors of the flip-flop and usually range from a few kΩ to tens of kΩ. $R_k$ is small resistor its value is normally two orders of magnitude smaller than $R_1$ and $R_2$. The voltage $u_0$ is attenuated by the ratio $R_0/R_k$ ($R_0$>>$R_k$) and is fed to flip-flop sensor.

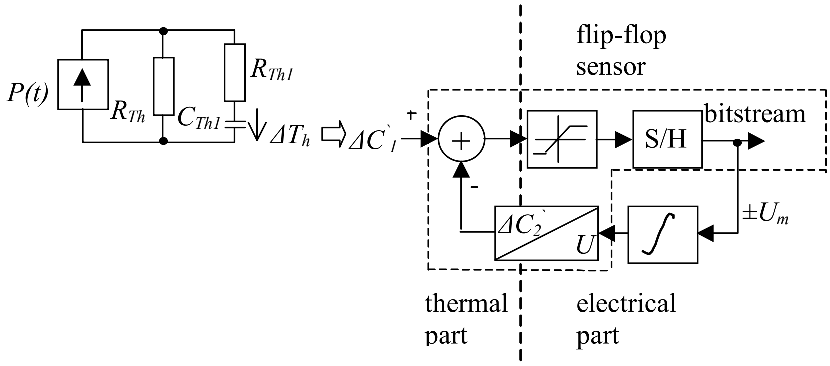


Figure 4b: Equivalent block diagram

By adjusting $u_0$, the asymmetry due to components in the flip-flop can be compensated, thus bringing the flip-flop sensor into 50% state [1]. The two outputs of the flip-flop are connected to an integrator. $C'_1$ represents a measured capacitor. Equivalent block diagram is shown in Fig.4b, where on the left side can be seen an equivalent thermal diagram.

In Fig.4b $R_{Th}$ is a thermal resistance of the environment, $R_{Th1}$ is a thermal resistance and $C_{Th1}$ is a thermal capacitance of the capacitor $C'_1$ [5]. In our case was as the source of heat used a resistor $R_T$ (see Fig.5).

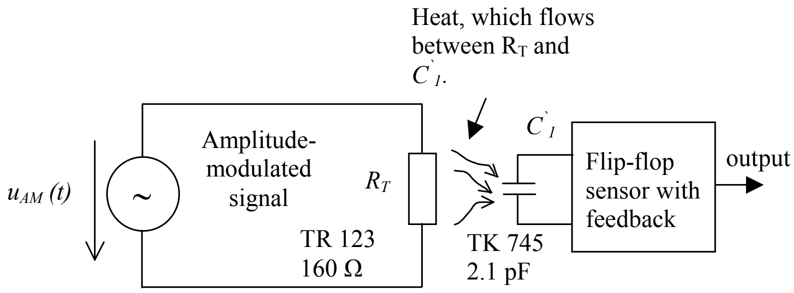


Figure 5: Resistor $R_T$ as the source of heat

In Fig.5 can be seen, that the source of the amplitude-modulated signal $u_{AM}$ is connected to the resistor $R_T$ on which arises an electrical power $P(t)$ as function of the time.

If we assume $R_{Th} << (R_{Th1} || C_{Th})$, so then the equivalent thermal diagram from the Fig.4b has the form shown in Fig.6.

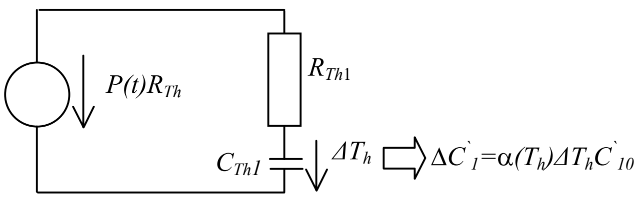


Figure 6: Equivalent thermal circuit

The thermal circuit in Fig.6 can be solved by using analogy with the solving of the electrical circuits [5] and for the thermal change $\Delta T_h$ we have:

