Simple Mathematical Model for Recognition and Visualization of Atrial Enlargement Using the DECARTO Technique

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Abstract. *For diagnostic recognition of the atrial enlargement with the use of a corrected orthogonal lead system, a simple mathematical model is proposed to provide visualization of the electrophysiological and anatomical characteristics of the heart atria. The atrial depolarization process is depicted in the form of a map on spherical surface enclosing the atria, with indication of the atrial wall size parameter on the same map. To obtain quantitative estimation for enlargement of each atrium, efficient integral parameters are used. This method incorporated into the DECARTO (dipole electrocardiotopography) technique is illustrated by cases with normal atria, biatrial enlargement, and one-sided atrial enlargement.*

Keywords: Dipole Electrocardiotopography, Atrial Enlargement, ECG Modeling

1. Introduction

In the previously proposed technique of dipole electrocardiotopography (DECARTO) [1,2], pictorial representation of the ventricular excitation process is provided on the basis of the electric heart vector components measured with a corrected orthogonal lead system. Major functional characteristics of the electrophysiological process in the ventricles are represented in a map-like form projected onto an image sphere enclosing the heart. To visualize in a similar way properties and estimate characteristics of the atria during the P wave of electrocardiogram, a simplified mathematical model is considered and used to construct atrial decartograms facilitating recognition of the atrial enlargement.

2. Subject and Methods

The electrogenic region of atrial myocardium is represented by a mathematical model in the form of a hollow shell. At each time instant t of the atrial depolarization, the shell is approximated by a sphere with external and internal radii, R_{Ep} and R_{En}, corresponding to the epicardium and endocardium radii in the region of depolarization front. In this simplified model, it is assumed that during the main part of the depolarization period (the P wave of electrocardiogram) the depolarization front is spreading tangentially with respect to the atrial wall surface and coincides

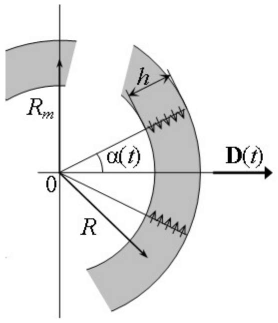

Fig. 1. Schematic diametral section of the model with the depolarization wave region at the time instant *t*. **D**(*t*), heart vector

with a ring-shaped uniform electric double layer (Fig. 1).

Then it can be shown that the magnitude of the heart vector (proportional to the total dipole moment of the depolarization front) at time instant *t* has the magnitude

$$D(t) = k\pi \sin^2 \alpha \left[R_{\mathrm{Ep}}^2(\alpha) - R_{\mathrm{En}}^2(\alpha)\right],$$

(1)

where *k* is a constant characterizing the bioelectric generator intensity relative to the excitation front area, and $\alpha = \pi t / T_\mathrm{p}$, where T_p is the total duration of the atrial depolarization and *t* is the time measured from the beginning of the P wave. Also,

$$D(t) = 2k\pi R(\alpha) h(\alpha) \sin^2 \alpha, \qquad (2)$$

where *R* is the mean radius and *h* is the thickness of the model wall. *R* and *h* depend on the angle *α* between the spherical model radius coinciding in direction with the heart vector and the radius defining position of the depolarization front. At the time instant t_m, when the depolarization front lies in the diametral plane of the model (for $\alpha = \pi/2$),

$$D(t_m) = 2k\pi R_m h_m \ ,$$

(3)

where R_m and h_m are the mean radius and wall thickness at the regions of wall being crossed by the depolarization front at the instant t_m. The corresponding ratio of the mean radii is expressed as

$$R_1(t) = \frac{R(\alpha)}{R_m} = \frac{1}{h_1(\alpha)\sin^2 \alpha} \frac{D(t)}{D(t_m)},$$

(4)

where the relative thickness of the model wall $h_1(\alpha) = h(\alpha)/h_m$ is approximated empirically on the basis of additional information.

Further simplification of the model is gained by the supposition that the depolarization starts in the region of the model pole corresponding to the negative direction of the heart vector at the time instant t_m and terminates in the opposite region. The boundaries of the ring-shaped depolarization fronts lie in the planes perpendicular to the mean heart vector.

On the image sphere which center and spatial orientation correspond to the assumed spherical model, the isochrones are placed in accordance with the positions of the depolarization front at equal time intervals during the P period. The instant t_m is chosen as the midpoint of this period.

For each isochrone, the relative mean radius of the model wall is calculated by Eq. (4), and in the map-like data representation this parameter is clearly indicated by respective graphical means.

To obtain an overall description of symmetric and asymmetric atrial enlargements, integral parameters of the atrial size are used. These parameters are calculated for the first half and the second half of the P interval, when mainly depolarization of the right atrium and left atrium takes place, respectively, as follows:

$$S_R = \int_0^{T_P/2} R_1(t)D(t_m)dt, \qquad S_L = \int_{T_P/2}^{T} R_1(t)D(t_m)dt.$$

(5)

Comparison of these values with corresponding values for normal cases enables relative estimation of enlargement of each atrium.

3. Results and Discussion

The presented method was experimentally tested with the use of the Frank lead system on 49 males and females aged between 16 and 72 years, including a group of 15 normal persons and groups with reliably verified right atrial, left atrial, and biatrial enlargement (10, 9, and 15 persons respectively). For all subjects, the atrial decartograms with distribution of the atrial size characteristic R_1 and values of integral parameters S_R and S_L were obtained.

