On calibration uncertainty estimation in ratio-metric measurements

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Abstract. *The formula for uncertainty estimation of the result of a ratio-metric measurement is derived. The result uncertainty decisively depends on the knowledge of the statistical correlation between the measurements by two measuring chains, assumingly of same kind and same metrological quality. A calibration procedure is stipulated and a practical example is treated, based on some recently published data. The same approach could be used in other ratio-metric measurements and such measurement calibration procedure as well.*

Keywords: ratio-metric measurement, measurement uncertainty, calibration uncertainty

1. Introduction

Measurement uncertainty analysis and calculation (older term measurement error analysis) is in general governed by the "Guide to the Expression of Uncertainty in Measurement "GUM" of 1995 [1] and subsequent standards. This topic is explained in numerous textbooks and papers on Measurement Science, e.g. [2–4].

Some measurement methods are based on determination of the ratio of two measurements of the same kind, i.e. measuring some sort of energy transmissibility through a dynamical system, for example input and output voltages, two acoustic pressures, two vibratory accelerations, etc. Hence for this measurement two measuring chains of the same kind and of similar metrological properties are used, consisting of sensor of a specific physical quantity, instrumentation and the evaluating instrument or computer system. The result of the measurement is a pair P_i of measured values x_i, y_i as particular realization of random set ξ, η of all possible values. The resultant ratio $z_i = x_i/y_i$ is computed and taken into account as the single measurement result, i.e. realization of the random variable $\zeta = \xi/\eta$ at the point P_i, with absolute uncertainty u_ζ.

The measurement system is usually calibrated beforehand – the two sensors mentioned are subjected to a stable and reproducible etalon signal, generated by a calibrating device and the pair of so obtained values x_c, y_c is subjected to the division procedure. The required result, due to the nature of the calibration process has to be unity; any deviation from this value is due to the calibration uncertainty. A major deviation from unity would indicate malfunction. This calibration procedure can be independently repeated couple of times under controlled conditions and in this way the absolute uncertainty of variable $\xi - u_\xi$ and of value $\eta - u_\eta$ respectively can be obtained. The calibration procedure is schematically depicted in Fig. 1.

What concerns the uncertainty influence (both A-type uncertainty due to statistical variance of repeated measurements and B-type uncertainty due to influence of the ambient) on each of the two measuring chains of the same kind the influence is not independent of each other. So, for example, if the ambient temperature increases the value measured by each of the measuring chains would be changed in the same manner, albeit with slightly different magnitude. Hence the ratio would be influenced by this change. Hence, it can be inferred, that the variables ξ, η are not statistically independent, but there is a certain degree of correlation, which has to be taken into further account [4, 5].

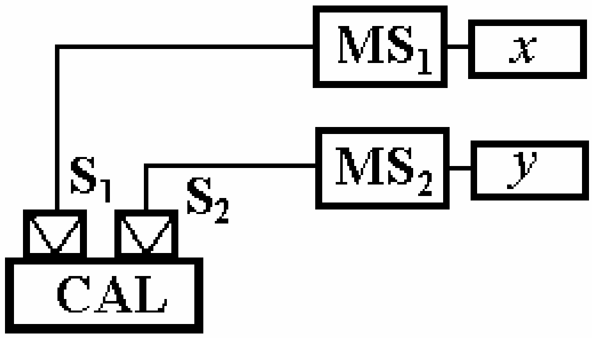

Fig. 1. Ratiometric calibration procedure (S_1, S_2 - sensors; MS_1, MS_2 – measuring systems; x, y – readouts)

2. Uncertainty analysis

The absolute uncertainty analysis is based on the standard approach – the ratio ζ of the two measurements ξ, η is expanded into a Taylor series around the point P_i and only the linear terms are further considered:

$$\zeta \approx \left(\frac{\xi}{\eta}\right)_{P_i} + \left(\frac{\partial(\xi/\eta)}{\partial\xi}\right)_{P_i}(x-\xi) + \left(\frac{\partial(\xi/\eta)}{\partial\eta}\right)_{P_i}(y-\eta). \tag{1}$$