$$\Delta T_h(j\omega) = \frac{P(j\omega)R_{Th}}{1 + j\omega R_{Th1}C_{Th1}} \tag{4}$$

and for $\omega R_{Th1}C_{Th1} >> 1$ it follows

$$\Delta T_h(j\omega) = \frac{P(j\omega)R_{Th}}{j\omega R_{Th1}C_{Th1}} \tag{5}$$

where $P(j\omega) = \int_{-\infty}^{\infty} P(t)e^{-j\omega t}dt$ and $\Delta T_h(j\omega) = \int_{-\infty}^{\infty} \Delta T_h(t)e^{-j\omega t}dt$ .

The final capacitive change can be derived through equations (1), (5). The result is

$$\Delta C_1^{`}(j\omega) = \frac{\alpha(T_h)P(j\omega)R_{Th}C_{10}^{`}}{j\omega R_{Th1}C_{Th1}} \tag{6}$$

The final block diagram of the sigma-delta modulator with flip-flop sensor is then shown in Fig.7 [3], where $\Delta C_2$ represents a change of the capacitance $C_2$ of the flip-flop sensor [3].

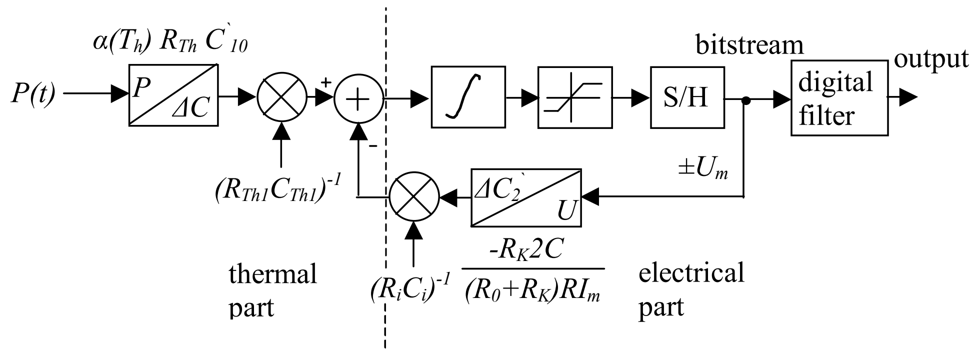


Figure 7: Final block diagram of the sigma-delta modulator with flip-flop sensor

The sigma-delta modulator with flip-flop sensor is in more detail described in reference [3].

## 3 Principle of the method

The principle of the measurement of the thermal characteristic of the given capacitor is based on the measurement of the capacitive change as the function of the temperature. The thermal change can be achieved by using the resistor $R_T$ as source of heat (see Fig.5). In our case was source of the amplitude-modulated signal connected to the resistor $R_T$. If we assume the modulation signal with square shape so then the electrical power, which arises on the resistor $R_T$, has the square shape too [3]. From the equation (5) then it follows a triangular shape of the thermal change $\Delta T_h$ as a function of the time [5].

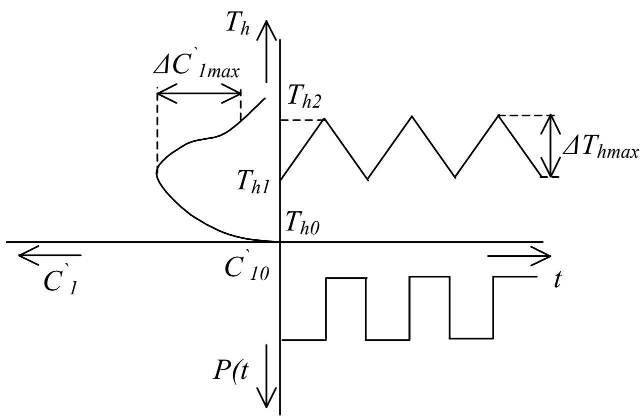


Figure 8: Principle of the measurement of the thermal characteristic of a capacitor

The principle of the measurement of the thermal characteristic of a capacitor is obvious from the Fig.8.

