ERROR AND UNCERTAINTY REDUCTION – CHALLENGE FOR A MEASURING SYSTEMS DESIGNER

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Abstract: *The existing classification of errors is unpractical because of its incoherence. The consequent splitting of two notions "error" and "uncertainty" removes disorder and makes designer work more clear. The basic rules for error and uncertainty reduction are presented in the paper. It is emphasised that these rule are completely different from each other.*

1. Historical background

The idea that all measurement errors should be divided into two groups: systematic errors and random errors, appeared many years ago, in the early period of the measurement science establishing. The conception seems to be very simple and clear:

- Errors with the values not changing or changing in a known manner belong to the systematic errors.
- Errors with the values changing in an unexpected manner belong to the random errors.

Otherwords: systematic errors are deterministic ones, while random errors are random ones. It is really clear from the theoretical point of view. Unfortunately, such a classification is not practical. There are two reasons for it:

1. The definition of the error is purely theoretical
   $$E = X^* - X \tag{1}$$
   where X* is a measured value and X is a true value. The true value is never known. So E value is never known too. It means that the systematic error exists only in the theory.
2. The second reason is even more serious. Among systematic errors there are both recognised and unrecognised components. There is no problem with the recognised components. They can be taken into consideration in a correction process. The problem is with these unrecognised. According to the theory they are still deterministic. For the user of the measuring system, however, they have to be treated as a random component. So, the random component appears as a part of a systematic error. That is why the whole clarity of the error classification fails.

The curiosity of the above given classification was, of course, noticed by the community of persons working on the measurement technology. In order to overcome the problem, the so called "most probable" value was introduced instead of the "true value". The effect was horrible. The error lost its theoretical correctness and did not become more practical. Therefore, the completely different method for indicating the accuracy of measuring instruments and systems was introduced in practice. The method consists in determining the limits of all components influencing the result of measuring process. Unfortunately, the limit was called the "limit error" while it has nothing on the definition of the error. It is rather near to the conception of uncertainty presented in the Guide for Expression of Uncertainty in Measurements [1]. It is worth noticing that the Guide formulates only some rules for expressing uncertainty. These rules are based on statistics of random processes. It does not mean that the conception of uncertainty is identical with the conception of random errors. Uncertainty is related to the calculated mean value. Random error is related to the unknown value. That is the difference. One can expect that it is a reason why the Guide's Authors - the prominent persons in the world of metrology- avoided like a plague the use of the word "error" in the Guide text.

2. Practical definition of the error

The remarks given above should not be understood as cancelling the definition of the error. The definition must be only changed. Let us introduce a new definition of a measurement error in the form of

$$E = X^* - X^{**} \tag{2}$$

where X** is no longer the "true value" but it is the "better known" value of the measurand. The word "known" has to be underlined in that definition. If we are not able to achieve – by calculation, by calibration or by any other means – this better known value, the error does not exist. In such a case only the uncertainty exists as a representation of our lack of knowledge. According to the definition (2) the error is always deterministic, has no random component and can be used in the measuring practice for introducing the correction factors to the measuring system. Commercially available measuring systems are currently equipped with a lot of hardware and software means for correction. It is the designer who has to find out the error sources, to define their influence and to develop the methods of correction.

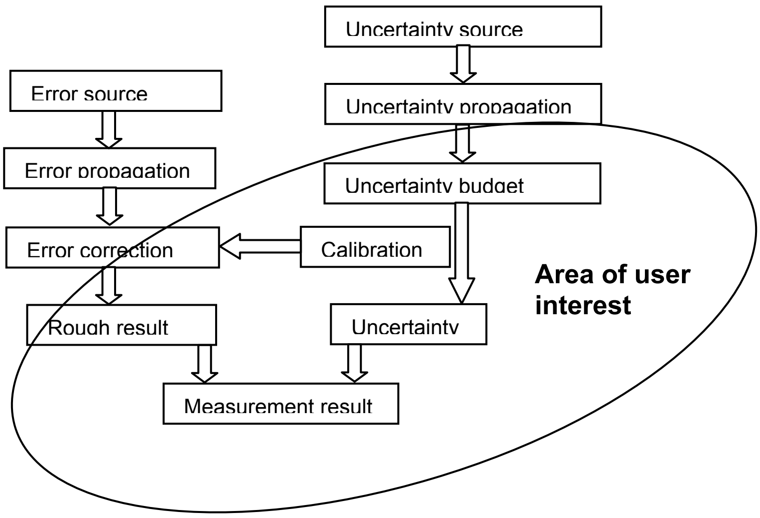

Fig.1. Error and uncertainty analysis forms two independent challenges for the measuring system designer. Only a little part of that analysis is to perform by the user.

