ANALYSIS OF THE ACCURACY OF THE GEARS' MEASUREMENT WITH THE CMM

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Abstract. *Producers of the co-ordinate measuring machines (CMM) deliver many additional measuring programs for the special purposes. There are programs for the measurement of the gears. It is of great importance to carry out the accurate metrological analysis of the measuring process and the final error of the gears' measurement with the CMM. Because it appeared impossible to analyze existing algorithms, the author created his own version and carried out the precise metrological analysis of the accuracy of basic gears' deviations measurement. The proposed algorithms, software and metrological analysis of the CMM's gears measurement accuracy is a basis for the complex measurement of the gears. The complex measuring method includes modern measuring devices and single flank measurement simulation. The final results are cinematic and dynamic deviations of the gear, the complex parameters describing the whole gear.*
Keywords:. Coordinate Measuring Machines and Technic, Gear measurement.

1. INTRODUCTION

The worked out program for the involute cylindrical gears measurement enables to determine all characteristic values with their deviations describing the examined gear. The values are following: basic radius, profile angle, involute profile of any tooth, circumferential pitch (left or right), tooth thickness, tooth line, topography of the tooth. The examined deviations of those values are following: basic radius deviation, profile angle deviation, deviation of the involute profile of any tooth, deviation of the circumferential pitch (left or right), deviation of the collected circumferential pitch (left or right), tooth thickness deviation, tooth line deviation, deviation of the tooth side profile. Data required for measurement and calculation those values are following: number of the teeth, module, nominal profile angle. The detailed algorithm of the gears' measurement which has been applied to the software for the CMM was described in the article [5].

2. ACCURACY OF THE COORDINATE SYSTEM DETERMINATION

Coordinate system of the gear is defined in the measuring space of the CMM using the gear's orifice or roller, gear's plane and the recess between two teeth. The initial point of the coordinate system is determined by the circle measurement and the calculations according to the algorithm described in [1].

Coordinates of the center of circle determine following equations:

$$x_0 = \frac{\sum dy_i^2 \sum dx_i\left(dx_i^2 + dy_i^2\right) - \sum dx_i dy_i \sum dy_i\left(dx_i^2 + dy_i^2\right)}{2\sum dx_i^2 \sum dy_i^2 - 2\left(\sum dx_i dy_i\right)^2}$$

$$y_0 = \frac{\sum dx_i dy_i \sum dx_i\left(dx_i^2 + dy_i^2\right) - \sum dx_i^2 \sum dy_i\left(dx_i^2 + dy_i^2\right)}{2\sum dx_i^2 \sum dy_i^2 - 2\left(\sum dx_i dy_i\right)^2}$$

and then a radius is calculated as follows:

$$R = \sqrt{\sum\left(dx_i^2 + dy_i^2\right) - \left(x_0^2 - y_0^2\right)}$$

The dimensions (the diameter and the position) calculated on the basis of the measuring points deviate depending of the number and place of the points on the detail's surface. Numerous researches and

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experiments with drilled and milled details prove that a relatively stable result ensure more than 16 measuring points [2]. The results of the deviations in the orifices with diameter 30-100 mm are presented in the Table 1.

Table 1. Deviations dependent on the number of measuring points

Number of points	Deviations of the center μm	Deviations of the diameter μm
4	9.0	12.7
8	1.3	1.8
16	0.8	1.1
32	0.5	0.7
64	0.35	0.5
125	0.25	0.35
250	0.18	0.25
500	0.12	0.17
1000	0.09	0.13

The results prove that the calculation of the center and the radius require minimum 16 measuring points (recommended 32) in order to reach relatively stable results. For the following analysis were 16 chosen points which should ensure error of the center no more than 1 μm. The number of simulation have been carried out in order to test the accuracy of the determination of the circle center. The assumption was, that the circle (the orifice or the roll of the gear) had been manufactured exactly according to the project and required accuracy. The parameters for the analysis: error of the CMM U=3,5 +L/250, radiuses of the circles: 25 mm, 50 mm, 100 mm and 200 mm, the tolerance of the circle: 21 μm and 36 μm, geometrical tolerance: ovality and lobing as the most frequent error, the same weight, accidental distribution of the CMM's error and its sensor's error.

