ERRORS DUE TO SYNCHRONIZATION EFFECTS IN MEASUREMENT OF FREQUENCIES

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Abstract: *Synchronisation appears if two frequency generators are coupled. Because this coupling is inevitable a frequency shifting even outside the synchronisation range leads to errors as investigated in detail in the paper. Especially the errors caused by this effect were investigated and consequences for the praxis of high-precision frequency measurement are deduced.*

INTRODUCTION

Frequency measurement normally compares two frequencies: the unknown frequency Ω_l and the reference frequency Ω_0 using converter (mixer) principle. The reference frequency Ω_0 in most cases is quartz-generated, e.g. this frequency may be assumed to be constant.
Because of the inevitable coupling of the two frequency sources the problem is described by a differential equation of *Hills* type (rheolinear system) leading to two effects: a synchronization – often desirable – and an inevitable frequency shifting $\Delta\Omega^*$ outside of the synchronization range. In the literature the last mentioned effects and therefore these errors are neglected that means the difference frequency is assumed to be

$$\Delta\Omega = \Omega_l - \Omega \tag{1}$$

The paper deals especially with these problems and leads to new results important for praxis of high-precision measurements.

1. SUBJECT & METHODS

1.1 Theory of linear system with time-variable parameters (rheolinear systems)

Because there are always capacitances between the two frequency sources of the mixer coupling between both is inevitable. Due to this fact the coupled voltage leads to two consequences: the amplification of the frequency generator Ω_l and due to the varactor effect of the transistor the frequency deciding capacitance is varied. Therefore the differential equation is

$$y``(t) + a_1(t)\, y`(t) + a_2(t)\, y(t) = x(t) \tag{2}$$

where $a_1(t)$ considers the variation of the damping because of the varying amplification and $a_2(t)$ the variation of the capacitance because of the varactor effect.
For we are only interested in the Eigenvalues the solution of the homogenous *Hill* equation is sufficient [1]

$$z(t)`` + \Phi(t)\, z(t) = 0 \tag{3}$$

To a theorem of *Floquet* [2] the following solutions are existing

$$z(t) = e^{\mu t} f(t) + e^{-\mu t} g(t) \tag{4}$$

The mathematical treatment is very difficult even for the special case of a periodic $\Phi(t)$, the so-called *Mathieu* differential equation [3]. Here the Diagram of *Ince* and *Strutt* as shown in Figure 1 demonstrates instable regions with real parts of the exponent μ in eq. (4).

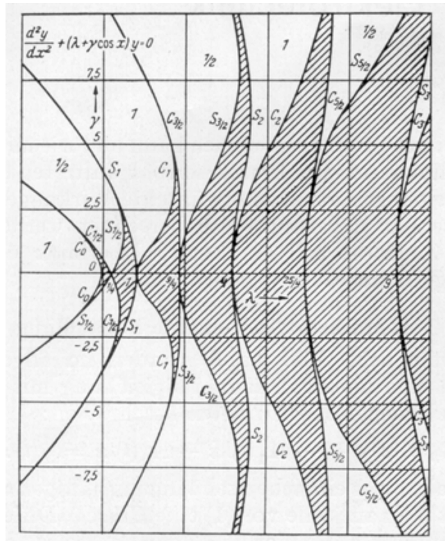

Figure 1. Diagram of *Ince* and *Strutt*

We will use another approach to get informations of the behaviour of the system and especially of the characteristic exponent μ based on physical considerations and approximations as in principle described in [4].

2.1. Investigations using physical considerations

To apply energy and phase investigations instead of equation (3) we use the so-called *Mathieu*-equation

$$z(t)'' + \Omega_0^2(1+\sigma \sin\omega t)\, z(t) = 0 \tag{5}$$

describing an oscillator with time-varying capacitance. Following investigations given by *Wenke* [5] and *Erdelyi* [6] we get a current in phase with the voltage and one with 90° phase difference leading to a negative damping and to an additional capacitance. This additional capacitance generates a frequency shift, the possible maximum yields the so-called locking or synchronization range $\Delta\Omega_o$. Because this parameter may be easily measured we will use him to describe the problem instead of σ in equation (5). As given in details in [7] the exponent μ is

$$\mu = \sqrt{\Delta\Omega_0{}^2/4 - \overline{\Delta\Omega^2}} \tag{6}$$

where $\overline{\Delta\Omega}$ means the deviation to the resonance frequency Ω_o.