## 4 Experiments

Proposed method was tested on the capacitor, which catalog thermal characteristic is shown in Fig.9 and its value was 2.1 pF.

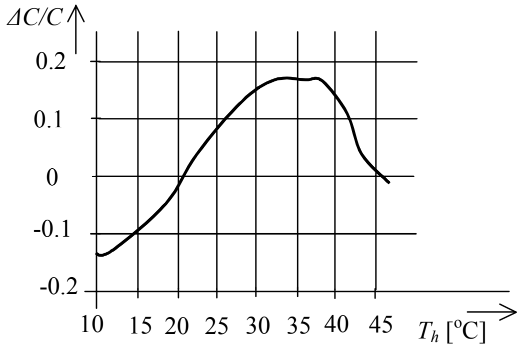


Figure 9: Catalog thermal characteristic of the capacitor

As it was described above the source of the amplitude-modulated signal was connected to the resistor $R_T$ (see Fig.5). The amplitude-modulated signal can be described by formula:

$$u_{AM}(t) = U_{N0}[1 + mx(t)]\sin(2\pi f_{N0}t) \tag{7}$$

where $U_{N0}$ is an amplitude of the carrier signal, $m$ is an index of the amplitude modulation, $f_{N0}$ is a value of the carrier frequency and $x(t)$ is a modulation signal in our case a periodical signal with the square shape. In the case of our experiment $U_{N0}$=3.5 V, $m$=0.3, $f_{N0}$=35 kHz.

The experimental test circuit was realized by Fig.4a, so that the values of the parameters were set as follow: $R=R_1=R_2$=6.8 kΩ, $R_k$=10 Ω, $R_0$=1.8 kΩ, $C_i$=10 nF, $R_i$=10 kΩ and $C$ =387 pF. The flip-flop sensor was controlled by a current pulse according to Fig.3, while $\delta_1=\delta_2$=100 ns, $I_m$=1.17 mA and $T$=70 µs.

The thermal characteristic of a capacitor can be measured by using two methods. First method is based on the measurement of the voltage $u_0$ (see Fig.4a) as a function of the time and the capacitive change $\Delta C_1^{`}$ can be calculated by formula:

$$\Delta C_1^{`} = -\frac{\Delta u_0 R_K 2C_i}{(R_0 + R_K)R_i I_m} \tag{8}$$

because $\Delta u_N=-\Delta u_0 R_K/(R_0+R_K)$.

The measured voltage $u_0$ as function of the time for the capacitor, which catalog thermal characteristic is shown in Fig.9, can be seen in Fig.10a and in more detail is shown in Fig.10b. The final capacitive change $\Delta C_1^{`}$ it is then possible to calculate from the equation (8).