Then the sum of squares addition is performed, as given by the error propagation law [1–3, 5]:

$$u_\zeta^2 = \left(\frac{\partial(\xi/\eta)}{\partial\xi}\right)_{P_i}^2 u_\xi^2 + \left(\frac{\partial(\xi/\eta)}{\partial\eta}\right)_{P_i}^2 u_\eta^2 + 2\left(\frac{\partial(\xi/\eta)}{\partial\xi}\right)_{P_i}\left(\frac{\partial(\xi/\eta)}{\partial\eta}\right)_{P_i} \mathrm{cov}(\xi,\eta)\, u_\xi u_\eta, \tag{2}$$

where the partial derivation values (the so called sensitivity coefficients) are taken for the particular paired values at point P_i and variable cov(ξ,η) denotes the statistical correlation between the variables ξ and η. When the respective partial derivations are evaluated, then:

$$u_\zeta^2 = \left(\frac{1}{y}\right)_{P_i}^2 u_\xi^2 + \left(\frac{x}{y^2}\right)_{P_i}^2 u_\eta^2 - 2\left(\frac{1}{y}\right)_{P_i}\left(\frac{x}{y^2}\right)_{P_i} \mathrm{cov}(\xi,\eta). \tag{3}$$

This can be expressed also as:

$$u_\zeta^2 = \left(\frac{x}{y}\right)^2\left(\frac{u_\xi}{x}\right)^2 + \left(\frac{x}{y}\right)^2\left(\frac{u_\eta}{y}\right)^2 - 2\left(\frac{x}{y}\right)^2\left(\frac{u_\xi}{x}\right)\left(\frac{u_\zeta}{y}\right)\rho_{\xi,\eta}, \tag{4}$$

where $\rho_{\xi,\eta}$ is the correlation coefficient between variables ξ and η.

If relative uncertainties δ_ζ, δ_ξ, δ_η are introduced, then the expression (4) can be rewritten as:

$$\delta_\zeta^2 = \left(\frac{u_\zeta}{z}\right)^2 = \left(\frac{u_\xi}{x}\right)^2 + \left(\frac{u_\eta}{y}\right)^2 - 2\left(\frac{u_\xi}{x}\right)\left(\frac{u_\eta}{y}\right)\rho_{\xi,\eta}, \tag{5a}$$

i.e.:

$$\delta_\xi^2 = \delta_\xi^2 + \delta_\eta^2 - 2\delta_\xi\delta_\eta\rho_{\xi,\eta}, \tag{5b}$$

wherefrom the definitions of respective relative uncertainties δ_ζ, δ_ξ, δ_η are obvious.

Three distinctive cases can be resolved:

i. No statistical correlation between the two measuring chains, i.e. $\rho_{x,y} = 0$:

$$\delta_\zeta = \sqrt{\delta_\xi^2 + \delta_\eta^2}\ , \tag{6a}$$

which is essentially the standard relative uncertainty propagation formula of a result obtained as a ratio of two independent measurements. If, as assumed, both measurement chains are of the same kind and their measurement uncertainty (obtained by an independent approach, or stated by the manufacturer in the documentation) is δ_m, then $\delta_\zeta = \sqrt{2}\delta_m$.

ii. If full statistical correlation between the two measuring chains is assumed, then $\rho_{x,y} = 1$, and from formula (5b) follows:

$$\delta_\zeta = \sqrt{\delta_\xi^2 + \delta_\eta^2 - 2\delta_\xi\delta_\eta} = \left|\delta_\xi - \delta_\eta\right|. \tag{6b}$$

If, as assumed, $\delta_\xi = \delta_\eta = \delta_m$, then from formula (6b) follows $\delta_\zeta = 0$ and so also $u_\zeta = 0$. This result is true only if the validity of the $\rho_{x,y} = 1$ assumption is reasonably justified.

iii. A specific case corresponds to $\rho_{x,y} = 0.5$ and $\delta_\xi = \delta_\eta = \delta_m$. Then from (4b) follows: $\delta_\zeta = \delta_m$.