Typical atrial decartograms for a normal case and cases with atrial enlargement are illustrated in Fig. 2. The maximum heart vector magnitude $D_{\max}$ is indicated for each decartogram. This value, along with the density of isochrones, characterizing the total duration of the atrial depolarization, is useful for diagnostic interpretation of the data. It is seen that the asymmetric enlargement of the atria is characterized by extremal values of R_1 in the corresponding region.

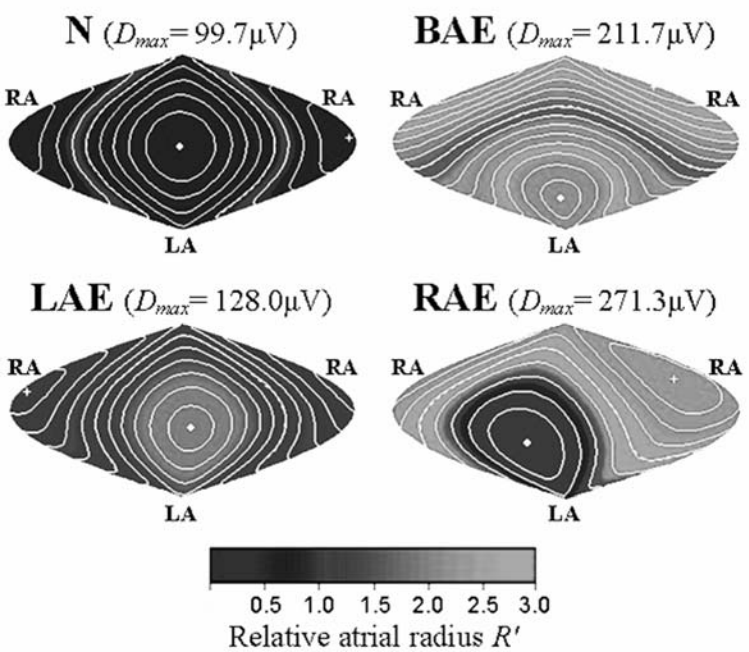

Fig. 2. Atrial decartograms for typical cases of norm (**N**), biatrial enlargement (**BAE**), left atrial enlargement (**LAE**), and right atrial enlargement (**RAE**). The image sphere enclosing the atrial model is cut along the right meridian, unrolled, and projected onto the plane in the equiareal format with indicated distribution of the relative atrial radius R_1. The isochrones (light lines) are drawn at the interval 10 ms. +, o, regions of beginning and termination of the atrial depolarization. **LA**, **RA**, regions of the left and right atrium, respectively.

Heuristic visual examination of the atrial decartograms for all cases studied shows that the increase of the atrial myocardium volume results in explicitly seen changes of the distribution of radial size parameter R_1, as compared to the normal decartograms, while these variations essentially depend on the particular enlarged region of the atria.

The dependence between the integral parameters relative to their values averaged over the normal group is presented in Fig. 3. It shows that the diagnostic groups are rather well distinguishable.

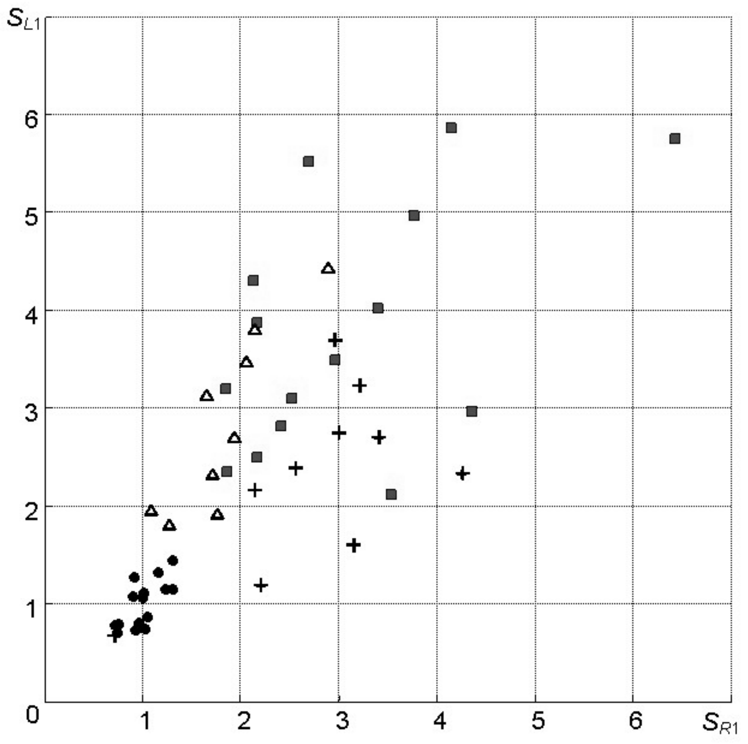

Fig. 3. Dependence between the integral atrial parameters S_{L1} and S_{R1} relative to the averaged normal values for the cases of norm (●), biatrial enlargement (■), left atrial enlargement (Δ) and right atrial enlargement (+).

Thus, the proposed visual representation and quantitative estimation of the atrial size facilitate accurate identification of the heart state at atrial overloads of various types.

Acknowledgement

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