The user of the measuring system has normally almost no opportunity to recognise the values of the error. In some cases he is able to calibrate his instrument, and it is his only way for finding the values of the errors. The uncertainty data are much more interesting for the user because they enable him to prepare an uncertainty budget. One can conclude that for the designer both the error and the uncertainty are of the great importance. Error and uncertainty propagation became the key problem in the design of measuring systems. The essential task in that design is to minimise the propagation coefficients. There exist a lot of methods to do it, but only two of them are common for errors and uncertainties. They consist in:

1. Finding such a principle of operation which is not affected by the influence variables. It is almost unrealistic method.
2. Not allowing the variables to influence the sensitive parts of the measuring system by screening, thermal insulation, vibration dumping etc.

All other methods for lowering the propagation coefficients are different for error propagation and for uncertainty propagation. Structural methods are effective for minimising error propagation. Their effectiveness for minimising uncertainty propagation is much lower, if any.

3. Structural methods for error reduction

In order to facilitate the considerations the transfer function of the sensor or transducer is reduced to the linear form with only one influence variable ΔV causing the error.

$$Y_1 = SX + W\,\Delta V + k\,X\Delta V \quad (3)$$

where X and Y_1 are the input (measured) and output quantities, respectively, ΔV is the influenced variable, S is the transducer sensitivity, W and k represent the propagation factors for the additive and multiplicative part of the error, respectively.

Structural methods are based on the error compensation principle. The placement of the compensating elements depends on the structure. The principal structures used in measuring systems are: Linear (cascade) (Fig.2A), differential (Fig. 2B) and ratiometric (relative) (Fig.2C).

The transfer function of the compensator in a linear structure is presented as

$$Y = S_k Y_1 + W_k\Delta V + k_k Y_1\Delta V, \quad (4)$$

where S_k, W_k i k_k are the compensator sensitivity to the input variable, the additive and multiplicative error, respectively. Then

$$Y = S_k SX + W_k\Delta V + S_k W\Delta V + k_k SX\,\Delta V + S_k kX\Delta V + k_k W\Delta V^2 + k\,k_k X\Delta V^2. \quad (5)$$

For compensation of both additive and multiplicative errors two equations have to be fulfilled simultaneously: $\frac{S_k}{k_k} = -\frac{S}{k}$ *and* $\frac{W_k}{S_k} = -W$ (6)

which is hard to achieve in practice. Such a structure was used in the past. Now it is rather replaced by the numerical error correction.

On the contrary, the differential structure has many advantages and is widely used. According to Fig. 2B the compensator transfer function is reduced to

$Y_2 = W_k \Delta V,$ (7)

because $X = 0$. Then:

$Y = SX + (W_1 - W_k)\,\Delta V + kX\Delta V,$ (8)

which indicates the opportunity for additive error compensation when

$W_1 = W_k$ (9)

and no possibility for mutiplicative error compensation. The same conclusion may be drawn from analysis of the structure presented in Fig. 2B$_1$ where two identical sensors operating in opposite directions are used.

$Y_1 = SX + W\Delta V + kX\Delta V$ (10)

$Y_2 = -SX + W\Delta V - kX\Delta V.$ (11)

Then

$Y = 2SX + 2kX\Delta V$.(12)

The most perspective is ratiometric structure (Fig. 2C). It has not been used before because of the difficulties in analog realisation of quotients. Today's measuring systems enable numerical dividing. Analysing two sensors with transfer functions

$Y_1 = S_1 X + W_1\,\Delta V + k_1 X\Delta V$ (13)

$Y_2 = S_2 X_0 + W_2\,\Delta V + k_2 X_0\,\Delta V)$ (14)

and assuming additionally the absence of additive errors $(W_1 = W_2 = 0)$ one obtains

