

## Influence of Digital Signal Processing on Precision of Power Quality Parameters Measurement

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**Abstract:** *The paper deals with errors, which rise up during Fast Fourier Transform (FFT) realising harmonic analysis for power quality measurements, if it is performed by digital signal processors (DSP). For FFT realisation there exist different algorithms based on the same principal but differing in the calculation operations sequence. The paper compares results obtained by radix-2 decimation in frequency and decimation in time algorithms. Then it gives possible improvements for overall error suppression.*

**Keywords:** *digital signal processing, power quality, harmonic analysis, FFT*

### 1. Introduction

Demand for power quality measurement rises very rapidly during last years. So it is necessary to produce measuring instruments for this purpose at low price and low energy consumption. One of the tasks of these devices is also harmonic analysis of power voltages and currents often also in three phase systems. According to valid standards analysis must be performed continuously without time gaps and in real time. These requirements lead to design of these devices on the base of fix-point DSP circuits. The application of processing element with limited word width may create difficulties with resulting precision of such a system.

### 2. Background of harmonic analysis errors

There are several sources of errors in DSP harmonic analyser. Principal block diagram of the structure of this device is shown in

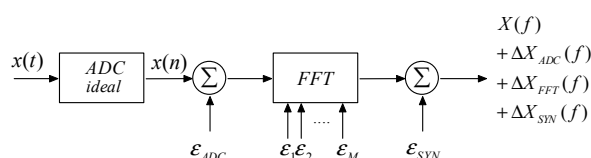


Fig. 1: Harmonic analyser block structure.

fig. 1, which also the sources of errors are given in. Generally the input analog signal is converted into digital form by AD converter (ADC) and digital samples are processed by FFT algorithm in DSP processor. The errors of results are influenced by inadequate sampling ( $\epsilon_{SYN}$ ), quantisation error of ADC ( $\epsilon_{ADC}$ ) and truncation errors of FFT algorithm ( $\epsilon_i$ ). All three sources create the resulting error components marked as  $\Delta X_{SYN}$ ,  $\Delta X_{ADC}$  and  $\Delta X_{FFT}$  respectively. The estimation of influence of ADC quantisation error was presented in [1]. Basic principals of  $\Delta X_{FFT}$  estimation are given in [2]. Now we will present the analysis of errors conditioned by different ways of FFT algorithm realisation.

To determine harmonic components of measured signal Fourier transform is used [3],[4]. It is defined by

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn} \quad 0 \leq k \leq N-1 \quad (2.1)$$

where  $x(n)$  is sample of input signal,  $X(k)$  is spectral component and  $W_N = e^{-j2\pi/N}$ .

This calculation is time consuming, therefore Fast Fourier Transform (FFT) was derived, which requires much less calculations. In praxis FFT radix-2 is used very often [4]. This version is based on

splitting the data sequence into two halves. Input data sequence may be split in time domain according to:

$$X(k) = \sum_{m=0}^{N/2-1} x(2m)W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)W_N^{(2m+1)k} \quad (2.2)$$

This procedure is called decimation in time (DIT). The second way of data sequence splitting is given by:

$$X(k) = \sum_{m=0}^{N/2-1} x(m)W_N^{mk} + \sum_{m=N/2}^N x(m)W_N^{mk} \quad (2.3)$$

This is called decimation in frequency (DIF).

By rearranging of operation steps in (2.2) and (2.3) we may get so called FFT butterflies, which show the data flow in elementary operation for both types of data decimation.

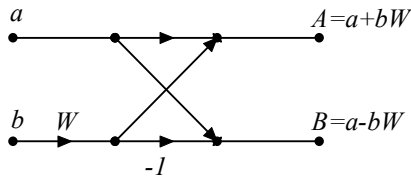


Fig. 2: DIT FFT butterfly.

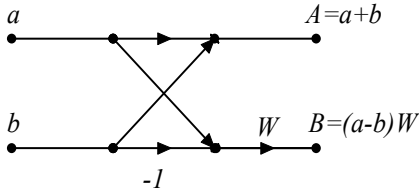


Fig. 3: DIF FFT butterfly.

