

Wavelet Correlation for Biomedical Shape Evaluation

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Abstract. We propose a method for multiscale evaluation of periodicity of geometrical patterns based on wavelet correlation. This method allows to characterize not only the period but also the scale of periodic components. An application for rotational symmetry estimation of shape of biomedical objects is described.

Keywords: Wavelet Transform, Biomedical Shape, Neurotoxicity Testing

1. Introduction

Wavelet based autocorrelation seems to be a useful tool for periodicity detection and description. Traditional autocorrelation applied to every single-scale wavelet coefficients enables to detect not only the position of periodic phenomena but also its size. Application of this method to shape description of biomedical object gives information about its symmetry and forms an efficient tool for evaluation of hypothesis about morphology changes induced by neurotoxicity.

2. Subject and Methods

Symmetry detection for planar shapes represented by closed contours can be replaced by periodicity evaluation of contour in parametric form using autocorrelation. We propose an extension of standard autocorrelation for detection of multiple periodic components by means of continuous wavelet transform. This method allows to discriminate between local and global phenomena using additional parameter called scale (see Fig.1).

The term “wavelet autocorrelation” stands for standard autocorrelation applied on coefficients of continuous wavelet transform (CWT).

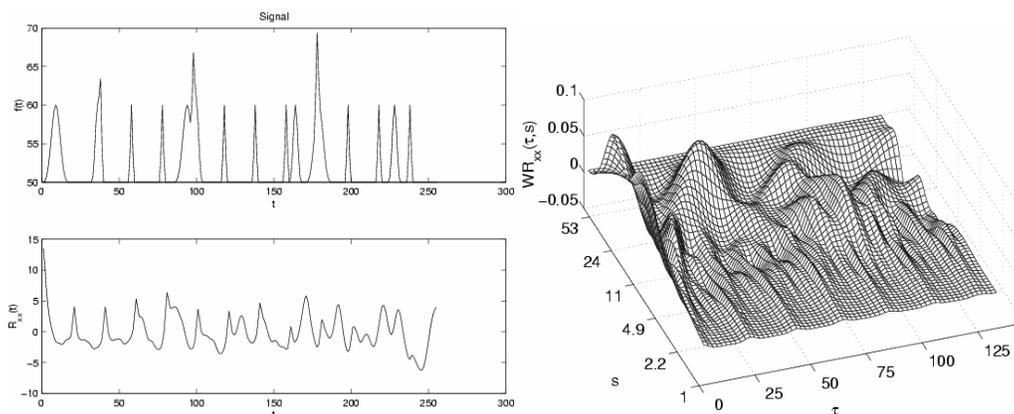


Fig. 1. Standard autocorrelation (left) and wavelet autocorrelation WR_{xx} of the periodic signal with periodical events at different scales (right)

The correlation is computed for every single scale of CWT, we look for periodicity in every band-limited portion of the evaluated signal. In continuous case the autocorrelation WR_{xx} can be described as:

$$WR_{xx}(\tau, s) = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} \overline{Wf(x, s)} Wf(x + \tau, s) dx \quad (1)$$

Where

Wf - coefficients of CWT of signal f ,

s - scale

τ - time shift

Unbiased estimation of wavelet autocorrelation for discrete signal with length of N can be expressed as:

$$WR_{xx}(\tau, s) = \frac{1}{N - \tau} \sum_{n=0}^{N-1} Wf(n, s) Wf((n + \tau), s) \quad (2)$$

Due to noise, the unbiased estimate gives poor performance for large τ (short signal segments in inner product (2)). We overcome this problem by using periodic autocorrelation; the parametric form of shape contour is periodic with the period equals to length of the parametrized curve. Problem with signal borders is also eliminated using periodic autocorrelation:

$$WR_{xx}(\tau, s) = \frac{1}{N} \sum_{n=0}^{N-1} Wf(n \bmod N, s) Wf((n + \tau) \bmod N, s) \quad (3)$$

The resulted wavelet autocorrelation WR_{xx} gives information about the periodicity in different scales (see Fig.1).

The general shape of contours is an ellipse; consequently main periodicity at large scales with period equals 2 is detected. For suppression of this unwanted effect we use a subtraction in wavelet domain. From wavelet coefficients of evaluated contour is subtracted a low-pass filtered contour represented the overall shape.

Last processing step comprises of maxima searching in WR_{xx} and their thresholding with respect to $WR_{xx}(0, s)$, which represents the energy contents of the signal at scale s . The result is a list of detected periodic events in the form of pairs (period, scale).