According to the analysis (based on the software STATISTICA) with mean square circle and normal distribution, it was proved:

- for **32 measuring points** the error of the calculation of the center of the circle is **1 +/- 1μm** (minimum error 0, **maximum 2μm**),
- for **16 measuring points** the error of the calculation of the center of the circle is **1.5 +/- 1.5μm** (minimum error 0, **maximum 3μm**).

The error of the calculation of the center of the circle will influence on the whole following measuring process, procedures and the accuracy of the setting movement, and the measuring movement (measuring points). In the further analysis 32 measuring points were distributed in uniform way on the circle, and the maximum error of the calculation of the center was considered +/- 2 μm. Determination of the recess between two teeth was carried out by means of measurement of several points in the reference diameter area on both sides of the recess, which was placed as accurately as possible in the *y*-axis of the CMM. The point of recess' symmetry was calculated according to the correction procedure applied when the thickness of the tooth and the pitch is being measured. The difference is that the chosen point should lay as close as possible to the reference diameter. Those two points are the base for the line 2D in the main plain (it should be defined on the gear's plane and placed the way, that the gear stayed in the negative values of the *z*-axis). The line becomes the main axis of the measured gear's coordinate system (fig. 1.).

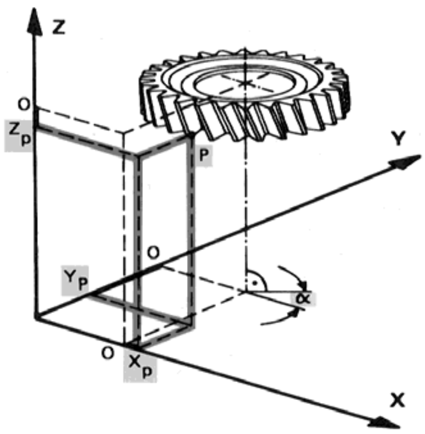

Fig. 1. A gear in the co-ordinate space of the CMM

3. MEASUREMENT OF THE TOOTH PROFILE

The software distinguish between the error of the profile caused by the geometrical error of the involute (fig. 2b) and caused by the diameter of the base circle. In case the involute was drawn correctly but with incorrect base circle, the software calculates the error of the base circle. The *correction I* was applied which corrected the measuring point (the coordinates of the center of the ball registered by the

CMM and transferred to computer and program) into the real point of the contact between the sensor's ball and the measured tooth. It is the correction of the diameter of the ball (determined during the calibration) along the vector orthodox to the measured surface. Methods, strategy and the procedures of the measurement of the base circle, involute profile and their errors are described in [5], [6].

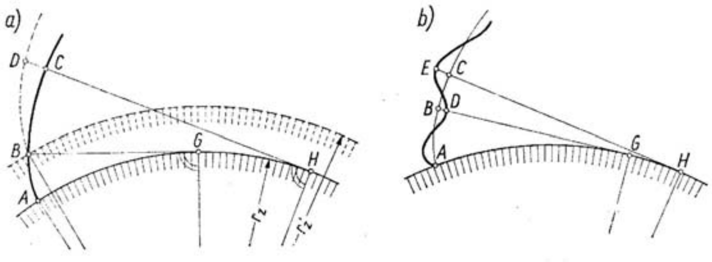

Fig. 2. The error of the tooth profile: a) involute based on the incorrect base circle (bigger diameter), b) geometrical error of the involute (waviness)

The simulation carried out for numerous gears with different modules and teeth numbers confirmed the correctness of the *correction I*. The differences of the expected and calculated deviations of the base circle and involute were analyzed. The simulation contained each error independently and in superposition with others. The differences (errors of the measurement of deviations) were compared to the DIN 3926. The analysis proved that the errors of the deviations' determination were below 10% of the acceptable values for the accuracy class of the measured gear, which is required by the metrological postulate. Deviations of the base circle and involute were compared to DIN 3926.

4. MEASUREMENT OF THE GEOMETRICAL VALUES

According to the definition the measurement of the tooth thickness and the pitch should take place on the reference circle. The measuring process of the CMM disables collection of the points laying both on the reference circle and on the side of the tooth. This is because all geometrical values of the measured gear have their deviations which are unable to be calculated before the measurement.