In case that all operands are in discrete form we may express them in forms:

$$a = \bar{a} + \Delta a \quad b = \bar{b} + \Delta b \quad W = \bar{W} + \Delta W \quad (2.4)$$

and the results of operations:

$$A = \bar{A} + \Delta A \quad B = \bar{B} + \Delta B \quad (2.5)$$

where  $\bar{a}, \bar{b}, \bar{W}, \bar{A}, \bar{B}$  represent correct values,  $\Delta a, \Delta b, \Delta W$  are errors due to quantisation of butterfly input data and  $\Delta A$  and  $\Delta B$  are butterfly result errors. The last ones may be estimated by error transfer theory. For DIT they will be

$$\begin{aligned} \Delta A &= |\Delta a| + |\Delta b \bar{W}| + |\bar{b} \Delta W| \\ \Delta B &= |\Delta a| + |\Delta b \bar{W}| + |\bar{b} \Delta W| \end{aligned} \quad (2.6)$$

and for DIF they will be

$$\begin{aligned} \Delta A &= |\Delta a| + |\Delta b| \\ \Delta B &= |\Delta a \bar{W}| + |\Delta b \bar{W}| + |(\bar{a} - \bar{b}) \Delta W| \end{aligned} \quad (2.7)$$

If we compare both groups of formulas we will see, that errors are different for DIT a DIF realisations of FFT. We may suppose that DIF gives us result with lower errors. Further we will try to approve this assumption by FFT data flow simulation.

### 3. Errors due to Fix-Point realisation

For verification of expected properties we have created mathematical model of real fix-point DSP algorithm of both types of FFT calculations. Our model simulates 16-bit fix-point processor with 16-bit arithmetic unit and 32-bit multiply accumulator. So its resolution is  $LSB=2^{-15}$ . If we want to obtain valid results we had to create model with optimal realisation of FFT procedure from the point of view of numerical truncation errors.

To express errors of whole fix-point FFT calculations procedure, we used as a reference FFT spectrum values calculated by 32-bit floating point arithmetic. For this arithmetic the difference between DIF and DIT lies on level of  $2.0e-15$ . We see that this value is much lower than fix-point resolution and errors. Then errors of FFT spectrum were calculated by formulas:

$$\begin{aligned} ERR_{DIT}(k) &= Y_{XDIT}(k) - Y(k) \\ ERR_{DIF}(k) &= Y_{XDIF}(k) - Y(k) \end{aligned} \quad (3.1)$$

where  $X_{XDIT}(k)$  is amplitude frequency spectrum calculated by fix-point DIT FFT procedure and  $X_{XDIF}(k)$  is amplitude frequency spectrum by fix-point DIF FFT.

For the first tests we used 256 samples of one period of harmonic signal with 0,8 FS amplitude. The error spectra of both

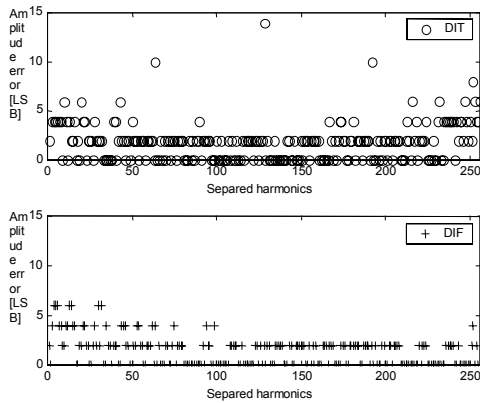


Fig. 4.: Amplitude errors generated by DIT and DIF realisation of FFT procedure.

procedures are shown in fig. 4, in which one can see:

- DIT algorithm gives almost symmetrical error spectrum, but errors of separate harmonics are larger than errors of DIF calculation. For DIT the maximum error is as high as 14 LSB, while for DIF it is only 6 LSB.
- DIF error spectrum is not symmetrical. The errors of harmonics are lower in the second half of spectrum than in the first half.

These results verify our expectation resultant from (2.6) and (2.7).

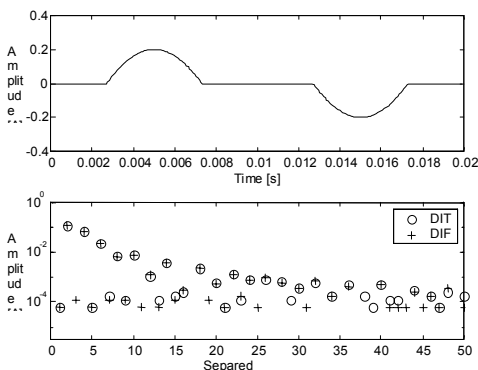


Fig. 5: Real current signal and first 50 harmonics of its FFT spectrum.