3. Results

The described method was tested on synthetic and real shapes. The scale discriminating property is illustrated on Fig. 2, Fig.3 and Fig.4. According to size of periodic phenomena on the contour, the maximum of wavelet autocorrelation shifts along the scale axis. Period is indicated by maximum in τ axis similarly as in standard autocorrelation.

We tested the proposed method also on contours of dorsal root ganglia (see Fig. 5) from experiments of neurotoxicity testing [1,2]. This method enables detection of hidden symmetries.

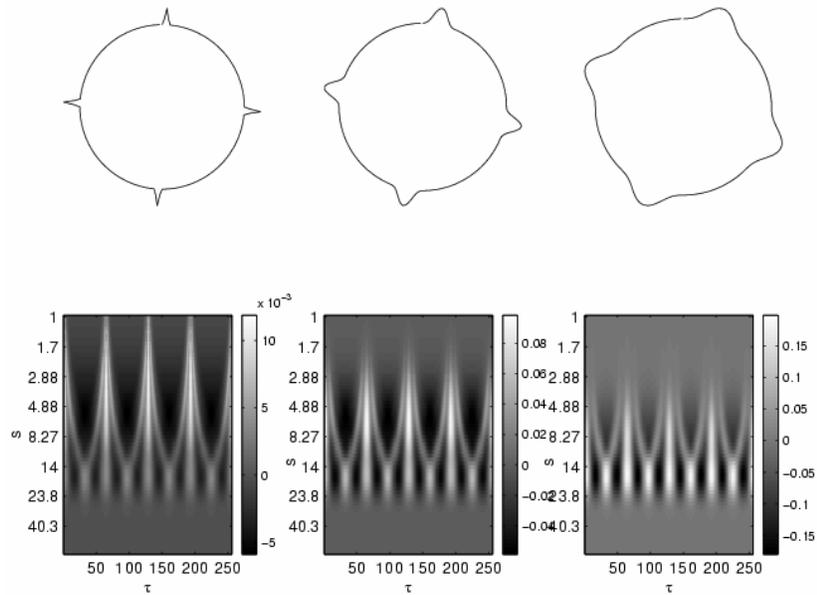


Fig. 2. Periodicity in shape at different scales (top), resultant wavelet autocorrelation (bottom)

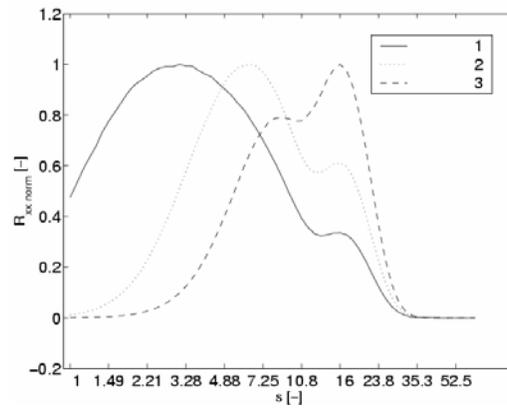


Fig. 3. Relation between scale and WRxx maximum for different shapes from Fig.2

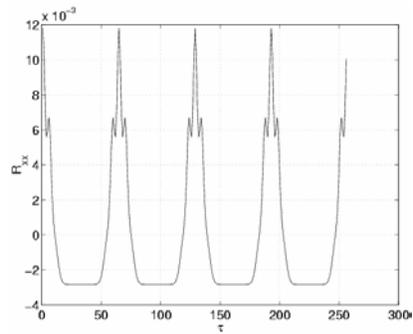


Fig. 4. Wavelet autocorrelation for fixed scale

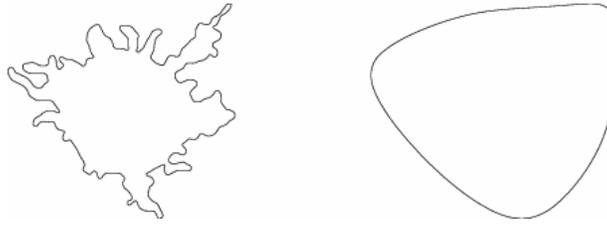


Fig. 5. The contour of biomedical shape (dorsal root ganglion) with detected rotational symmetry (left), low-pass filtered contour (right)

4. Conclusions

Autocorrelation applied on coefficients of continuous wavelet transform represents a useful tool for periodicity detection and description. This method enables to detect not only the position of periodic phenomena but also its size. Application of this method to shape description of biomedical object gives information about rotational symmetry.

Undoubtedly a certain extent of resemblance of this method to classical cepstral analysis exists. The main difference between both methods lies in fact that the proposed method does not use the logarithm of spectrum, as it is required by the definition of cepstral transform. In fact the calculation of logarithm of spectral function could cause problems due to the existence of segments of spectra having zero value.

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