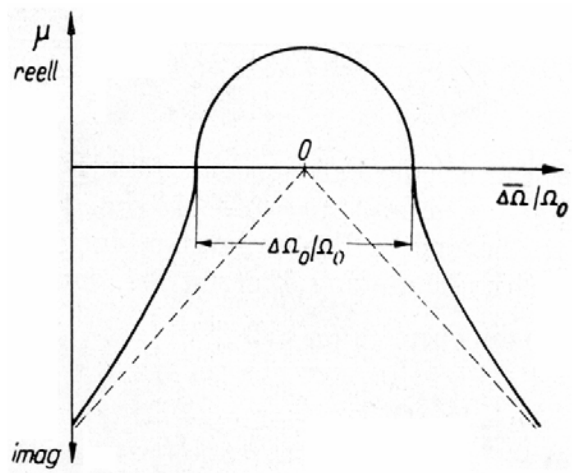

Fig. 2. Course of the characteristic exponent μ

Figure 2. shows the course of μ as a function of the normalized difference frequency. Inside the synchronisation range $\Delta\Omega_o$ the exponent μ is real, that means a negative damping and not a frequency difference (synchronization). Outside this range μ is imaginary, that means a frequency deviation (error) in comparison with the ideal difference frequency $\Delta\Omega = \Omega_1 - \Omega_0$.

2. RESULTS AND DISCUSSION

To gain the error between the ideal difference frequency $\Delta\Omega$ adequate to equation (1) and the real difference frequency we use equation (6) with $\overline{\Delta\Omega} = \Delta\Omega$

$$\mu = \sqrt{\Delta\Omega_o^2/4 - \Delta\Omega^2} = j\sqrt{\Delta\Omega^2 - \Delta\Omega_o^2/4} \tag{7}$$

So the real difference frequency follows

$$\sqrt{\Delta\Omega^2 - \Delta\Omega_o^2/4} = \Delta\Omega\,[1 - 0{,}5(\Delta\Omega_o^2/4\,\Delta\Omega^2)] \tag{8}$$

and the relative error is

$$-\,0{,}5(\Delta\Omega_o^2/4\,\Delta\Omega^2) \tag{9}$$

That means: If for instance the difference frequency is by the factor 100 greater than the synchronization range $\Delta\Omega_0$ the relative error still runs to $0{,}125 \cdot 10^{-4}$! In praxis the permissible error in high precision frequency or time measurement may be less than 10^{-10} to 10^{-12} [8]. From this fact it follows that the synchronization range has to be smaller than $0{,}3{*}10^{-4}$ to $0{,}3{*}10^{-5}$ of the difference frequency.

In praxis the synchronization range should be measured before frequency measurement. If $\Delta\Omega_o$ is known equation (7) can be used for error-correction.

If it is not possible to know the synchronization range $\Delta\Omega_o$ one can use the measurement of the distortions to gain $\Delta\Omega_o$, because there exists a relationship between the distortion factor and the synchronization range. The reasons for the distortion are relaxations in the neighbourhood of the synchronization border [9]. Here further investigations are necessary.

3. CONCLUSION

At the beginning an introduction to the theory of systems with time varying parameters – so-called rheolinear systems – is given. It is shown that a characteristic exponent μ appears meaning either a negative damping or a frequency deviation depending on the value of μ. Using phase and energy investigations the course of this parameter in relation to the difference frequency can be gained. From these results follows a frequency error even outside the synchronization range. This result is of great importance for high-precision frequency or time measurement. The relation of this error with the synchronization range is gained and so it is possible to correct this error if the synchronization range is known. If it is not possible to measure the synchronization range the distortion factor can be used to gain the frequency error for there exists a connection between both. Here further investigations are useful as well as concerning consequences to other fields of measurement.

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