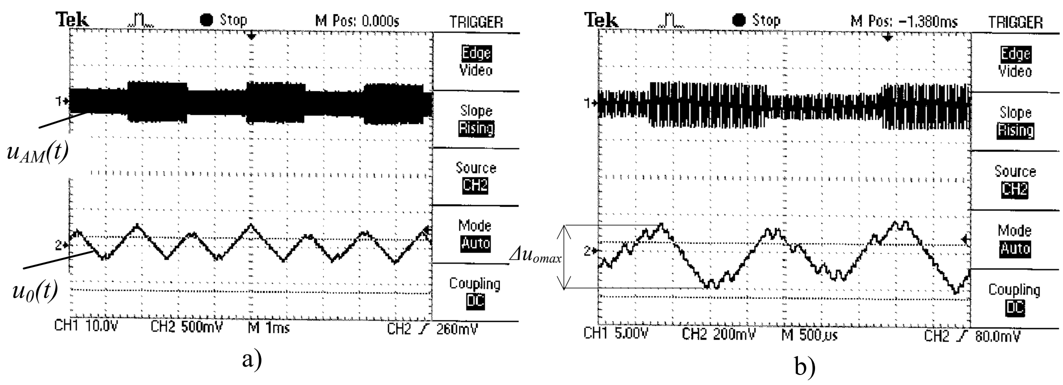


Figure 10: Measured voltage $u_0$ as the function of the time

It is clear that the voltage $u_0$ is the function of the time, but from the equation (5) it follows that for $\omega R_{Th1}C_{Th1} >> 1$ the thermal change $\Delta T_h$ is given by equation:

$$\Delta T_h(t) = \frac{R_{Th}}{R_{Th1}C_{Th1}} \int_0^t P(\tau)\, d\tau \qquad (9)$$

From the equation (9) it is obvious that the time axis $x$ can represent the thermal axis, but the thermal parameters $R_{Th}$, $R_{Th1}$, $C_{Th1}$ must be known.

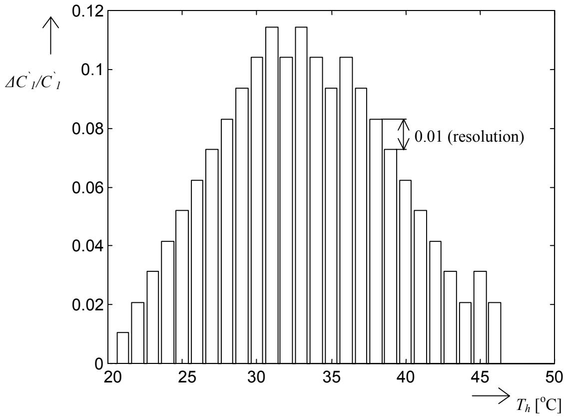


Figure 11: Final thermal characteristic of the capacitor

The principle of the second method is based on the processing of the bitstream from the output of the flip-flop sensor (see Fig.4b). The final thermal characteristic given capacitor measured by second method is shown in Fig.11. A reverse counter was used in this case as digital filter, so that its output information was processed in *MATLAB*.

In Fig.11 can be seen the final thermal characteristic as function of the temperature $T_h$, so that the test circuit was calibrated for given type of the resistor $R_T$ and measured capacitor $C^{`}_1$. Quantization error in Fig.11 can be reduced by higher sampling frequency $f=1/T$ (see Fig.3).

For the verification of the theoretical considerations was made an experimental circuit by *SMT* (see Fig.12).

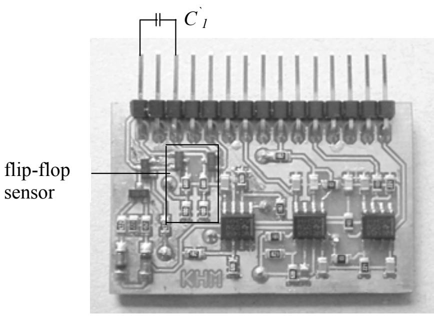


Figure 12: A photography of the experimental circuit

Some important parameters of the experimental circuit are shown in Tab.1.

| Parameters | Values |
|---|---|
| Supply voltage | ±9 V |
| Power consumption | 75 mW |
| Frequency of the control pulses | 14.3 kHz |
| Resolution (see Fig.11) | 0.01 |

Table 1: Some important experimental parameters

## 5 Conclusions

A new method for the measurement of the thermal characteristics of the capacitors has been presented, which is based on using of the flip-flop sensor in the structure of the sigma-delta modulator. The main property of this system is its ability to measure the thermal characteristics of the capacitors in range a few pF. The validity of the theoretical considerations was proved by laboratory experiments.

It is true that the proposed system will be hard to calibrate as a board-level product, however, it may well prove an interesting technique to assist with process characterisation test-masks, where such a technique could be used on-chip, and where the limitations will be much less of an issue.

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