To conclude - ratiometric measurement with measurement chains of the same kind, having same metrological properties (i.e. same relative measurement uncertainty) assuming total uncertainty correlation, would be free of any uncertainty. If the value of $\rho = 0.5$ would be stipulated then $\delta_\xi = \delta_\eta = \delta_\zeta = \delta_m$. The exact knowledge of the correlation coefficient is hence crucial for proper uncertainty assessment.

The evaluation of the correlation coefficient value $\rho_{x,y}$ can be furnished experimentally - by repeating independently the calibration procedure N-times, recording the set of measured values x_i, y_i, $i = 1, 2, \ldots, N$ and subjecting these values to analysis, as described e.g. in [2, 5].

3. Calibration uncertainty assessment example

Introduction

The following example is pertinent to evaluation of vibration influence on the human body, seated in an upright position in suspended and cushioned seat, used for various mobile working means, e.g. industrial trucks, agricultural and forest tractors, earth moving machinery, lorries, busses, on-road trucks, railway engines and carriages, etc. The evaluation procedure is specified in the general standards ISO 30326-1 and ISO 30326-2, describing the test codes to be followed. Hitherto none of these standards deals directly with uncertainty assessment. Only recently some large-scale inter-laboratory comparisons were undertaken, resulting in a rather wide spread of results [6]. Also a Standard on instrumentation used (so called "Human vibrometers") – ISO 8041:2005 is in preparation, in which some mandatory constraints on the calibrator uncertainty are given. Specifically the Annex A quotes a part of the "GUM" [1], to be followed in estimating calibrator uncertainty. In further some, hitherto available, data [6] will be used for uncertainty assessment for this class of applications.

Calibration procedure

The calibration is furnished by mounting the two, accelerometers of the same kind and similar metrological properties onto the calibrator device and subjecting to a calibration run, as depicted in Fig. 1. Due to this nature of calibration it can be assumed, that the measurements are correlated and have the same tendency. Alternatively, it could be assumed, that the measurements are not correlated at all as the worst case. The calibrator uncertainty influences both accelerometers in the same way and is not correlated with either of these. Hence:

i. For totally un-correlated measurements $\rho_{x,y} = 0$ and measurement uncertainty is governed by formula (6a). The calibrator uncertainty has to be excluded, as contributing to both in the same manner. If measurement chain relative uncertainties identity is assumed then $\delta_{zp} = \sqrt{2}\delta_m$. If the value of $\delta_m = 4$ % as given in [6] is used for the calibration procedure follows: $\delta_C = \sqrt{2} \cdot 0.04$, i.e.: $\underline{\delta_C \approx 5.66\ \%}$.

ii. For fully correlated measurements which posses the same tendency, $\rho_{x,y} = 1$; hence the uncertainties of two identical measuring chains would cancel out and: $\underline{\delta_C = 0.0\ \%}$.

iii. The real measuring system uncertainty would be between the above bounds. To be more specific it could be reasonably assumed that the correlation between the two measuring chains would be somewhere between 0.5 and 0.8 [7]. Then the calibration relative uncertainty estimation would be between 2.5 % and 4 %.

4. Conclussion

Formula for relative uncertainty assessment of the result of a ratio-metric measurement has been derived. The result uncertainty decisively depends on the knowledge of the statistical correlation between the measurements by two measuring chains, assumingly of the same kind and of the same metrological properties. A calibration procedure has been stipulated and a method on estimating the statistical correlation in a particular case was hinted at. A practically important case was treated using some recently published data. The same approach could be used in other ratio-metric measurement calibration process as well.

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