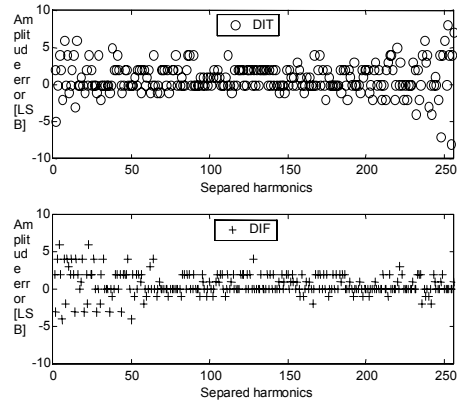


Fig. 6: Errors of harmonics by DIT and DIF realisation of FFT.

In praxis clear harmonic signal is present in power network only very rarely. Obvious wave-shapes of current waves are similar as is shown in fig. 5 (diode) and 7 (triac). In their upper part one can see time waveforms and in lower parts corresponding spectra of first 50 harmonics.

Graphical representations of amplitude errors are shown in fig. 6 and fig. 8 respectively. Also in this case the diagrams approve our expectation, according which DIF gives better results than DIT.

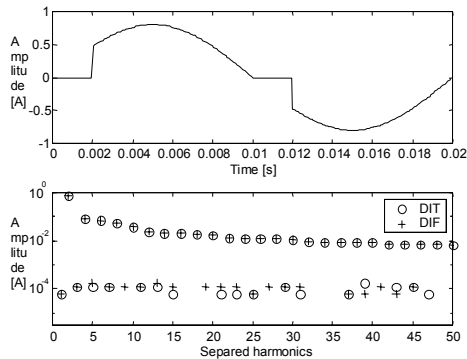


Fig.7: Real example of voltage signal and its spectrum.

#### 4. Possible improvements

When we considered the symmetry of magnitude DFT spectrum and non-symmetric dependence of errors shown in figures 4, 6 and 8 as well as the fact, that errors have both

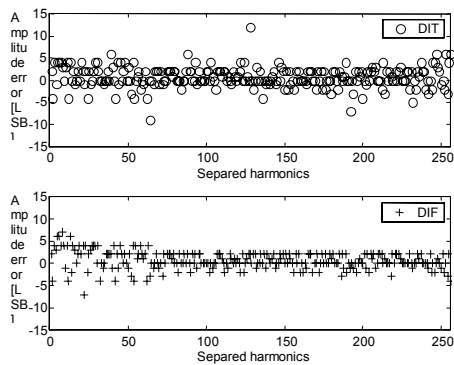


Fig. 8: Errors of harmonics of signal from fig. 7 by DIT and DIF realisation of FFT.

polarities, we found out, that it could be advantageous to use upper part of DIF spectral values or to calculate the mean value of symmetric components to improve the precision of harmonic analysis.

For verification of proposed improvement of amplitude harmonic analysis we simulated both procedures. The results are shown in table 1, where lower part means the lowest 50 components, upper part highest 50 components of spectrum and mean represents the average of corresponding lower and upper components. It is evident, that our assumptions were fully approved.

Table 1: RMS error of 50 harmonic components in LSB.

	Signal	Lower part	Upper part	Mean
DIF	sinus	3.30	1.35	2.03
	diode	2.74	1.07	1.56
	triac	3.32	1.47	1.94
DIT	sinus	2.89	3.37	2.86
	diode	2.34	3.42	1.99
	triac	2.67	2.67	2.19

### 5. Conclusions

For realisation of FFT transformation by DSP processors there exist several techniques, which require almost the same processing power. These techniques use different order of calculation operations. If they are performed by fix-point processors

with limited word length, the errors of calculated values may depend upon operation sequence. Our simulations showed that there are significant differences in error magnitudes as well as their distributions even for two basic FFT algorithm realisations – DIF and DIT. It was shown that DIF algorithm has lower errors, so it could be recommended for measurement purposes. Error, which rises by DIF algorithm in upper half of the spectrum, is 2 LSB smaller than in lower part. So we proposed to use the former property for error suppression of harmonic analysis. The errors by DIT realisation do not have similar dependence on the order of harmonics and it is significantly higher. Even in this case proposed modification of FFT calculation helps to decrease overall error.

Presented results were obtained for 256 samples FFT of one period signals. Results for 8 or 10 periods and more samples are quite similar.

### Acknowledgement

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### References

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