The Influence of Covariances on the Uncertainty of Temperature Measurement by Resistance Thermometer

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Abstract. *Results of the measurements shall be presented with the corresponding uncertainties. In accordance with the international accepted documents the covariances should be taken into account if possible. In this paper the standard platinum resistance thermometer (SPRT) calibrated at the defining fixed points (DFPs) is considered. This thermometer is used for temperature measurement in the range from 0 °C up to 660 °C. Value of temperature is determined in accordance with the International temperature scale 1990 (ITS – 90) by using of inverse reference function. The law of propagation of uncertainties for the interpolation of temperature between the DFPs is applied. In the conclusion is the discussion concerning the influence of covariances caused by the triple point of water (TPW) to the result of measurement, specifically to the evaluation of the uncertainties.*

Keywords: Temperature measurement, propagation of uncertainties, covariances, SPRT calibration

1. Introduction

In most cases especially concerning the calibration practice it is necessary to measure the temperature at high level of accuracy. It means not only to determine precise value of temperature but to determine the corresponding uncertainty as well. Here it is reasonable to think of the influence of the covariances on the evaluation of the uncertainties. In the case of calibration of the SPRT at the DFPs it is necessary to use the inverse deviation function for evaluation of the measured temperature. Evaluation of uncertainties depends on the way of presentation of the calibration results in the calibration certificate. The way of introduction of covariances to the evaluation of uncertainties can be various. It depends on the level of information concerning not only to the measuring process of temperature but the process of SPRT calibration as well. In this paper are particularly considered the covariances caused by the measurement of SPRT resistance at the triple point of water, e. g. there are taken into account the measurements at the same TPW during the calibration.

2. Measurement of temperature by calibrated SPRT

Standard platinum resistance thermometer calibrated at the defining fixed points is used for the measurements of temperature on the highest level of accuracy. This procedure covers the case of temperature measurement in the temperature range from 0 °C up to 660 °C. Attention was aimed at the evaluation of uncertainty and to the discussion of influence of covariances to the resulting uncertainty. There was assumed that all data from calibration could be accessible for the evaluation of uncertainty. The value of measured temperature for the mentioned temperature range is determined by means of the inverse reference function in accordance with the document ITS-90 [1]

$$T - 273{,}15 = D_0 + \sum_{i=1}^{9} D_i \left[\frac{W_{\mathrm{rT}} - 2{,}64}{1{,}64} \right]^i \qquad (1)$$

The values of the coefficients D_i are given in [1] and the values W_{rT} are determined from the deviation function for the corresponding temperature range.

$$W_{\mathrm{rT}} = W - \Delta W \tag{2}$$

where

ΔW is the deviation function from the calibration of the SPRT,

W is ratio of the resistances defined as follows

$$W = \frac{R}{R_{\mathrm{TPW}}} \tag{3}$$

where

R is the resistance of the SPRT corresponding to the temperature T,

R_{TPW} is the resistance of the SPRT corresponding to the temperature of TPW T_{TPW}.

The value of measured temperature T is determined by putting of $W_{\mathrm{r}T}$ into the equation (1). For the evaluation of temperature the matrix form is used [3]. It means that the coefficients (a, b, c [1] see eq. (6)) of the deviation function are calculated by means of the matrix form and as a result is the matrix of coefficients and the matrix of uncertainties of the coefficients and the covariances between the coefficients [3, 4].

The uncertainty of the measured temperature is determined by the following equation:

$$u(T) = A_{\mathrm{r}T}\, u(W_{\mathrm{r}T}) \tag{4}$$

where

$u(W_{\mathrm{rT}})$ is the combined standard uncertainty of the resistance ratio $W_{\mathrm{r}T}$,

$A_{\mathrm{r}T}$ is the sensitivity coefficient. This coefficient is determined by derivation of the inverse reference function (1).

$$A_{\mathrm{r}T} = \frac{\partial f(T)}{\partial W_{\mathrm{r}T}} = \sum_{i=1}^{9} \frac{i \cdot D_i}{1{,}64} \left[\frac{W_{\mathrm{r}T} - 2{,}64}{1{,}64} \right]^{i-1} \tag{5}$$

On the basis of equation for the deviation function [1]:

$$W_T - W_{\mathrm{r}T} = \Delta W_T = a[W_T - 1] + b[W_T - 1]^2 + c[W_T - 1]^3 \tag{6}$$

is determined the uncertainty $u(W_{\mathrm{r}T})$:

$$u^2(W_{\mathrm{r}T}) = u^2(W_T) + u^2(\Delta W_T) - 2u(W_T, \Delta W_T) \tag{7}$$

where

$u(W_T)$ is combined standard uncertainty of the resistance ratio W_{T} based on the measurement of temperature T by the calibrated thermometer and the resistance value measured at the TPW taken from the calibration certificate

$u(\Delta W_T)$ is the uncertainty of the resistance ratio deviation,

$u(W_T, \Delta W_T)$ is the covariance between the resistance ratio W_T and determination of the resistance ratio deviation ΔW_T based on using the same values of resistance R_{TPW} determined by the calibration (it could be arithmetic mean of the SPRT resistances measured at the TPW after the measurements at the DFPs (Al, Zn, Sn), e. g. R_{TPWAl}, R_{TPWZn}, R_{TPWSn} which should be taken from calibration or it could be the last measured value of R_{TPW} during the calibration). In this case the change of R_{TPW} value is not considered from the time of calibration.