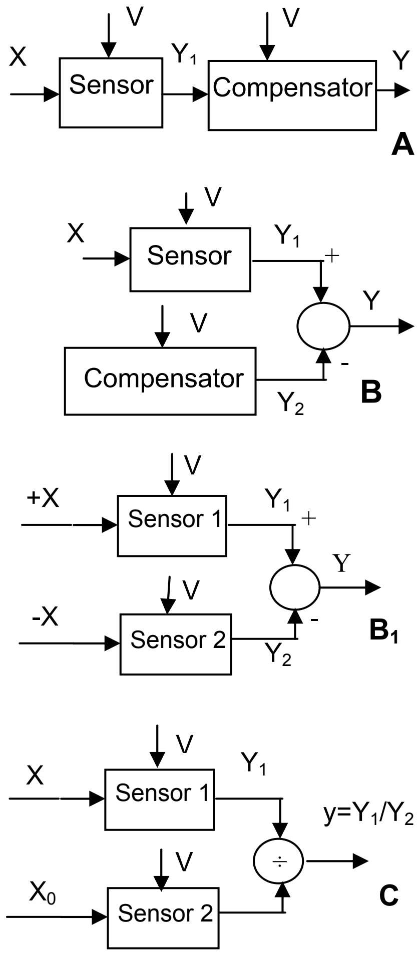

Fig.2. Basic structures for error compensation

$$y = \frac{X}{X_0}\frac{S_1 + k_1\Delta V}{S_2 + k_2\Delta V}. \tag{15}$$

When the both sensors are identical then

$$\frac{S_1}{k_1} = \frac{S_2}{k_2} \tag{16}$$

and the multiplicative error is eliminated. When the additive error appears ($W_1 \neq W_2 \neq 0$) the structure becomes more complicated, and some additional but realistic assumptions should be made:

$$\frac{W_2\Delta V}{X_0} << S_2, \quad and \quad k_2\Delta V << S_2 \quad then \tag{17}$$

$$y = \frac{S_1}{S_2}\frac{X}{X_0} + \left(\frac{W_1}{S_2X_0} - \frac{S_1W_2}{S_2^2X_0^2}X\right)\Delta V + \frac{S_1}{S_2}\frac{X}{X_0}\left(\frac{k_1}{S_1} - \frac{k_2}{S_2}\right)\Delta V, \tag{18}$$

so neither additive nor multiplicative errors are cancelled, however, they are strongly reduced. Taking into account the identity of the sensors $S_1 = S_2 = S$ *and* $W_1 = W_2 = W$ the both errors are equal to zero only when $X = X_0$

$$y = \frac{X}{X_0} + \frac{W}{SX_0}\left(1 - \frac{X}{X_0}\right)\Delta V. \tag{19}$$

More detailed considerations dealing with the error propagation are presented in [2]

4. Uncertainty propagation

Uncertainty propagation derives from the rules of statistics and is well described in the Guide. There is no reason to cite all these rules once more here. The random character of uncertainties justifies the question if the structural cancelling of uncertainty components is possible in general. Only external noise sources influencing in the same way both sensitive parts of differential or ratiometric structures may be treated as correlated ones, and therefore may be reduced with respect to their correlation coefficient. It is a seldom case. In practice the sources of uncertainties are in internal random effects like electronic noises, in external random effects like electromagnetic disturbance transferred into the circuit and in not enough precise error correction or compensation. That later effect caused by our lack of knowledge is also a random one. Therefore the methods for uncertainty reduction should be different from those used for error reduction. In general they are based on two modes of operation: averaging and filtering. Averaging is described as a continuous time process

$$Y(t) = \frac{1}{W}\int_{t-t_0}^{t} w(\tau)y(\tau)d\tau \quad \text{or as a numerical algorithm} \quad Y_k = \frac{1}{W}\sum_{k-n}^{k} w_i Y_i \tag{20}$$

where w(t) and w_i are continuous weighting functions or numerical weighting coefficients, respectively, and W denotes integral or sum of that function. The simplest on line (real time) averaging with w(t) = 1 (w_i = 1) has its physical interpretation as a mean value. It is evident that the delay occurs in that case. Many complicated weighting functions have been proposed in order to reduce the uncertainty without affecting the median value of the averaged zone [3] [4]. The windows used are no more rectangular. The procedures are commonly called “smoothing” and are used off line. From the formal point of view, each averaging formulates a new measurand, different from that defined before averaging. It is evident in the simplest case of rectangular window, but it is also true in all other cases.

Filtering in the sense of frequency band limiting process is described by similar equations. The weighting function is replaced by the pulse response of the filter and the fold integral or fold sum is used. The value after filtering consists of the input values multiplied by some coefficients. It also leads to the redefinition of the measurand.

5. Conclusion

In order to make the definitions of errors and uncertainties practically useful these two notions should be treated as completely separate ones. The definition of error presented in Eq. (2) gives such an opportunity. The aim of the measuring system designer is to reduce both errors and uncertainties. The means for error reduction are completely different from those for uncertainty reduction. It is an additional argument for deep distinction between error and uncertainty. In most cases the change of measurand definition follows the uncertainty reduction. The user of the measuring system should be informed about it because he has practically no opportunity to check it himself. He should be informed too, about all uncertainty components in order to enable him a proper calculation of the uncertainty budget . The information about the error sources, error propagation, and error reduction applied to the system is rather useless for him. The designer of the measuring system has to focus his attention on both errors and uncertainties but the strict distinction between them is highly recommended.

References

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