The software contains especial algorithm called *correction II*. Its task is to move the measured point (after *correction I* placed close to the reference circle) to the reference circle. Number of the calculations was carried out before the appropriate methods and algorithms were worked out. Both simulations and metrological analysis confirmed the correctness of the introduction of *correction II*.

Measurement of the tooth line contains the collection of the measuring points along the tooth width, with the same procedures and algorithms, changing the measuring plane in the *z*-axis, the errors are similar.

The whipping of the gear is defined in the program as the difference between the position of the center of the circle containing circles inscribed into recesses, and the center of the coordinate system. The vector of the direction is added (fig. 3). Circles inscribed into recesses between the teeth are calculated with measuring points C collected for the pitch and tooth thickness measurement. Those two points enable to create two circles with certain diameter. This diameter means the diameter of the measuring roll usually used for the measurement tooth thickness. For the measurement of whipping are applied the circles with their centers most distant from the center of the coordinate system. The final error depends on errors of the measuring points C (value not important) and errors of calculation the circle based

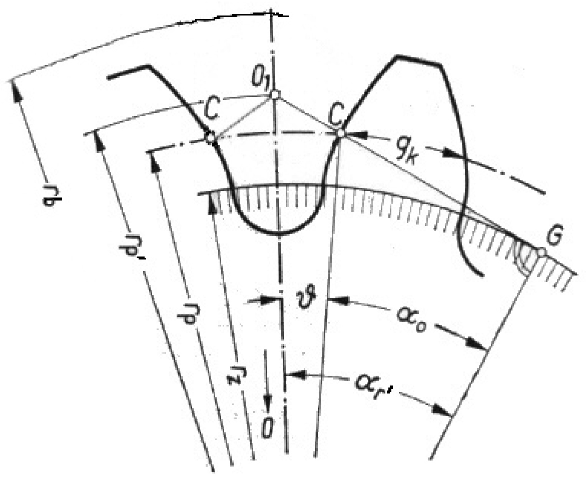

Fig. 3. Collecting of the measuring points for the measurement of whipping

on the centers of the circles inscribed into recesses. The result contains coordinates of the center of the circle, its distance from the coordinate system center and the angle to the *y*-axis.

The measurement of the topography of the tooth side is the superposition of the algorithms and procedures of the measurement of involute and the tooth line.

5. CONCLUSIONS

The measuring algorithms and calculation procedures have been thoroughly analyzed. For the gears with modules from 1 to 12, tooth number from 17 to 257, profile angle α_o=20º and correction coefficient x=0 the simulation have been carried out. Under simulation were theoretical – geometrically ideal gears, and the gears with known deviations as well. During the simulation the correctness of the identification of those deviations was checked, and their errors determined. This all proved that the algorithms were worked out properly, and in particular that the *correction I and II* were needed and calculated properly.

The greatest errors during the defining the coordinate system appear when the base circle is being measured. Maximum error according to the simulations is below 2 μm. The error of the coordinate system center projects directly on the measuring points and the start points from which the sensor goes to the measuring points.

Correction I – moving the coordinates of the measuring ball center into the point where it touches the detail – was chosen and tested by simulation so the error of the touch point identification is minimum. The correctness of the calculation the deviations of the base circle and involute was proven. Deviations were below the assumption for the certain class, their errors were about 10% of the acceptable value of the deviation, which is metrologically correct.

Correction II – moving of the measuring point placed on the tooth surface after *correction I* into the place where the base circle intersects with the real tooth surface. The task of the operation is to reach the measuring points for the calculation of the pitch (left and right), tooth thickness and the whipping, as well the deviations of those values from their nominal. The function of the 3rd degree describes the relation of the correction parameters to the number of teeth. The *correction I* minimized the error of the measuring of particular deviations, its value is below 0.5 μm. Some influence on the results has the simplification which states, that the errors are adding not align the base circle, but align the normal to the tooth surface in the point C. The simplification is, however, acceptable, because the analysis proved very small values of the influence on the final result. The only significant influence on the measurement of the pitch had the accuracy of the coordinate system center determination. According the definition, this circle is used for the measurement of the pitch, tooth thickness, and the points for calculation of the whipping.

The analysis proved that the mentioned error (2 μm for 32 measuring points) enables the correct measurement of the gears of 5th class (3 μm for 16 measuring points – 6th class), because its value is below 10% of maximal acceptable deviation.

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