Remark:
Covariance $u(W_T, \Delta W_T)$ is omitted in the case of possibility to measure the value of R_{TPW} during the measurement of temperature T. In this case is assumed the independent measurement of R_{TPW} at TPW that is not identical with the TPW of calibration.

In the event of consideration of covariance between the resistance ratio W_T and determination of the resistance ratio deviation ΔW_T the following equation is derived:

$$u^2(W_T) = \frac{u^2(R_T) + W_T^{\,2}\; u^2(R_{\text{TPW}T}) - 2W_T\, u(R_T, R_{\text{TBV}T})}{R_{\text{TPW}T}^{\;2}} \tag{8}$$

When the values of resistance R_{TPW} are coming from the different sources (e.g. there is not considered correlation between the measured of resistance of SPRT at T and at TPW during the SPRT calibration) then the equation (8) can be simplified

$$u^2(W_T) = \frac{R_{\text{TPW}}^2 u^2(R_T) + R_T^2\; u^2(R_{\text{TPW}})}{R_{\text{TPW}}^4} \tag{9}$$

There is possible to derive the following expression for the evaluation of uncertainty for measured temperature T:

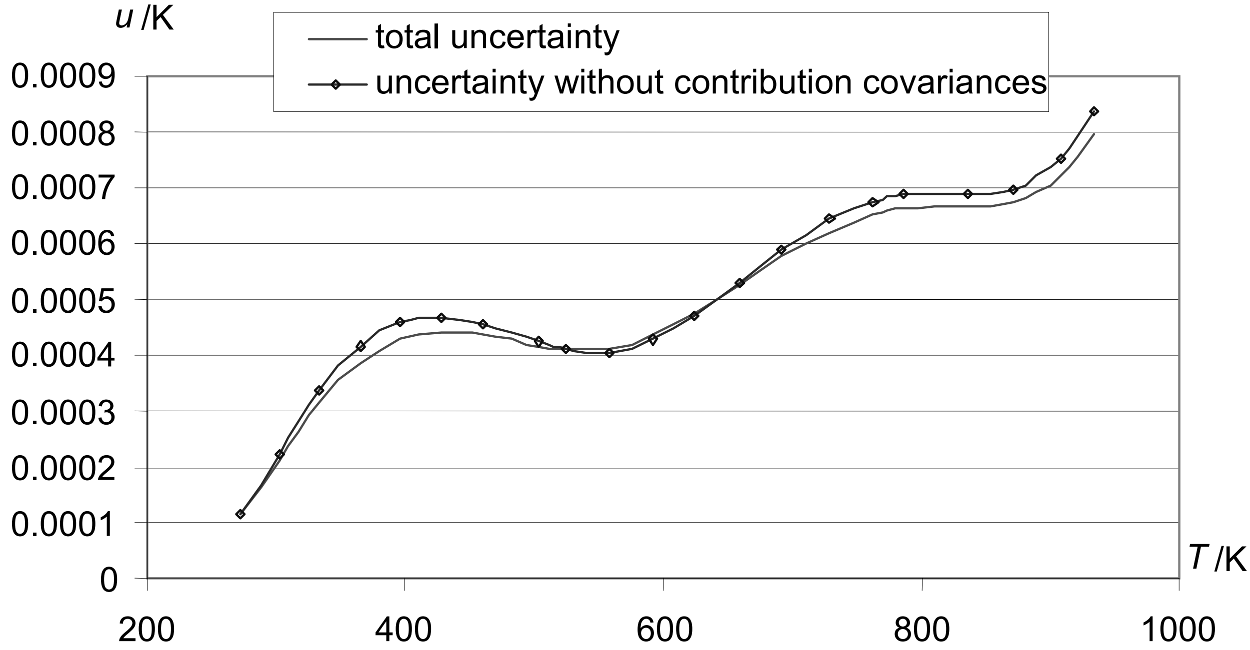
$$u^2(T) = A_{\text{r}}^2\, \frac{1}{R_{\text{TPW}}^2} u^2(R) + u_{\text{cal}}^2(T) = u_{\text{meas}}^2(T) + u_{\text{cal}}^2(T) \tag{10}$$

where

$u_{meas}(T)$	is the uncertainty related to the measurement of temperature T by means of calibrated thermometer,
$u_{cal}(T)$	is the uncertainty coming from the calibration

It means that the resulting uncertainty includes the component of the uncertainty coming from the calibration and the component related to the measurement of resistance R related to the measured temperature T.

The following figure presents the situation when the uncertainty from the calibration is considered with the covariances (total uncertainty) and without them. There are the differences between the uncertainties for actual measured data at the defining fixed points. The curves relate to the propagation of the uncertainties between the defining fixed points.



3. Discussion

The highest level of accuracy of temperature measurement with the SPRT calibrated at the DFPs does not very sensitive to the influence of covariances. Differences between the uncertainties with covariances and without them are negligible (approximately up to 10 % of the uncertainty value). Uncertainties of resistance measurements has growing tendency with increasing of temperature. Between the fixed points there was applied the law of propagation of uncertainties. There is strong influence of mathematical definition of the temperature scale.

Covariances between the R_{TPW} taken from the calibration certificate and deviation ΔW_T also from the calibration generally cause decreasing of uncertainty values. But for the conditions taken in this example this is relatively small (negligible) and it does not need to take attention to it. Besides this the value of R_{TPW} is changing in time and it could not be taken always from the calibration certificate. This is further reason not to consider the covariance induced by the value of R_{TPW}.

References

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